Analysis of Maximum Channel Capacity for Linear Antennas Using Method of Moments

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1. Introduction

Multi-antenna technique, generally known as MIMO (Multi Input Multi Output), can contribute to the improvement of the wireless communication performances by using diversity techniques and spatial-multiplexing. Due to a great need for broadband mobile communication, more compact terminals employing such multi-antenna technique are being required. However, it is commonly known that setting the antenna length and the antenna spacing to less than $1/4\lambda$ and $1/2\lambda$ respectively makes antenna performance degraded in multi-antenna systems. Therefore, it is necessary to take the antenna size into account.

As a fundamental limit of antenna performance, the Chu limit is widely recognized. This evaluation method is derived by assuming the energy is homogeneously distributed inside imaginary sphere including an intended antenna, which is called Chu's sphere. By treating this as an electric circuit, a lower bound of Q factor is defined [1]. Recently, there are various kinds of debates about fundamental boundary on MIMO channel capacity derived by associating electromagnetic theory with information theory [2]. However, these arguments deal with equivalent electromagnetic wave from enclosed region including imaginary sources. Accordingly, it is difficult to apply these limitations to evaluation of material linear antenna performance.

In this paper, a maximum channel capacity between two linear antennas is revealed using MoM (Method of Moments). To evaluate the potential channel capacity, we calculated the S-parameter among the segments of MoM by assuming each segment as small dipole. By using SVD (Singular Value Decomposition) of this S-parameter based on MoM, the available eigenmodes can be identified. The optimal excitation for the highest capacity can be estimated by using this process. In the following part of this paper, an approach of the channel capacity analysis by using MoM and numerical results are reported in detail.

2. Definition of MoM-based channel

At first, a couple of dipole antennas in Fig. 1 are divided into segments by using MoM. These segments obtained in MoM can be regarded as an array of the small dipole antennas. When a dipole is divided into N small dipoles, two facing dipole antennas can be recognized as $N \times N$ multi-antennas systems. Figure 2 depicts the $N \times N$ multi-antennas systems. By transforming impedance matrix of these multi-small antennas into S-parameter, we obtain $\boldsymbol{S}_{TT}$ and $\boldsymbol{S}_{RR}$, which are matrices representing reflection to Tx and Rx respectively. Also, $\boldsymbol{S}_{RT}$ and $\boldsymbol{S}_{TR}$ which are matrices representing incidence power between Tx and Rx can be obtained. Characteristic impedance Z_0 used in parameter transformation is fixed at 50Ω. By using SVD, $\boldsymbol{S}_{RT}$ corresponding to a channel matrix can be expressed as

$$\boldsymbol{S}_{RT} = \boldsymbol{U}_{RT}\boldsymbol{\Lambda}_{RT}\boldsymbol{V}_{RT}^{H} \qquad (1)$$

where $\boldsymbol{V}_{RT} = [\boldsymbol{v}_{RT,1}, \cdots, \boldsymbol{v}_{RT,N}]$ and $\boldsymbol{U}_{RT} = [\boldsymbol{u}_{RT,1}, \cdots, \boldsymbol{u}_{RT,N}]$ represent transmitting and receiving weight vectors, respectively, and $\boldsymbol{\Lambda}_{RT} = \mathrm{diag}(\sqrt{\lambda_{RT,1}}, \cdots, \sqrt{\lambda_{RT,N}})$ is singular matrix. By using these values, it became possible that the power distribution among the eigenmodes can be independently controlled. We define this scheme as MMT (Multi Mode Transmission). And by using eigen weight obtain from (1), reflection to Tx and Rx for each eigenmode are defined as,

$$\begin{cases} |\Gamma_{TT,i}|^2 = \| \boldsymbol{S}_{TT} \boldsymbol{v}_{RT,i} \|_2^{\ 2} \\ |\Gamma_{RR,i}|^2 = \| \boldsymbol{u}_{RT,i}^H \boldsymbol{S}_{RR} \|_2^{\ 2} \end{cases} \qquad (i = 1,2 \cdots N) \tag{2}$$

where $\|\cdot\|_2$ represents Euclidean norm. If we assume that ideal matching is possible, the eigenvalue with the ideal matching, i.e., matched eigenvalue, is written as

$$\lambda'_{RT,i} = \frac{\lambda_{RT,i}}{(1-|\Gamma_{TT,i}|^2)(1-|\Gamma_{RR,i}|^2)}. \qquad (i = 1,2 \cdots N_m) \tag{3}$$

Where N_m is defined as the number of the matchable eigenmodes. This means $N - N_m$ eigenmodes are not matchable since their reflections are almost 1 and division by zero occurs in (3). In order to avoid this problem, we have a constraint, which is explained later in section 3.

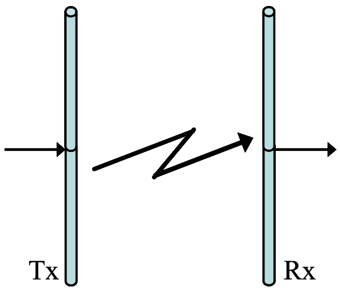


Fig. 1 Dipole antenna.

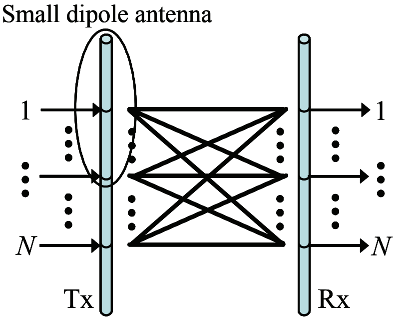


Fig. 2 $N \times N$ small dipoles model.

3. Numerical results

3.1 Eigenmode analysis

Figure 3 shows a model to analyze channel characteristics. In this model, two parallel dipole antennas are placed and we assume near field communication model which have no scatters between Tx and Rx. The parameters used in this analysis are indicated in Table 1. Figure 4 and 5 show amplitude and phase of eigen-weight of the first, second and third eigenmodes. Figure 6, 7 and 8 show eigenvalue distribution in dB, reflection for each eigenmode to Tx and Rx side, respectively. As these figure shows, the first eigenmode appears outstandingly and this means that only single mode is actually available for half-wave length dipole antenna.

Table 1 Parameter of analysis.

Antenna length, l	0.5λ
Segment length, Δl	0.05λ
Radius, a	0.001λ
Distance, d	1.0λ

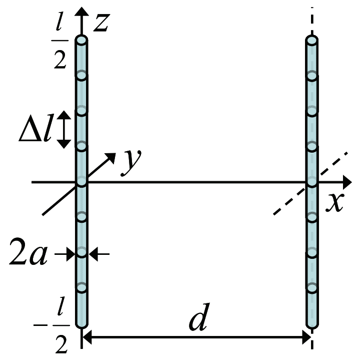
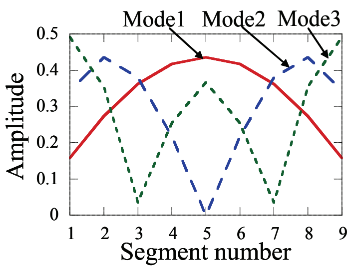
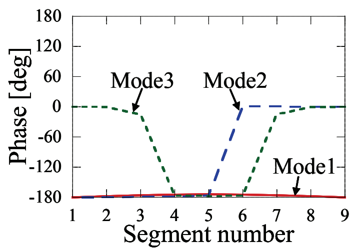


Fig. 3 Analysis model.

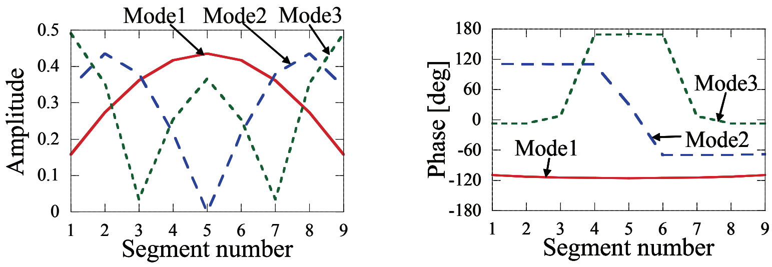


(a) Amplitude



(b) Phase

Fig. 4 Transmitting weight.



(a) Amplitude (b) Phase

Fig. 5 Receiving weight.

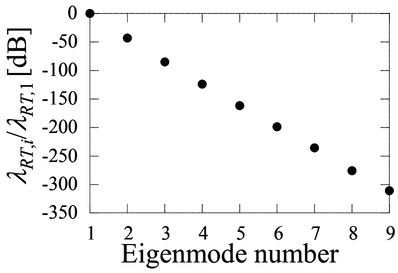


Fig. 6 Eigenvalue distribution.

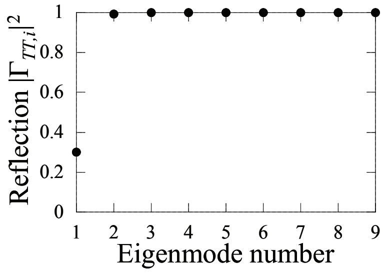


Fig. 7 Reflection @ Tx.

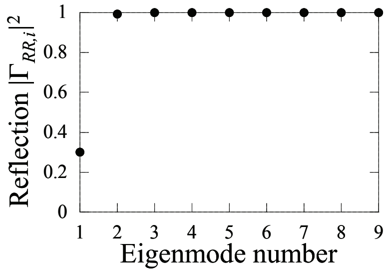


Fig. 8 Reflection @ Rx.

3.2 Channel capacity without matching

Next, we analyze the variation of the channel capacity with extension of the antenna length. Then, the antenna length l is extended from 0.1λ to 2.0λ at 0.1λ intervals. The channel capacity with MMT is calculated by applying WF (Water Filling Algorism) to each eigenmode. For comparisons, both of the capacities with SMT (Single Mode Transmission) and with dipole antennas are calculated. Where, SMT uses only the first eigenmode. However, matching is unconsidered. The noise power is determined by SNR with half-wave length dipole antenna. The signal power changes depending on the antenna length. And total transmitting power is set to 1W.

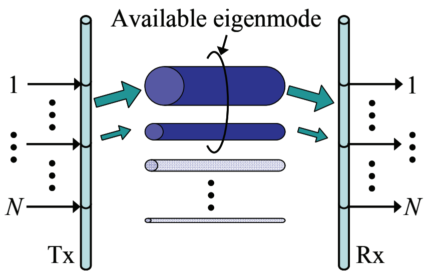


Fig. 9 Multi Mode Transmission.

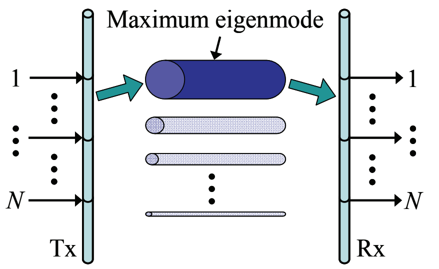


Fig. 10 Single Mode Transmission.

Figure 11 and 12 show NAE (the Number of Available Eigenmode) and channel capacity versus antenna length, respectively. NAE represents the number of used eigenmodes based on WF. As Fig. 11 shows, it is found that NAE increases step-by-step with extending the antenna length and increasing SNR. Figure 12 shows channel capacities of dipole antenna and SMT reach a peak at l is 0.5λ. On the contrary, the channel capacity with MMT increases as the antenna length gets to longer.

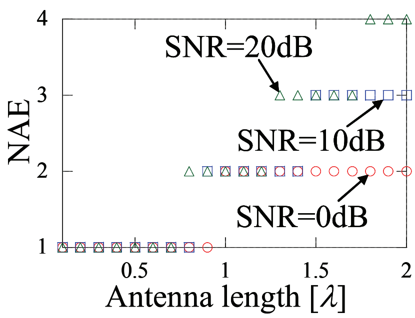
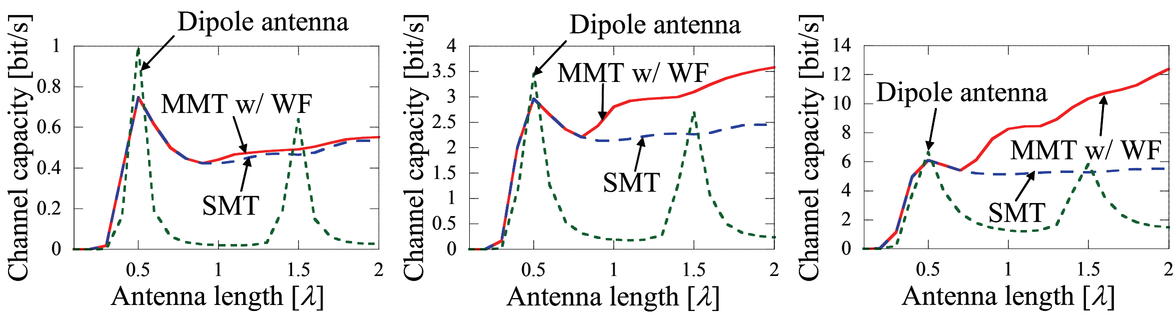


Fig. 11 NAE versus antenna length.



(a) SNR=0dB (b) SNR=10dB (c) SNR=20dB
Fig. 12 Channel capacity versus antenna length.

3.3 Transmission power with matching

We now calculate channel capacity taking ideal matching for each eigenmode into account. Here, to avoid division by zero for (3), we introduce a threshold value of reflection. The threshold value is set to that of the reflection of the first eigenmode at l = 0.1λ. That is, the eigenmodes with higher reflection than this threshold are discarded. Figure 13 shows transition of matched eigenvalue derived from (3). There are the first to fifth eigenmodes. However, eigenvalue of non-matchable eigenmode at particular antenna length is not plotted. In Fig. 13, it is found that the number of matchable eigenmodes increases with extension of the antenna length as well as their eigenvalue. Here, the transposition of eigenvalue between mode1 and mode2 occurs at l = 1.1λ. So we newly defined the order of the eigenmodes with considering matched eigenvalues. Figure 14 shows transmission powers of SMT and dipole antennas with consideration ideal matching. It can be seen that, the transmission powers of SMT and dipole antenna demonstrate a similar increment tendency in the region $l \leq 1.1\lambda$. On the other hand, SMT constantly exceeds dipole antenna in the region $l > 1.1\lambda$. This means that SMT with the optimal antenna excitation can realize the channel capacity exceeding that of the dipole antenna.

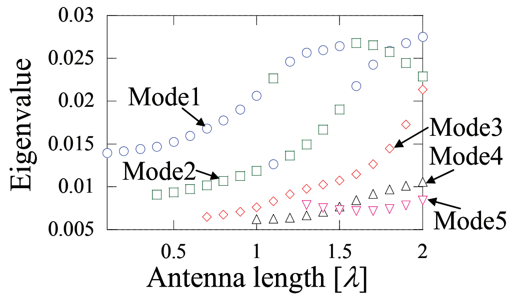


Fig. 13 Transition of eigenvalue.

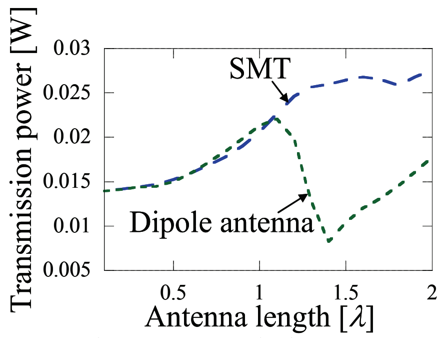


Fig. 14 Transmission power with matching.

4. Conclusion

In this study, we analyzed the channel capacity based on S-parameter which was calculated with MoM. Numerical results showed that the channel capacity with MMT increases with extending the antenna length. And it was revealed that, for the antenna whose length is over 1.1λ, the channel capacity with SMT constantly exceeds that with dipole antenna with consideration of ideal matching. Finally, these results in this paper proved that the optimal antenna excitation will provide the channel capacity over that of the conventional center-fed dipole antenna.

Acknowledgement

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[2] M. D. Migliore, “On Electromagnetics and Information Theory,” IEEE Trans. Antennas and Propagat., Vol.56, No.10, pp.3188-3200, Oct. 2008