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A spiking neuron model and its pulse-coupled network of a self-organizing map

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Abstract—In this paper, we present a resonate-and-fire type spiking neuron model and parameter update rule to memorize the input frequency as a resonance frequency. We also present a pulse-coupled network of the spiking neuron models and show its SOM function by numerical simulation.

1. Introduction

Many spiking neuron models have been presented and investigated [1, 2, 3, 4, 7, 8, 9]. For example, resonate-and-fire models have been used to investigate spike-based information coding and processing functions and implemented on electric circuits [2, 4, 7, 8, 9]. Also, resonate-and-fire models have been used to construct pulse-coupled neural networks (PCNNs) whose application potential include image processing based on synchronization phenomena [5, 6, 10]. Also, a self-organizing map (SOM) using digital phase-locked loops (DPLLs), which can be regarded as a resonate-and-fire models, was presented [11, 12].

In this paper, first, we propose a subthreshold oscillating neuron (SO) model that can be regarded as a piecewise linearized version of simple Izhikevich model or a generalized version of Mitsubori-Saito (MS) model. Second, we present a parameter update rule based on synchronization phenomena. Third, we prototype a self-organizing network of spiking neurons (SSN) using the SO models and show its simulation result. We conclude that the SSN has a basic SOM function from the simulation result. We emphasize that typical implementations of an SOM need a minimum value detector for implementation of the winner-take-all (WTA) process but the SSN doesn't because of replacing the WTA process by competitive process. We also emphasize that the SSN memorizes input pulse train intervals to a natural frequency of the SO model without a phase detector unlike [12].

2. A spiking neuron model

In this section, we introduce a new spiking neuron. It is a nonlinear subthreshold oscillating neuron model which is easily implemented by an electronic circuit. We use the model as a network element in the next section. Let $t \in \mathbb{R}_+ = \{t | t \in \mathbb{R}, t \geq 0\}$ be a continuous time. Then we define a periodic input $u(t)$ whose period is $T \in \mathbb{R}_+$ i.e.,

$$u(t) := \sum_{n=1}^{\infty} \delta(t - nT) \tag{1}$$

where δ is the Dirac's delta function. We use two state variables v, $r \in \mathbb{R}$ and one weight parameter $w \in \mathbb{R}$. v corresponds to a membrane potential and r corresponds to a recovery variable of Izhikevich's simple model (2003) respectivety, and w corresponds to a synaptic weight. For simplicity, we use a state vector $X = [v, r]^T \in \mathbb{R}^2$ and a weight vector $W = [w, 0]^T \in \mathbb{R}^2$. The subthreshold dynamics of a subthreshold oscillating spiking neuron (SO) model is described by the following equation.

$$\begin{aligned} \dot{X} &= AX + B + Wu \\ &\begin{cases} A = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \text{ for } v < \eta \\ \\ A = \begin{bmatrix} -a & -b \\ b & a \end{bmatrix}, B = \begin{bmatrix} 0 \\ 2a\eta \end{bmatrix} \text{ for } v > \eta \end{cases} \end{aligned} \tag{2}$$

First, we consider an autonomous and subthreshold oscillating case, i.e., $W = [0, 0]^T$ and v stays less than a firing threshold. We use three parameters $a, b, \eta \in \mathbb{R}$ to characterize the subthreshold behavior. In this paper, we fix $(a, \eta) = (-4\pi, -20)$. So the control parameter is b. Fig:1 shows waveforms and phase space trajectories. We can see that the SO model oscillates periodically. We can obtain the approximate period $P \in \mathbb{R}$ by the following SO model's eigen values λ.

$$\lambda = a \pm bi \text{ for } v < \eta \tag{3}$$

$$\lambda = \pm bi\sqrt{1 - \left(\frac{a}{b}\right)^2} \text{ for } \eta < v, \tag{4}$$

We assume that $|a|$ is much smaller than $|b|$, i.e., $(a/b)^2 \simeq 0$. Then, the natural period P is approximated by the parameter b, i.e.,

$$P \simeq b/2\pi. \tag{5}$$

Secondly, we consider an autonomous and firing case, i.e., $W = [0, 0]^T$ and v is greater or equal to the firing threshold $\theta \in \mathbb{R}$. Let $t_n \in \mathbb{R}_+$ be the n-th moment when the state v reaches the firing threshold θ, i.e., $t_1 = min\{t | v(t) \geq \theta, t \geq 0\}$, $t_{n+1} = min\{t | v(t) \geq$

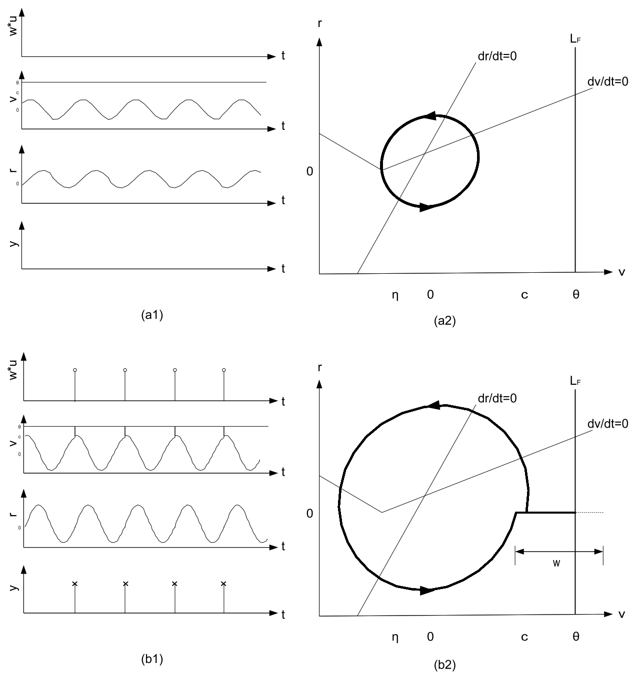

Figure 1: Typical behaviors of the SO models. (a1) shows time series of the states , input and output in autonomous case. (a2) shows its trajectories. (b1) shows time series of the states , input and output in non-autonomous case. (b2) shows its trajectories. (a, b, η, θ, c) $= (-4\pi, 20.0{\cdot}\pi, -20, -60, 40)$. Input period $T = 1/20$.

$\theta, t > t_n\}$. Let $t^+ = \lim_{\epsilon \to +0} t + \epsilon$. At the moment $t = t_n$, the state vector X is reset to $[c, r]^T$, i.e.,

$$X(t^+) = \begin{bmatrix} c \\ r(t) \end{bmatrix} \text{ if } v(t) \geq \theta \qquad (6)$$

where $c \in \mathbb{R}$ is a reset parameter. We refer to such a reset as a self-firing. An output of the SO model is given by

$$y(t) := \sum_{n=1}^{\infty} \delta(t - t_n) \qquad (7)$$

Third, we consider a non-autonomous case, i.e., $W \neq [0, 0]^T$. We define $t^- = \lim_{\epsilon \to -0} t + \epsilon$ and $t^{++} = \lim_{\epsilon \to +0} t^+ + \epsilon$. At the moment $t = nT$, the state vector X jumps to $X + W$ by an input pulse $u(nT)$, i.e.,

$$X(t^+) = X(t^-) + W \text{ if } t = nT \qquad (8)$$

If the state $v(t^+)$ overs the firing threshold θ, the state vector X is reset to $[c, r]^T$, i.e.,

$$X(t^{++}) = \begin{bmatrix} c \\ r(t^-) \end{bmatrix} \text{ if } v(t^+) \geq \theta \qquad (9)$$

We refer to such a reset as a compulsory-firing. The exact solution of the state $X(t)$ can be obtained by solving the piecewise linear equation in Eq(2).

2.1. Parameter update rule

We expand the SO model to memorize a period of the input $u(t)$ by updating the parameter b. We update the parameter b when a compulsory-fire occurs as the follows.

$$\text{if} \quad t = t_n = mT, \text{ then}$$
$$b(t^{++}) = b(t^-) + g(r(t^-)) \qquad (10)$$
$$g(r) := \begin{cases} \alpha r & \text{for } L_l < r < L_u \\ 0 & \text{otherwise} \end{cases} \qquad (11)$$

We refer to such an update for $g(r) \neq 0$ as valid update and refer to the parameter b as a dynamic parameter. Fig:2 shows a bifurcation diagram of the dynamic parameter b for the input period T.

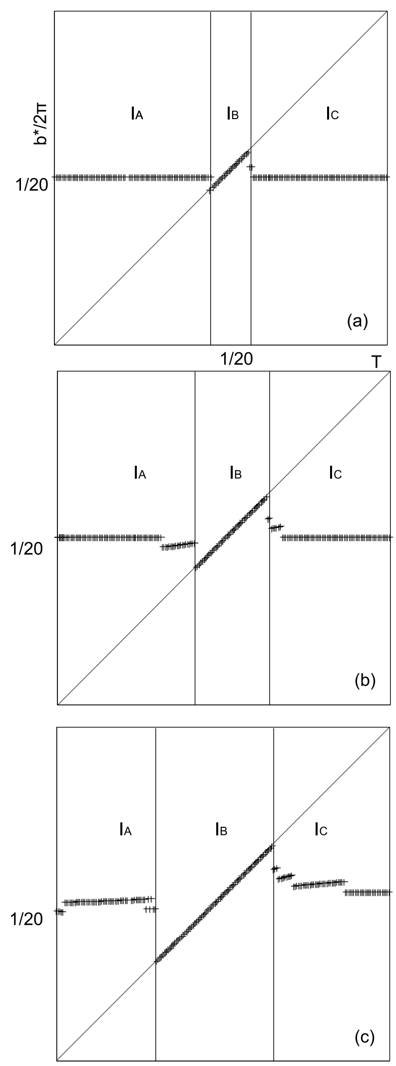

Figure 2: A typical bifurcation diagram of the dynamic parameter b for the input period T. [a, η, θ, c,α, L_l, L_u] = [-4π, -20, -60, 40, $-\pi/60$, -30, 30]. Initial parameter b_0 is fixed to $20.0 \cdot 2\pi$. (a) shows $w = 15$ case. (b) shows $w = 30$ case. (c) shows $w = 100$ case.

As shown in Fig:2, the input period T can be split into I_A, I_B and I_C. From Fig:2, if the input period T is close to $P(0) = b(0)/2\pi$, then $b^*/2\pi := \lim_{t\to\infty} b(t)/2\pi$ is near T, i.e.,

$$b^*/2\pi \fallingdotseq T \text{ for } T \in I_B \tag{12}$$

3. A pulse-coupled neural network

In this section, we introduce the self-organizing network of spiking neurons (SSN) interconnected with the spike train. Let $i \in \mathbb{N}$ be an index of each spiking neuron and $t_{n,i} \in \mathbb{R}_+$ be the n-th firing moment of the i-th neuron. Then we use state variables $v_i, u_i \in \mathbb{R}$, weight parameter $w_i \in \mathbb{R}$ and dynamic parameter $b_i \in \mathbb{R}$. For simplicity, we use a state vector $X_i = [v_i, u_i]^T \in \mathbb{R}^2$ and a weight vector $W_i = [w_i, 0]^T$. The single neuron is described by the following equations.

$$\begin{aligned}\dot{X}_i &= A_iX_i + B + W_iu\\ &\begin{cases} A_i = \begin{bmatrix} a & -b_i \\ b_i & a \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \text{ for } v < \eta \\ \\ A_i = \begin{bmatrix} -a & -b_i \\ b_i & a \end{bmatrix}, B = \begin{bmatrix} 0 \\ 2a\eta \end{bmatrix} \text{ for } v > \eta \end{cases}\end{aligned} \tag{13}$$

At the n-th firing moment of the i-th neuron i.e., $t = t_{i,n}$, the state vector X_i is reset to $[c, r_i]^T$, i.e.,

$$X(t^+) = \begin{bmatrix} c \\ r_i(t) \end{bmatrix} \text{ if } t = t_{n,i} \geq \theta \tag{14}$$

The output y_i of the i-th neuron is given by

$$y_i(t) := \sum_{n=1}^{\infty} \delta(t - t_{n,i}) \tag{15}$$

In the SSN, the SO models are connected by the following two processes, competitive process and parameter update process.

1. The competitive process is described by two update rules of weight w_i.

   (a) At the n-th input moment i.e., $t = nT$, the weight w_i jumps to $w_i + \delta w$, i.e.,

   $$\begin{aligned}&\text{if} \quad t = nT \text{ then}\\ &\qquad w_i(t^+) = w_i(t^-) + \delta w, \text{ for all } i\end{aligned} \tag{16}$$

   (b) At the n-th firing moment of the j-th neuron i.e., $t = t_{n,j}$, the weight w_i jumps to $w_c \in \mathbb{R}$, i.e.,

   $$\begin{aligned}&\text{if} \quad t = t_{n,j} \text{ then}\\ &\qquad w_i(t^+) = w_c, \text{ for all } i \text{ except } i = j\end{aligned} \tag{17}$$

2. The parameter update process is given by a update rule of dynamic parameter $b_i, b_{i\pm1}$.

   (a) At the moment $t = t_{n,j}$, we update dynamic parameter $b_j, b_{j\pm1}$ when the j-th neuron's compulsory-fire occurs, i.e.,

   $$\begin{aligned}&\text{if} \quad t = t_{n,j} = mT \text{ then}\\ &\qquad \begin{cases} b_j(t^{++}) = b_j(t^-) + g(r_j(t^-)) \\ b_{j\pm1}(t^+) = b_{j\pm1}(t^-) + \delta b \end{cases}\end{aligned} \tag{18}$$

   (b) In this paper, we set 10 neurons in line

   $$\delta b := \begin{cases} 0.1 & \text{if } b_i - b_{i\pm1} > 0 \\ -0.1 & \text{if } b_i - b_{i\pm1} < 0 \end{cases}$$

Now we simulate the SSN of the SO models by dynamics. We select input period from $\{T_A := 1/20, T_B := 1/22, T_C := 1/24\}$ randomly every two seconds. We set 10 neurons in line and initial parameter $b_i(0) = 20 \cdot 2\pi$ for all i. Fig:3(a) shows time waveforms of the indices of the firing neurons. Fig:3(b) shows time waveforms of the dynamic parameter b_i for all neurons. Fig:3(c) shows the dynamic parameter b_i at the end of the simulation. We can see that the dynamic parameters are clustered into N_A, N_B and N_C in Fig:3(c). So, we can say that the SNN learns the input periods by the dynamic parameter b_i.

We have also simulated the Kohonen's SOM numerically. We have confirmed that the Kohonen's SOM and the SSN have similar functions. Further detailed analysis will be presented in another manuscript.

4. Conclusion

We have presented the subthreshold oscillating (SO) model and the self-organizing network of spiking neurons (SSN). First, we have shown the behavior of the SO model. Secondly, we have checked the behavior of the dynamic parameter b using the bifurcation diagram under large weight w. Third, we have presented the simulation result of the SSN and checked the basic SOM function. Future problems include classification and analysis of bifurcation phenomena and the performance of the network and application to pulse-based signal processing.

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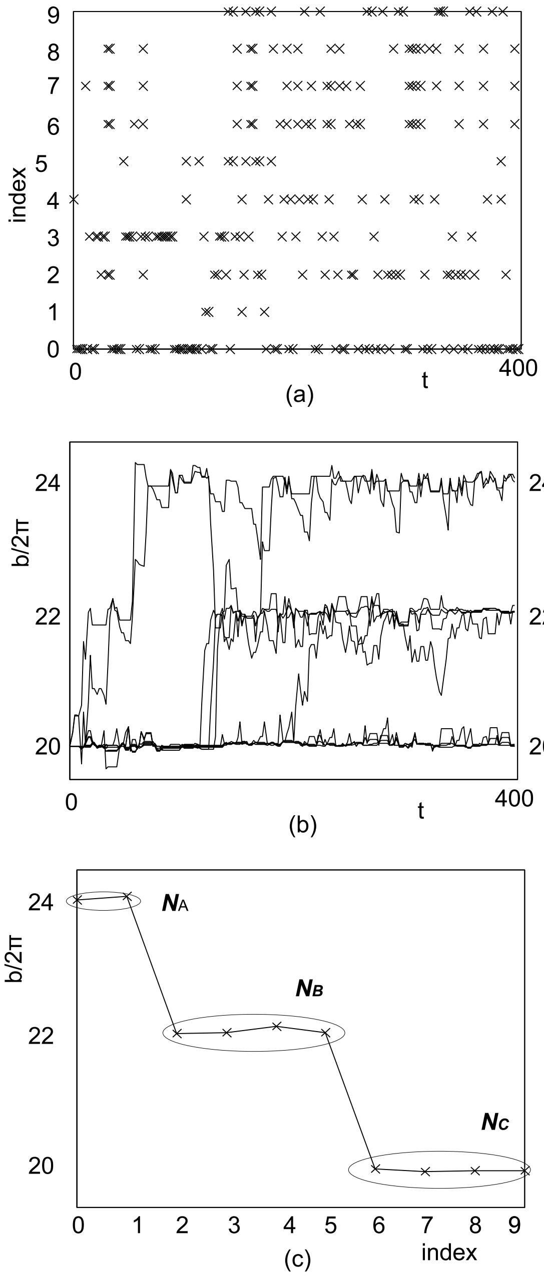

Figure 3: A typical result of the ONLINE simulation of the SSN. This SSN is composed of 10 neurons. (a) shows the firing moments $t_{n,i}$ for all i in $t-i$ plane. (b) shows the transition of the dynamic parameter b_i for all i in $i-b$ plane. (c) shows the dynamic parameter b_i in a steady state in $i-b$ plane. $[a, \eta, \theta, c, \alpha, L_l, L_u, \delta w, w_c] = [-4\pi, -20, -60, 40, -\pi/60, -30, 30, 5, 5]$. Initial dynamic parameter $b_i(0)$ is fixed to $20.0{\cdot}2\pi$ for all i. Simulation is done for 200 seconds and the input intervals are selected randomly from $\{T_A = 1/20, T_B = 1/22, T_C = 1/24\}$ every 2 seconds.

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