Highly Nonlinear Electrostatic MEMS Frequency Up-Converter for Energy Efficient Harvesting

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Abstract– The design of a MEMS frequency up-converter is described in this paper, the MEMS device relies on the highly nonlinear phenomena of electrostatic pull-in and hysteresis in order to multiply the input frequency by a significant factor, possibly up to several orders of magnitudes. An analytical solution of the performance of the device is obtained using a spring-mass model, and the optimal values of physical parameters are also derived analytically. This frequency up-converter is designed in view of a specific application which is to provide efficient energy harvesting for cardiac medical implants.

1. Introduction

The highly appealing prospect of supplying ultra-low power electronics with their energy by integrating an energy scavenging device is most often impaired by the low efficiency of energy harvesters, this issue is further exacerbated by the lack of high frequency environmental excitation where mechanical-to-electrical energy conversion is more efficient. In literature several frequency up-conversion MEMS devices have been suggested [1]-[3]. In this work, an efficient electrostatic MEMS frequency up-converter is presented operating on the principle of electrostatic pull-in.

2. Modelling

The MEMS device used in this work is depicted schematically in Fig. 1, where it consists of two MEMS resonators: the first is a low frequency resonator hereon labeled seismic resonator with a natural frequency within the bandwidth of the input excitation, and represented in Fig. 1 by the mass-spring system denoted M_S and K_S respectively. The second high frequency resonator, is depicted with the lumped elements K and M in Fig. 1.

The seismic mass is covered by a dielectric layer of thickness h and a relative dielectric constant ε_r, and a constant voltage difference V is maintained between the two resonators whose respective proof mass constitute the electrodes of a parallel plate capacitor.

The attractive force between the two electrodes (formed by the masses M and M_S) is thus given by:

$$F_{elec} = \frac{\varepsilon_0 A V^2}{2\left(\frac{h}{\varepsilon_r} + X\right)^2} \tag{1}$$

where F_{elec}, ε_0, ε_r , A, X are the electrostatic force between the two plates, the vacuum permittivity, the relative permittivity of the dielectric, the electrode's surface area, and the separation between the proof mass respectively.

Upon external excitation of the seismic resonator the seismic mass moves to a sufficiently near distance to the high frequency resonator to cause a pull-in effect [4]. The high frequency oscillator is thereafter entrained by the seismic resonator as the latter returns to an equilibrium position. This entrainment proceeds until the electrostatic force is no longer sufficient to maintain the two electrodes stuck, and at which point and due to the hysteretic effect of electrostatic pull-in, the high frequency oscillator is released and oscillates with large amplitudes near its natural frequency ("near" because it is situated in an electrostatic potential well which results in nonlinear oscillations), these oscillations are thereafter converted to electrical energy using a separate transduction system.

It is important at this point to identify the effect responsible for converting energy into high frequency domain not as the impact caused by the pull-in, ideally that impact will carry very little energy, but by the entrainment and sudden release of the proof mass of the high frequency resonator as caused by the hysteretic behavior of the electrostatic force.

By noting the critical position at which the oscillator is released as X_{cr}, it is possible to express X_{cr} using the equilibrium of electrostatic and elastic force, i.e. $F_{elec} = F_{elastic} = KXcr$.

By assuming that the seismic resonator is driven near its natural frequency ω_0, and denoting its maximum displacement as δ. It is possible to express the equation describing the common motion of the two resonators (once they are stuck) as:

$$\ddot{X} + \frac{K + K_S}{M + M_S} X = \frac{K_S}{M + M_S}\delta \tag{2}$$

where δ is given as $\delta = Qa_0 \frac{M_S}{K_S}$, and the solution to the above differential equation is:

$$X(t)=\frac{Qa_0M_S}{K+K_S}\left(1-\cos(\omega_I t)\right) \quad (3)$$

The energy transferred from the seismic resonator to the high frequency oscillator is expressed as follows:

$$E_T=E_{Kinetic}+E_{elastic}-E_{elect}-E_{pull-in} \quad (4)$$

Where E_T represents the total energy transferred into high frequency oscillations, $E_{Kinetic}$ represents the kinetic energy of the high frequency resonator proof mass at the moment of pull-off, $E_{elastic}$ is the elastic energy stored in the spring of the high frequency oscillator at the moment of pull-off, E_{elec} represents the electrostatic potential well that the high frequency oscillator needs to overcome as it oscillates after detachment, finally $E_{pull-in}$ represents the energy lost during the pull-in phase between the two resonators. Note that E_{elec} need not be completely lost, however, equation (4) represents an upper limit for dissipation due to the electrostatic potential well and to electrical dissipation in charging the capacitance.

Finally by defining the separation dependent capacitance as $C(X)=\frac{\varepsilon_0 A}{\left(\frac{h}{\varepsilon_r}+X\right)}$ it is possible to express each of the energy terms in equation (4) as follows:

$$E_{elastic}\cong\frac{1}{2}KX_{cr}^2$$

$$E_{Kinetic}=\frac{1}{2}K\dot{X}^2$$

$$E_{elec}\cong\int_0^\infty F_{elec}dx=\frac{\varepsilon_0AV^2}{2}\left(-\frac{1}{\frac{h}{\varepsilon_r}}\right)$$

$$=-\frac{\varepsilon_0AV^2}{2\frac{h}{\varepsilon_r}}=-K\frac{h}{\varepsilon_r}X_{cr}$$

$$E_{pull-in}=\int_{BeforePull-in}^{AfterPull-in}Vidt=\int_{BeforePull-in}^{AfterPull-in}Vdq \quad (5)$$

$$\cong\int_0^\infty V^2dC(X)=-2K\frac{h}{\varepsilon_r}X_{cr}$$

where in the above sets of equations *i* represents the current provided by the constant voltage source to the variable capacitance during pull-in, and *q* is the charge.

In equation (5), $E_{pull-in}$ represents a certain upper limit on dissipation that includes mechanical energy lost upon impact, and energy dissipated in form of stored electrostatic charge in the variable capacitor formed by the two resonators' proof mass.

Therefore the total energy transferred into the high frequency resonator is given by:

$$E_T=KX_{cr}\left(\frac{X_{cr}}{2}-\frac{3h}{\varepsilon_r}\right)+\frac{1}{2}M\dot{X}^2 \quad (6)$$

By injecting the expression for the time dependent deformation into (6) and deriving with respect to time, it is possible to obtain the detachment length that maximizes E_T as:

$$X_{cr}=\frac{2Qa_0M_S}{K+K_S} \quad (7)$$

where *Q* is the quality factor of the seismic resonator, and a_0 is the excitation acceleration, this also corresponds to a zero kinetic energy, i.e. detachment takes place at the point where maximum oscillation amplitude is reached and the velocity is momentarily zero.

Finally if efficiency is defined as the ratio of the energy transferred to the high frequency resonator to that initially stored in the seismic resonator, then efficiency η is given as:

$$\eta=E_T/E_{Seismic}=E_T/\frac{1}{2}K_S\delta^2 \quad (8)$$

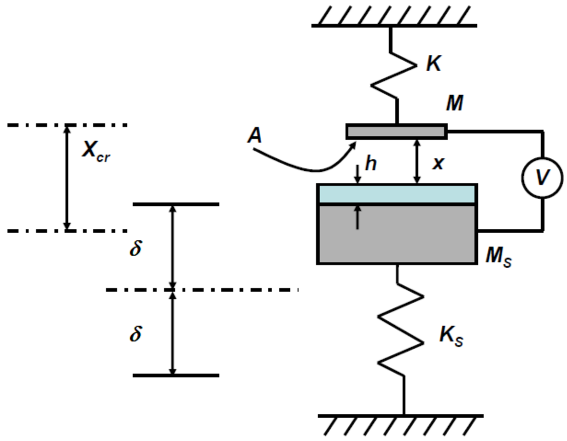

Fig. 1. Schematic representation of the MEMS frequency up-converter showing the main physical parameters and the magnitude of oscillation of the seismic mass (δ) and the detachment length (*Xcr*).

3. Simulation Results

As a case study, compliant with cardiac medical application, a system with the following properties is considered $M_S = 2\ g$, $\omega_0 = 45\ rd/s$, $K_S = 4\ N/m$, $Q = 10$, $a_0 = 0.1\ m/s^2$, $A = 10^{-6}\ m^2$, and $\varepsilon_r = 3.9$ (equivalent to that of SiO_2) resulting in $X_{cr} = 500\ \mu m$, $V = 7.5V$ for $h = 1\ \mu m$, and $V = 75V$ for $h = 10\ \mu m$.

The dynamics of the above system were obtained using a 1-dimensional finite time difference algorithm implemented in MATLAB. The time evolution of the system as shown in Fig. 2, where the graph shows the seismic resonator starting with an initial displacement amplitude $\delta = g$, where *g* is the initial gap between the two proof mass. The plot in Fig. 2 identifies the pull-in, the entrainment of the high-frequency proof mass along with the seismic resonator, and the detachment of the two electrodes (constituted by the proof mass) at the critical displacement. Once the two resonators are detached, most of the oscillation energy is transferred from the low-frequency towards the high-frequency resonator, as can be noted from the respective amplitudes of vibration after detachment.

The hysteretic loop behavior of the system is shown plotted in Fig. 3, the plot in this case is for the summation of forces acting on the proof mass in arbitrary units, plotted as a function of the normalized displacement of the high frequency resonator's proof mass.

Since the critical displacement sets the efficiency of the system, and since the point of detachment depends on the electrostatic force and hence on the applied voltage, there exist a dependence of the overall energy transfer on voltage. This dependence is shown in Fig. 4, where the efficiency is plotted as a function of the voltage (normalized to the optimal voltage).

The converted energy and the efficiency of the conversion, η, is also plotted in Fig. 5 as a function of K/K_S. Note that if the term (h/ε_r) is small compared to X_{cr}, the efficiency approaches 100% this is demonstrated in Fig. 4 when comparing the efficiencies of $h = 1\mu m$ and $h = 10\ \mu m$.

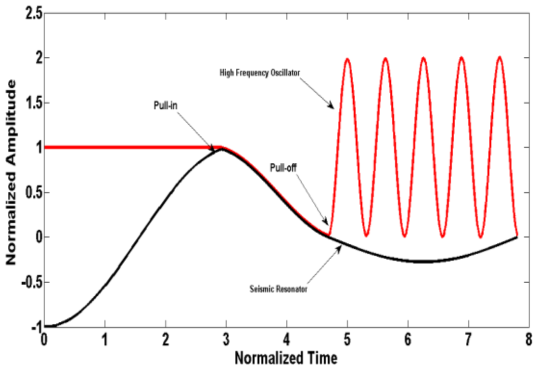

Fig. 2. Dynamics of the system where the seismic resonator (black) starts with an initial displacement, and the high frequency resonator (red) starts from rest, plotted as a function of time (normalized with respect to the period of the seismic resonator), the plot identifies the pull-in, the entrainment, the pull-off, and the post detachment oscillations showing the transfer of energy from the seismic resonator to the high frequency resonator.

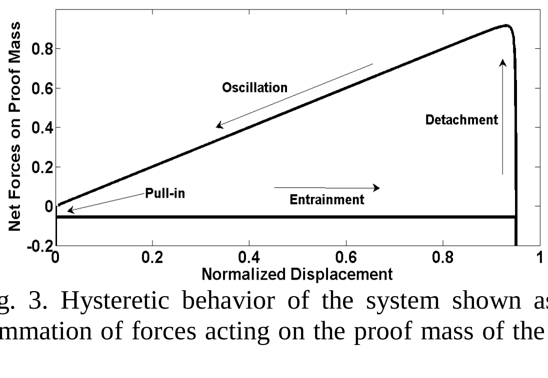

Fig. 3. Hysteretic behavior of the system shown as the summation of forces acting on the proof mass of the high frequency resonator plotted as a function of displacement, the diagram identifies, the pull-in, the entrainment, the release, and the eventual oscillations.

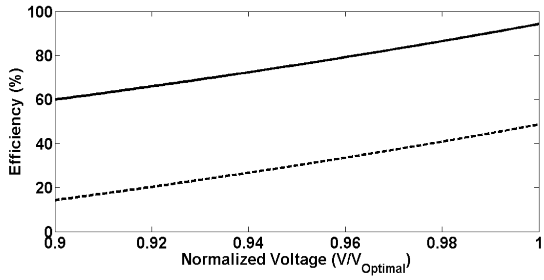

Fig. 4. Sensitivity of the low to high frequency conversion on variation in voltage normalized with respect to optimal voltage.

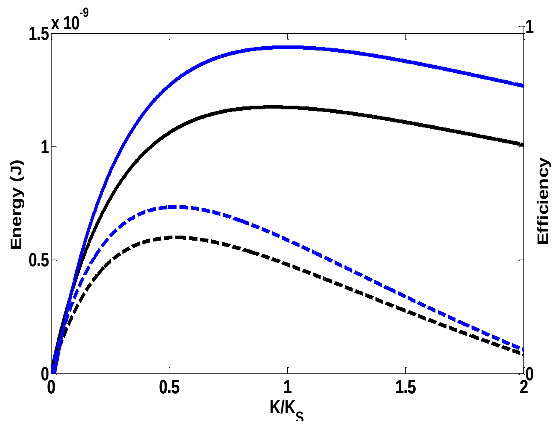

Fig. 5. Graphical representation showing the energy (black lines), and efficiency (blue lines) for h = 1μm (solid line), and h = 10 μm (dashed lines).

Acknowledgments

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