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Analyzing and Composing Music with Complex Networks: Finding Structures in Bach's, Chopin's and Mozart's

Chi K. Tse, Xiaofan Liu and Michael Small

Department of Electronic and Information Engineering, Hong Kong Polytechnic University, Hong Kong
URL: http://chaos.eie.polyu.edu.hk

Abstract—In this paper we study the network structure in music and attempt to compose music artificially. Networks are constructed with nodes and edges corresponding to musical notes and their co-occurrences. We analyze sample compositions from Bach, Mozart, Chopin, as well as other types of music including our local (Hong Kong) pop. We observe remarkably similar properties in all networks constructed from the selected compositions. Power-law exponents of degree distributions, mean degrees, clustering coefficients, mean geodesic distances, etc. are reported. With the network constructed, music can be created by using a biased random walk algorithm, which begins with a randomly chosen note and selects the subsequent notes according to a simple set of rules that compares the weights of the edges and/or the relative degrees of nodes. The newly created music from Mozart's network will be played in the presentation, along with the original piece.

1. Introduction

Music is a form of creative art which is often identified as a signature of a particular composer, a group of people, country and culture at different times in history. People from different parts of the world and in different eras have their own music. One fundamental question of interest is whether these different music share similar properties, and the implication of this question is whether a common process/rule exists in the human brain that is responsible for composing music.

The study of complex networks in physics has aroused a lot of interest across a multitude of application areas. A key finding is that most networks involving man-made couplings and connection of people are naturally connected in a scalefree manner, which means that the number of connections follows a power-law distribution [1]. Scalefree power-law distribution is a remarkable property that has been found across of a variety of connected communities [2]–[8] and is a key to optimal performance of networked systems [9].

In this paper we analysis a few distinct types of music, including classical, Russian folks and our local pop. Our approach is to treat a piece of music as a complex network and to evaluate the properties of the resulting network, such as degree distribution, mean degree, mean distance, clustering coefficient, etc. The purpose is to make an attempt to find out if different music would display uniformity or disparity in terms of network structure. Our results demonstrate, quite surprisingly, that different music types actually share remarkably similar properties. Our final task in this paper is to make an attempt to create "reasonably good" music[1] from the network that has been formed from given compositions such as Bach's and Mozart's. We basically find that if the same network property is retained, it is possible to compose music artificially and the remaining open problem is the choice of a particular sample from a large number of possible compositions. In composing a music, from a system's viewpoint, our human brain would have automatically performed a processing step that allows only compositions that satisfy certain network properties to emerge and finally pick the best composition according to the composer's subjective choice. Of course, we do not know exactly how the brain does that. As an interim trick, some rudimentary rules may come into play when selecting compositions.

2. Review of Networks

A network is usually defined as a collection of "nodes" connected by "links" or "edges" [2]. If we consider a network of musical notes, then the nodes will be the individual musical notes and a link between two nodes denotes that the two musical notes are neighbors in the score. The number of links emerging from and converging at a node is called the "degree" of that node, usually denoted by k. So, we have an average degree for the whole network. The key concept here is the distribution of k. This concept can be mathematically presented in terms of probability density function. Basically, the probability of a node having a degree k is $p(k)$, and if we plot $p(k)$ against k, we get a distribution function. This distribution tells us about how this network of musical notes are connected. Recent research has provided concrete evidence that networks with man-made couplings and/or human connections follow power-law distributions, i.e., $p(k)$ vs k being a straight line whose gradient is the characteristic exponent [3]–[8]. Such networks are termed *scalefree networks*.

[1]The authors have listened to the reconstructed music and find some of them very appealing. Samples to be played at the conference.

Figure 1: A crochet of middle C is a note (left), and a quaver of middle C is a different note (right). Both are considered as different nodes in a musical network.

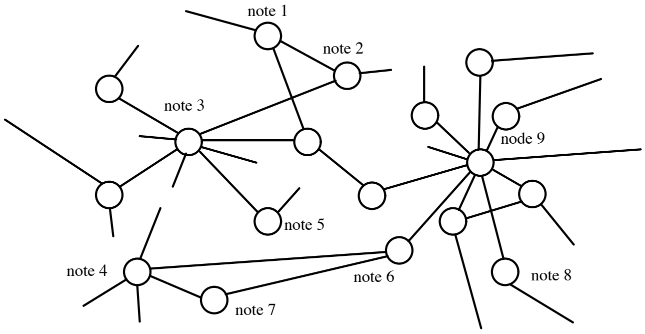

Figure 2: A network for music, where nodes are notes and edges are connections of two consecutively played notes. This one corresponds to Bach's sonatas.

3. Network Construction Based on Co-occurrence

A musical *note* is defined by its pitch and time value. For example, a crotchet of the middle C is considered as a note, and a quaver of the same middle C is a different note. See Fig. 1. Consider an 88-key piano keyboard. If we limit each key to have 20 possible time values (e.g., brieve, semibrieve, dotted minum, minum, dotted crochet, crochet, dotted quaver, quaver, dotted semi-quaver, semi-quaver, dotted demisemi-quaver, demisemi-quaver, etc. [10]), for instance, there are altogether 1760 possible notes.

For simplicity, we consider single-note scores where notes are to be played one after another, without simultaneous playing of two or more notes like a chord. Then, we may examine the way in which notes appear in the score for the purpose of constructing a complex network to represent the score.

To form a network, we need to define what *node* and *edge* are. For the purpose of constructing a network from a musical score, we consider notes as *nodes* as explained earlier. A piece of music can be considered as a sequence of notes and hence edges can be defined by connections from one note to another chronologically. That is, if note i starts at time T and note j ends at the same time, then an edge is established from note i to note j.

Suppose there are N nodes. Then, node i is connected to node j when node i is played and followed by node j, and the connection is directed from node i to node j. Eventually, a network is formed with each node connected to a number of other nodes, as shown in Fig. 2. Of particular interest is the number of edges emerging from a node, which is defined as the *degree* of that node and is denoted by k. Also, the *distance* between two nodes, d, which reflects how closely two nodes are connected, and the *clustering coefficient*, C, which reflects on the extent of interconnections of nodes, are also of importance. Furthermore, to probe into the structure of the network, the distribution of the degree will be considered.

In the following section we will examine the networks formed from music composed by Bach, Chopin and Mozart, as well as from Russian folks and local pop. A typical network formed using the method described above is shown in Fig. 3, which corresponds to Bach's sonatas.

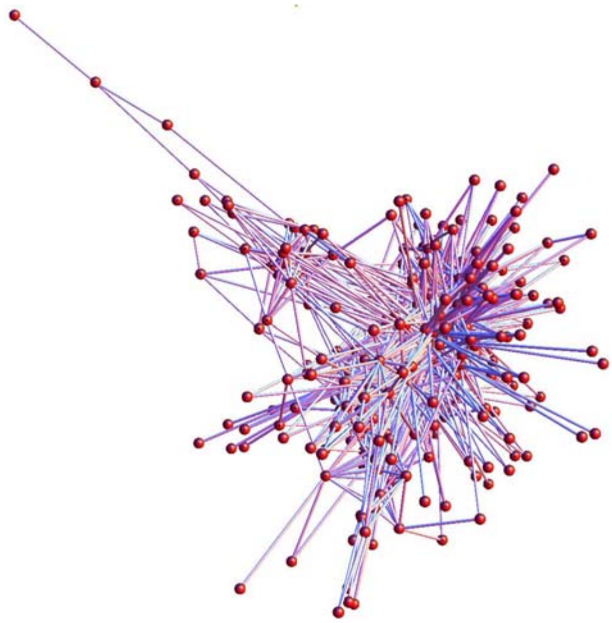

Figure 3: Network from Bach's sonatas.

Table 1: Simplified MIDI file format.

Time mark	Event	Note identity
Tick 1	Start	Pitch name 1
Tick 2	Start	Pitch name 2
Tick 3	End	Pitch name 1
Tick 4	End	Pitch name 2
Tick 5	Start	Pitch name 3
Tick 6	End	Pitch name 3
...	...	...

4. Analysis

The MIDI (Musical Instrument Digital Interface) format is used here for representation of music [11]. MIDI allows music to be stored in digital forms that can facilitate repeated performance at later times. Referring to Table 1, tick n is the time mark which indicates the time an event occurs. An event is either the start or end of a musical note.

For instance, pitch name 1 starts at tick 1 and ends at tick 3. In our study, MIDI files are created by direct conversion from the scores or from the actual real-time performance. In the case of actual real-time performance, the number of time values for a note can be much more numerous and will be truncated to a set of quantized values. Once the network is formed, we can compute the following parameters:

1. Number of nodes, N
2. Total number of edges, $\sum k$
3. Mean degree, $\bar{k}$
4. Mean shortest distance between nodes, $\bar{d}$
5. Clustering coefficient, C
6. Power-law exponent of degree distribution, γ

The number of nodes for a network can be found by simple counting. The mean degree can also be found relatively easily by taking the average over the degree values of all nodes in the network. The calculation of the mean minimum distance between nodes requires some computational efforts, and in this work we have adopted the Floyd-Warshall algorithm [12]. To find the clustering coefficient, we use the following formulas:

$$C_1 = \frac{3 \times \text{number of triangles in the network}}{\text{number of connected triples of nodes}}$$

$$C_2 = \frac{1}{N} \sum_i \frac{\text{number of triangles connected to node } i}{\text{number of triples connected to node } i}$$

The power-law exponent is the slope of the log-log plot of the degree distribution, $p(k)$ versus k, assuming that it is a straight line and thus reflects a scalefree distribution.

In our study, compositions from several composers and sources are considered, namely, selected Bach's violin works and "Well-Tempered Clavier" (WTC), Mozart's sonatas, Chopin's waltz, Russian folk and Cantonese pop music. Basically, we concatenate a number of pieces of the same type of works together to form a single set, from which a MIDI file is generated. Thus, six sets are created and hence six MIDI files are generated for evaluation. Complex networks are then constructed and the parameters are extracted for each network. Table 2 summarizes the results for the selected musical works. Some findings are worth noting: (i) the networks formed for the different musical works are found to be scalefree in their degree distribution, and the power-law exponents, γ, are surprisingly consistent and all fall in the range of 1 to 1.4, using a least-square-error estimation. The maximum fitting errors are 0.01 to 0.06 for sample sizes of around $N/2$ (as half of the data corresponding to very large and small k have been discarded) [13]. Fig. 4 shows the degree distributions plotted in a log-log scale. (ii) The clustering coefficients for MIDI data generated directly from musical scores (around 0.3–0.40 are found to significantly larger than those for MIDI data generated from real-time recordings (around 0.1). (iii) The mean degrees for MIDI data generated directly from musical scores are also much larger than those for MIDI data generated from real-time recordings.

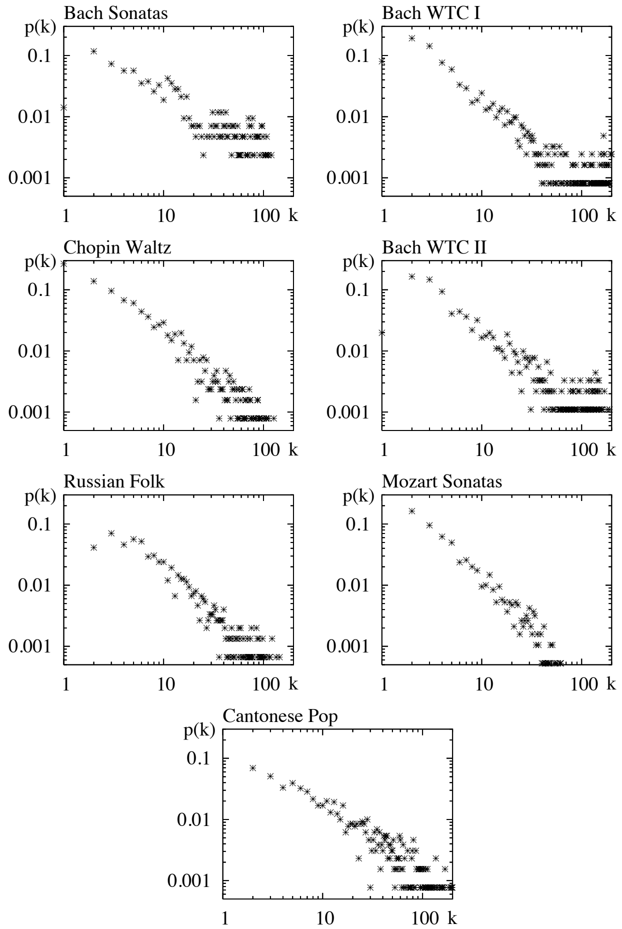

Figure 4: Degree distributions for different music. $p(k)$ versus k in log-log scale. Slopes are measured and reported as γ in Table 2.

5. Construction of Music: Artificial Composition

The properties of the musical networks have indicated some universality across different composers and styles. For instance, the power-law degree distribution is a specific manifestation of such universality. If different composers would come up with music displaying universality, then would it be possible that music can be created artificially by preserving the same or similar network parameters?

Suppose we form a network from the works of Mozart. Then, we may create a new score using the same set of nodes (notes) and connecting one after another following an algorithm that preserves some selected network parameters. Let us focus on the preservation of the scalefree degree distribution as it seems to be the most striking common feature. We take a simple approach in connecting the nodes (generating the sequence of notes), which is based on a *biased random walk* algorithm.

Algorithm 1: First, we begin with an arbitrarily chosen node (note) in the network. The next node in the sequence will be chosen among those connected to it. According to

Table 2: Results of network parameters found for selected works.

Music	N	$\sum k$	$\bar{k}$	$\bar{d}$	C_1	C_2	γ	fitting $\mathrm{error}_{\max}$
Bach's sonatas (MIDI from score)	425	9362	22.03	2.77	0.43	0.38	1.3169	0.0408
Bach's WTC (1) (MIDI from score)	1231	24529	19.93	2.78	0.36	0.33	1.3961	0.0099
Bach's WTC (2) (MIDI from score)	910	18665	20.51	2.81	0.32	0.31	1.2726	0.0168
Chopin's waltz (MIDI from real-time recording)	1271	12117	9.53	3.51	0.16	0.18	1.4392	0.0137
Mozart's sonatas (MIDI from real-time recording)	1897	7859	4.14	3.88	0.04	0.10	1.4191	0.0591
Russian folk (MIDI from real-time recording)	1501	13249	8.83	3.45	0.11	0.16	1.0767	0.0235
Cantonese pop (MIDI from real-time recording)	1298	23036	17.75	2.75	0.20	0.19	0.921	0.0455

the *strength* of a connecting edge,[2] we define the probability that this edge will be chosen. Then, the node connected to the chosen edge will be the next node. The process continues and a new score is thus created.

Algorithm 2: An alternative way to create a new score is as follows. Again, we begin with an arbitrarily chosen node (note) in the network. The next node in the sequence will be chosen among those connected to it. Here, according to the degrees of all connecting nodes, we define the probabilities that these nodes will be chosen as the next node. In this way, nodes are chosen one after another. The process continues and a new score is thus created.

Some samples of music generated from the musical networks can be downloaded from the the following website:

- http://cktse.eie.polyu.edu.hk/MUSIC/

Remarks – Musics generated from the above algorithms are far too numerous. Thus, filtering off "bad" music is important. Our initial consideration is the extent of duplication of any sequence of notes. Intuitively, a duplication-free sequence resembles a random sequence which is undesirable. Thus, we may incorporate a duplication measure in our algorithm to improve our compositions.

6. Conclusion

We have analyzed selected musical compositions in terms of co-occurrence network structures. Selected works from Bach, Chopin and Mozart, as well as from Russian folks and local pop, are analyzed, and networks are constructed according to the note-to-note connections of the musical scores. The networks have been found to be scale-free and their degree distributions have a similar power-law property with the values of the exponent equal to around 1.6. Such commonality suggests that the human brain composes music which naturally exhibits a scalefree degree distribution. We have therefore extended our study to reconstructing music and the basic criterion is to preserve the same power-law property. The resulting reconstructed music are still very numerous and not all sound appealing. An optimization (selection) process is needed to pick the finalist, and it will be a challenging task to study how the human brian does the selection in the process of composing music.

[2]The strength of an edge connecting two nodes is the number of times the two nodes are connected as the music is played in the original music from which the network was generated.

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