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Propagation Mechanism of Phase-Inversion Wave in In-and-Anti-Phase Synchronization on 2D Lattice Oscillator

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Abstract—We analyze synchronization phenomena on coupled oscillators systems as a ladder and a lattice. On the systems, we observed phase-inversion waves, which are phenomena of changing phase states between two adjacent oscillators from in-phase synchronization to anti-phase synchronization or from anti-phase synchronization to in-phase synchronization in steady state. Some characteristics of phase-inversion waves are propagations, penetrations, reflections, and disappearances. In this paper, we discover the phase-inversion waves in in-and-anti-phase synchronization. We clarify regions which the phase-inversion wave can be observed in in-and-anti-phase synchronization, and clarify a mechanism of propagation of a phase-inversion wave in in-and-anti-phase synchronization on the lattice system.

1. Introduction

A lot of synchronization phenomena can be observed in nature world. For example, there are biological clocks, schools of sardines, the synchronization of fireflies, and so on. Recently, synchronization phenomena are researched in various fields[1]-[2].

In our previous study, we observed synchronization phenomena on coupled oscillators system. This system is made by using van der Pol oscillators which are coupled by inductor as a lattice[3]. We predicted the time-series data by using this system including nine oscillators[4]. We observed phase-inversion waves on this system including over 25 oscillators. We analyzed a mechanism of disappearance between two phase-inversion waves. Further, we analyzed a mechanism of reflection when two phase-inversion waves arrive at a corner at same time. However, these phase-inversion waves are observed in double-in-phase synchronization which all oscillators synchronize to in-phase for a vertical direction and a horizontal direction. In other hand, on ladder system, the phase-inversion waves are observed in in-and-anti-phase synchronization[5]. In-and-anti-phase synchronization is in-phase and anti-phase synchronizations exist alternately.

In this study, we observe the phase-inversion waves in in-and-anti-phase synchronization. We clarify regions which the phase-inversion wave can be observed in in-and-anti-phase synchronization when N equals 9, and clarify a mechanism of propagation of a phase-inversion wave in in-and-anti-phase synchronization using instantaneous frequency of each oscillator and phase differences between adjacent oscillators on the lattice system.

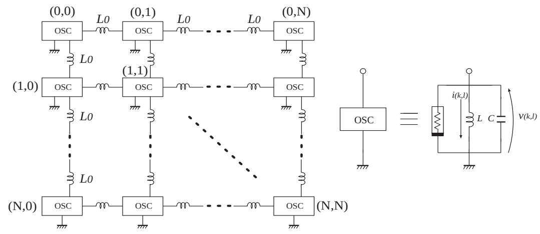


Figure 1: Circuit model.

2. Circuit model

The van der Pol oscillators are coupled by inductors L_0 as a lattice(see Fig. 1). The number of column and row of this system are assumed as "$N+1$" respectively. We name each oscillator OSC(k,l). A voltage of each oscillator is named $v_{(k,l)}$, and a current of inductor of each oscillator is named $i_{(k,l)}$(see Fig. 1). The circuit equations of this circuit model are normalized by Eq. (1), and the normalized circuit equations are shown as Eqs. (2)–(6).

$$i_{(k,l)} = \sqrt{\frac{Cg_1}{3Lg_3}}x_{(k,l)},\quad v_{(k,l)} = \sqrt{\frac{g_1}{3g_3}}y_{(k,l)},$$
$$t = \sqrt{LC}\tau,\quad \frac{d}{d\tau} = \text{“ } \cdot \text{ ”}, \alpha = \frac{L}{L_0},\quad \varepsilon = g_1\sqrt{\frac{L}{C}}. \tag{1}$$

[Corner–top] (left:(a,b)=$(0,1)$. right:(a,b)=$(N,N-1)$.)

$$\frac{dx_{(0,a)}}{d\tau} = y_{(0,a)}, \tag{2}$$
$$\frac{dy_{(0,a)}}{d\tau} = -x_{(0,a)} + \alpha(x_{(0,b)} + x_{(1,a)} - 2x_{(0,a)}) + \varepsilon(y_{(0,a)} - \frac{1}{3}y^3_{(0,a)}).$$

[Corner–bottom] (left:(a,b)=$(0,1)$. right:(a,b)=$(N,N-1)$.)

$$\frac{dx_{(N,a)}}{d\tau} = y_{(N,a)}, \tag{3}$$
$$\frac{dy_{(N,a)}}{d\tau} = -x_{(N,a)} + \alpha(x_{(N-1,a)} + x_{(N,b)} - 2x_{(N,a)}) + \varepsilon(y_{(N,a)} - \tfrac{1}{3}y^3_{(N,a)}).$$

[Center] ($0 < k < N$. $0 < l < N$.)

$$\frac{dx_{(k,l)}}{d\tau} = y_{(k,l)}, \tag{4}$$
$$\frac{dy_{(k,l)}}{d\tau} = -x_{(k,l)} + \alpha(x_{(k+1,l)} + x_{(k-1,l)} + x_{(k,l+1)} + x_{(k,l-1)} - 4x_{(k,l)}) + \varepsilon(y_{(k,l)} - \tfrac{1}{3}y^3_{(k,l)}).$$

[Edge]

(top:(a, b)=(0, 1).bottom:$(a, b)=(N, N-1)$.both:$0< l <N$.)

$$\frac{dx_{(a,l)}}{d\tau} = y_{(a,l)}, \tag{5}$$

$$\frac{dy_{(a,l)}}{d\tau} = -x_{(a,l)} + \alpha(x_{(a,l-1)} + x_{(a,l+1)} + x_{(b,l)} - 3x_{(a,l)}) + \varepsilon(y_{(a,l)} - \frac{1}{3}y^3_{(a,l)}).$$

(left:(a, b)=(0, 1). right:$(a, b)=(N, N-1)$. both:$0<k<N$.)

$$\frac{dx_{(k,a)}}{d\tau} = y_{(k,a)}, \tag{6}$$

$$\frac{dy_{(k,a)}}{d\tau} = -x_{(k,a)} + \alpha(x_{(k-1,a)} + x_{(k+1,a)} + x_{(k,b)} - 3x_{(k,a)}) + \varepsilon(y_{(k,a)} - \frac{1}{3}y^3_{(k,a)}).$$

The α corresponds to a coupling parameter. The ε corresponds to a nonlinearity of each oscillator. This system is simulated by the fourth order Runge-Kutta method and Eqs. (2)-(6). The phase-inversion waves are shown in Fig. 2. The Fig. 2–A expresses an attractor of each oscillator(current vs. voltage). The Fig. 2–B expresses itinerancy of phase difference by which sum of voltages of adjacent oscillators is shown along the time(sum of voltage vs. time).

3. In-and-anti-phase synchronization

In our circuit, an oscillator, which is not an oscillator on the edge, has four adjacent oscillators. When phase states between the oscillator and two of four oscillators are anti-phase synchronization, phase states between the oscillator and other two oscillators are in-phase synchronization. Oscillators on the edges stay in anti-phase synchronization in in-and-anti-phase synchronization. These phase states are called "in-and-anti-phase synchronization." The phase-inversion waves in in-and-anti-phase synchronization are classified into two patterns. Pattern A can be observed if N is an odd number. Odd number's phase-inversion waves propagate in vertical direction and horizontal direction respectively. Pattern B can be observed if N is an even number. Even number's phase-inversion waves propagate in vertical direction and horizontal direction respectively. Simulation results of pattern A and B show in Figs. 2 and 3 respectively. Figure 4 shows regions which the phase-inversion wave can be observed in in-and-anti-phase synchronization when N equals 9. The coupling parameter α and nonlinearity ε are changed from 0.050 to 1.0, every 0.050. The phase-inversion wave in in-and-anti-phase synchronization can be observed in region(i)(see Figs. 2 and 4). The complex phenomena on in-and-anti-phase synchronization can be observed in region (ii)(see Figs. 4 and 5).

We can observe some characteristics of phase-inversion waves in in-and-anti-phase synchronization. These characteristics are a propagation, a penetration, a reflection at an edge, and a reflection between two phase-inversion waves(see Figs. 2 and 3). These characteristics are shown in Table 1.

4. Mechanism

We analyze a mechanism of propagation of a phase-inversion wave. The mechanism is made clear by using instantaneous frequency of each oscillator and phase differences between adjacent oscillators. Figure 6 shows the signs

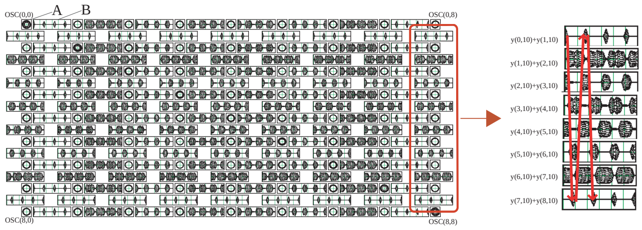


Figure 2: Pattern A - Phase-inversion waves on 9x9 oscillators(α=0.05 and ε=0.15).

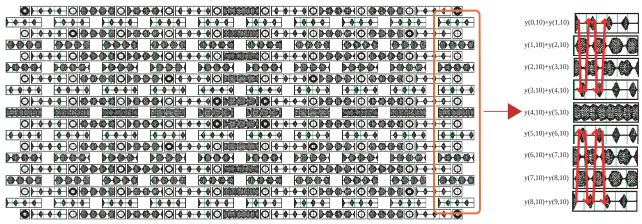


Figure 3: Pattern B - Phase-inversion waves on 10x10 oscillators(α=0.05 and ε=0.15).

of the initial values of the voltages and currents of each oscillator. The coupling parameter α is fixed as 0.05, and nonlinearity ε is fixed as 0.15. An equation of the instantaneous frequency of OSC(k, l) is obtained as follows(see Eq. (7)). The instantaneous frequency is named $f_{(k,l)}(a)$ where "a" expresses the number of times of the peak value of the voltage. Time of a-th peak value of the voltage of OSC(k, l) is assumed as $\tau_{(k,l)}(a)$(see Fig. 7). Similarly, $\tau_{(k+1,l)}(a)$ and $\tau_{(k,l+1)}(a)$ are decided.

$$f_{(k,l)}(a) = \frac{1}{\tau_{(k,l)}(a) - \tau_{(k,l)}(a-1)}. \tag{7}$$

Three frequencies are observed in this system. To consider of the synchronizations for the vertical direction and for the horizontal direction are needed, because this system is 2 dimensional array. The in-phase synchronization and the anti-phase synchronization exist. Therefore, three types of synchronizations are observed as follows:

1. OSC(k, l)–OSC(k, $l+1$), OSC(k, l)–OSC(k, $l-1$), OSC(k, l)–OSC($k+1$, l), and OSC(k, l)–OSC($k-1$, l): the in-phase synchronization.
2. {(OSC(k, l)–OSC(k, $l-1$), and OSC(k, l)–OSC(k, $l+1$) are a same phase synchronization state, and OSC(k, l)–OSC($k-1$, l), and OSC(k, l)–OSC($k+1$, l) are another phase synchronization state.} **or** {OSC(k, l)–OSC($k-1$,

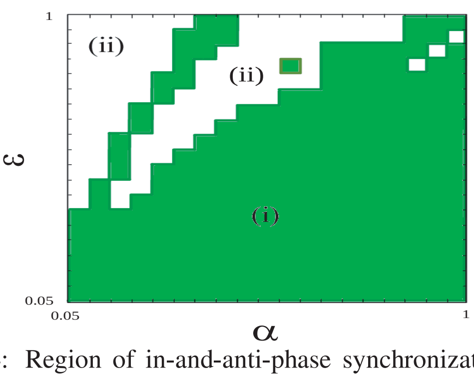


Figure 4: Region of in-and-anti-phase synchronization on 9x9 oscillators

Table 1: Characteristics of the phase-inversion waves on in-and-anti-phase synchronization.

Names of characteristics	*Phenomena*
Propagations	The phase-inversion waves propagate for vertical direction or horizontal direction. The vertical phase-inversion waves independently move from the horizontal phase-inversion waves.
Penetrations	Two phase-inversion waves arrive at an oscillator from vertical direction and horizontal direction, and each phase-inversion wave penetrates each other.
Reflections at an edge	When a phase-inversion wave arrives at an edge, the phase-inversion wave reflects and propagates to where they came from. Sometime this phenomenon is happened with penetration.
Reflections between two phase-inversion waves	When two phase-inversion waves coming from the opposite directions arrive to two adjacent oscillator at same time, the phase-inversion waves reflect and propagate to where they came from.

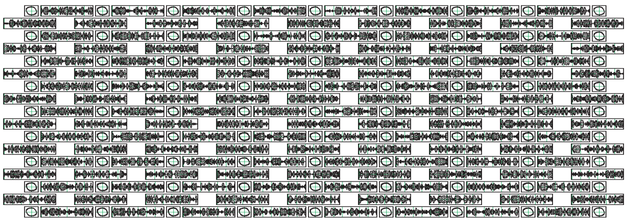


Figure 5: An example of complex phenomena in region(ii)(α=0.05 and ε=0.85).

	0	1	2	3	4	5	6	7	8
0	+	+	—	—	+	+	—	—	+
1	+	+	—	—	+	+	—	—	+
2	—	—	+	+	—	—	+	+	—
3	—	—	+	+	—	—	+	+	—
4	+	+	—	—	+	+	—	—	+
5	+	+	—	—	+	+	—	—	+
6	—	—	+	+	—	—	+	+	—
7	—	—	+	+	—	—	+	+	—
8	+	+	—	—	+	+	—	—	+

Figure 6: Sign of initial value of each oscillator of in-and-anti-phase synchronization.

l), and OSC(k, l)–OSC(k, $l-1$): are a same phase synchronization state, and OSC(k, l)–OSC($k+1$, l), and OSC(k, l)–OSC(k, $l+1$): are another phase synchronization state.} **or** {OSC(k, l)–OSC($k-1$, l), and OSC(k, l)–OSC(k, $l+1$): are a same phase synchronization state, and OSC(k, l)–OSC(k, $l-1$), and OSC(k, l)–OSC($k+1$, l): are another phase synchronization state.}

3. OSC(k, l)–OSC(k, $l+1$), OSC(k, l)–OSC(k, $l-1$), OSC(k, l)–OSC($k+1$, l), and OSC(k, l)–OSC($k+1$, l): the anti-phase synchronization.

An instantaneous frequency $f_{(k,l)}$ of OSC(k, l) is obtained in each synchronization-type. The 1st situational synchronization frequency is called f_{in-in}. The 2nd situational synchronization frequency is called $f_{in-anti}$. The 3rd situational synchronization frequency is called $f_{anti-anti}$. The phase difference is calculated as follows. A phase difference between OSC(k, l) and OSC($k+1$, l) and a phase difference between OSC(k, l) and OSC(k, $l+1$) are obtained. The phase differences are assumed as $\Phi_{(k,l)(k+1,l)}(a)$ and $\Phi_{(k,l)(k,l+1)}(a)$ respectively. The $\Phi_{(k,l)(k+1,l)}(a)$ and $\Phi_{(k,l)(k,l+1)}(a)$ are obtained by Eq. (8)(see Fig. 7).

$$\begin{aligned}\Phi_{(k,l)(k+1,l)}(a) &= \frac{\tau_{(k,l)}(a) - \tau_{(k+1,l)}(a)}{\tau_{(k,l)}(a) - \tau_{(k,l)}(a-1)} \times 360 \text{ [degree]} \\ \Phi_{(k,l)(k,l+1)}(a) &= \frac{\tau_{(k,l)}(a) - \tau_{(k,l+1)}(a)}{\tau_{(k,l)}(a) - \tau_{(k,l)}(a-1)} \times 360 \text{ [degree]}.\end{aligned} \tag{8}$$

4.1. Propagation mechanism

We can observe a phenomenon that a phase-inversion wave to a vertical direction in each column propagate in in-and-anti-phase synchronization. We fix the N as 19. Propagation mechanism is shown in Table. 2(see Fig. 8). In Fig. 8(a), the vertical axis is instantaneous frequency, and horizontal axis is time. In Fig. 8(b), the vertical axis is the phase difference, and the horizontal axis is time.

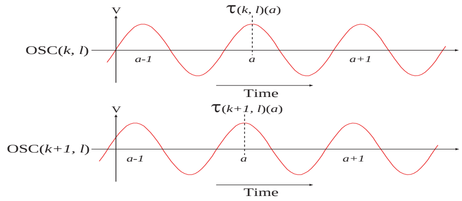


Figure 7: The detection method of frequencies and the phase differences.

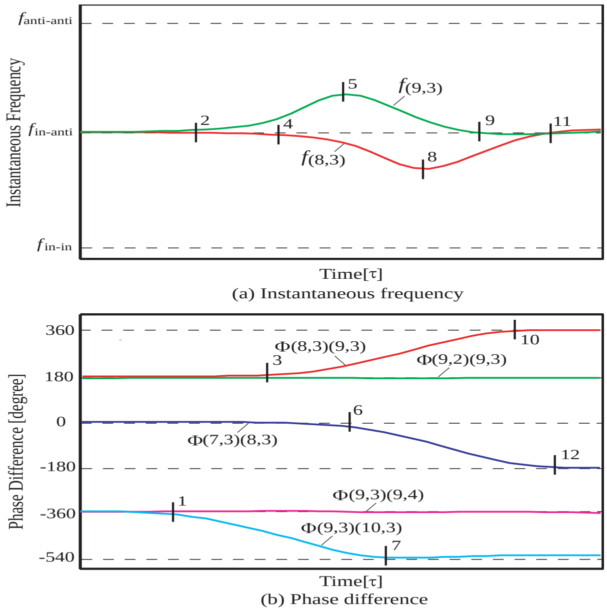


Figure 8: Transitions of phase difference and frequencies by propagation of a phase-inversion wave on in-and-anti-phase synchronization.

4.2. Comparison between a propagation in double in-phase synchronization and a propagation in in-and-anti-phase synchronization.

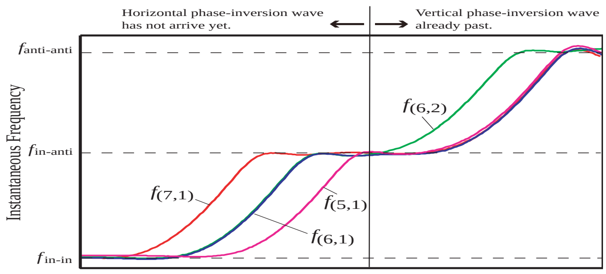
The frequency's itinerancies of propagation of the phase-inversion wave in double in-phase synchronization are Fig. 9. The frequencies are changed from f_{in-in} to $f_{anti-anti}$.

5. Conclusion

We discovered the phase-inversion waves in in-and-anti-phase synchronization. We clarified regions which the phase-

Table 2: Propagation mechanism of a phase-inversion wave(see Fig. 8).

no.	*Mechanism*
0	At this time, $\Phi_{(9,3)(9,4)}$ and $\Phi_{(8,3)(8,4)}$ are fixed the in-phase synchronization. $\Phi_{(9,2)(9,3)}$ and $\Phi_{(8,2)(8,3)}$ are fixed the anti-phase synchronization. In vertical direction, the phase-inversion wave, which changes synchronized state, arrives at the number of row is 10 from the number of row is 18.
1	A phase difference $\Phi_{(9,3)(10,3)}$ starts to change from the in-phase synchronization toward the anti-phase synchronization by a phase-inversion wave.
2	A instantaneous frequency $f_{(9,3)}$ starts to increase from $f_{in-anti}$ toward $f_{anti-anti}$, because $\Phi_{(9,3)(10,3)}$ starts to change from the in-phase synchronization toward the anti-phase synchronization and $\Phi_{(9,2)(9,3)}$ and $\Phi_{(8,3)(9,3)}$ are anti-phase synchronization and $\Phi_{(9,3)(9,4)}$ is in-phase synchronization.
3	$\Phi_{(8,3)(9,3)}$ starts to change from the anti-phase synchronization toward the in-phase synchronization by $f_{(9,3)}$.
4	$f_{(8,3)}$ starts to decrease from $f_{in-anti}$ toward f_{in-in}, because $\Phi_{(8,3)(9,3)}$ starts to change from the anti-phase synchronization toward the in-phase synchronization and $\Phi_{(8,3)(8,4)}$ and $\Phi_{(7,3)(8,3)}$ are the in-phase synchronization and $\Phi_{(8,2)(8,3)}$ is the anti-phase synchronization.
5	$f_{(9,3)}$ doesn't arrive at $f_{anti-anti}$ and $f_{(9,3)}$ starts to decrease toward $f_{in-anti}$ again, because $\Phi_{(9,3)(10,3)}$ starts to change from the in-phase synchronization toward the anti-phase synchronization and $\Phi_{(8,3)(9,3)}$ starts to change from the anti-phase synchronization toward the in-phase synchronization and $\Phi_{(9,3)(9,4)}$ is the in-phase synchronization and $\Phi_{(9,2)(9,3)}$ is the anti-phase synchronization.
6	$\Phi_{(7,3)(8,3)}$ starts to change from the in-phase synchronization toward the anti-phase synchronization by $f_{(8,3)}$.
7	$\Phi_{(9,3)(10,3)}$ arrives at the anti-phase synchronization and becomes fix.
8	$f_{(8,3)}$ doesn't arrive at f_{in-in} and $f_{(8,3)}$ starts to increase toward $f_{in-anti}$ again, because $\Phi_{(8,3)(9,3)}$ starts to change from the anti-phase synchronization toward the in-phase synchronization and $\Phi_{(7,3)(8,3)}$ starts to change from the in-phase synchronization toward the anti-phase synchronization and $\Phi_{(8,3)(8,4)}$is the in-phase synchronization and $\Phi_{(8,2)(8,3)}$ is the anti-phase synchronization.
9	$f_{(9,3)}$ arrives at $f_{in-anti}$ again, because $\Phi_{(9,3)(10,3)}$ arrives at the anti-phase synchronization and $\Phi_{(8,3)(9,3)}$ starts to change from the anti-phase synchronization toward the in-phase synchronization and $\Phi_{(9,3)(9,4)}$ is the in-phase synchronization and $\Phi_{(9,2)(9,3)}$ is the anti-phase synchronization.
10	$\Phi_{(8,3)(9,3)}$ arrives at in-phase synchronization and becomes fix, because $f_{(9,3)}$ arrives at $f_{in-anti}$ and becomes fix.
11	$f_{(8,3)}$ arrives at $f_{in-anti}$ again, because $\Phi_{(8,3)(9,3)}$ arrives at the in-phase synchronization and $\Phi_{(7,3)(8,3)}$ starts to change from the in-phase synchronization toward the anti-phase synchronization and $\Phi_{(8,3)(8,4)}$ is the in-phase synchronization and $\Phi_{(8,2)(8,3)}$ is the anti-phase synchronization.
12	$\Phi_{(7,3)(8,3)}$ arrives at anti-phase synchronization and becomes fix, because $f_{(8,3)}$ arrives at $f_{in-anti}$ and becomes fix.
The phase-inversion wave propagates by this mechanism.	



(a) Instantaneous frequency

Figure 9: Transitions of frequencies by propagation of a phase-inversion wave on in-and-in-phase synchronization.

inversion wave can be observed in in-and-anti-phase synchronization when N equals 9, and clarified a mechanism of propagation of a phase-inversion wave in in-and-anti-phase synchronization by using instantaneous frequency of each oscillator and phase differences between adjacent oscillators on the lattice system, and compared between a propagation in double in-phase synchronization and a propagation in in-and-anti-phase synchronization. The frequencies of the phase-inversion wave in double in-phase synchronization are changed from f_{in-in} to $f_{anti-anti}$. However, the frequencies of the phase-inversion wave in in-and-anti-phase synchronization are changed around $f_{in-anti}$, and can not change to f_{in-in} and $f_{anti-anti}$. We observed some characteristics of phase-inversion waves on in-and-anti-phase synchronization. These characteristics are a propagation, a penetration, a reflection at an edge, and a reflection between two phase-inversion waves.

Acknowledgements

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