# Cylindrical Multiuser Beam-Free Active Phased Array and Comparison with the Standard Multisector Antennas for Mobile Communication


#I. Yu. Sergeev
IZMIRAN, Russian Academy of Sciences
IZMIRAN, Troitsk, Moscow region, 142190, Russia, fje@mail.ru



## Abstract

A multiuser beam-free cylindrical adaptive active array is considered. The comparison of the standard multisector antennas with the beam-free array consisting of the same number of elements shows that the capacity (number of users operating on the same frequency) increases up to 2 times as against 6-sector antenna.



**Keywords :** Antennas, Phased Array, Beam-Free, Wireless Communication, Sector Antennas


## 1. Antenna Geometry and Main Formulas

Let us consider cylindrical active phased array consisting of $N$ elements ($\beta_n \neq \beta_i$ if $n \neq i$) placed in the same plane and $M$ signal sources determined by their signal complex amplitudes $A_m(t)$ and complex polarization vectors $\mathbf{P}_{Am}$ (Figure 1). The sources operate on the same frequency and are placed in the Fraunhofer zone at the angles $\varphi_m$ and $\vartheta_m$ correspondingly ($\varphi_m \neq \varphi_i$ if $m \neq i$). $A_m(t) = a_m(t)\exp(\mathrm{j}\,\xi_m(t))$, $a_m(t)$ are the real amplitudes, $\xi_m(t)$ are the phases, j is the imaginary unit. We assume that $\varphi_m$ are known. There are many possibilities to find $\varphi_m$ by the antenna itself (see, for example, [1]) and this problem is not considered in the paper.

The complex amplitude $S_n$ of the excitation of the antenna element $n$ is defined by the formula:

$$S_n(t) = \sum_{m=1}^{M}\left[k_m \cdot \left(\mathbf{P}_{Am} \cdot \mathbf{P}^*_{Fn}(\varphi_m - \beta_n, \vartheta_m)\right) \times q_n(\varphi_m - \beta_n, \vartheta_m) \cdot A_m(t) \cdot \exp\left(\mathrm{j}\,r\cos(\varphi_m - \beta_n)\right)\right], \qquad (1)$$

where $k_m$ are proportionality coefficients taking into account such constant parameters as distance between the signal sources and the antenna, etc., $\mathbf{P}_{Fm}$ are the complex polarization vectors of the antenna elements, $\left(\mathbf{P}_{Am} \cdot \mathbf{P}^*_{Fn}\right)$ are the polarization transmission coefficients, $q_n$ are the radiation patterns of the antenna elements taking into account the interaction between the other elements and structural of the antenna, $r = 2\pi R/\lambda$, $R$ is the cylinder radius. We will assume that the vectors $\mathbf{P}_{Fm}$ are collinear and do not depend on the fist argument. It takes place, for example, when antenna elements are identical and have axial symmetric radiation pattern. In such case we can simplify (1):

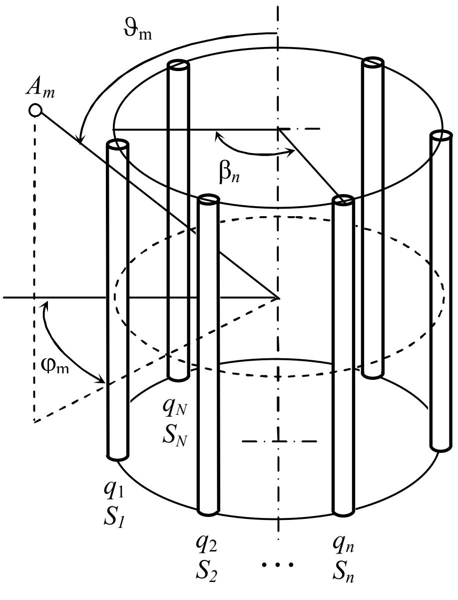


Figure 1: Antenna geometry.

$$S_n = \sum_{m=1}^{M} A_m(t) K_m G_n(\varphi_m, \vartheta_m). \qquad (2)$$

$K_m = k_m\left(\mathbf{P}_{Am} \cdot \mathbf{P}^*_{Fn}(\varphi_m - \beta_n, \vartheta_m)\right)$ and does not depend on the time and number of antenna element, $G_n(\varphi_m, \vartheta_m) = q_n(\varphi_m - \beta_n, \vartheta_m)\exp\left(\mathrm{j}\,r\cos(\varphi_m - \beta_n)\right)$. Further, we will usually omit the second argument in $G_n$

meaning $G_n(\varphi_m) = G_n(\varphi_m, \vartheta_m)$.

We will require that the antenna will be able to restore exactly signals up to time independent factor i.e. to find $KA_m(t)$, where $K$ is a constant. This requirement seems excessive but as it was shown in the paper [1] such approach results in the highest capacity in most cases. Eq. (2) shows that the exact signal restoration is feasible when $N \geq M$ and

$$rank \begin{bmatrix} G_1(\varphi_1) & \dots & G_1(\varphi_M) \\ . & \dots & . \\ G_N(\varphi_1) & \dots & G_N(\varphi_M) \end{bmatrix} = M . \quad (3)$$

Also (2) shows that the solution will be in the linear function class i.e. $A_m$ can be obtained as a linear combination of the element excitations: $A_m K_m = C_{m1}S_1 + C_{m2}S_2 + \dots + C_{mN}S_N$, where $C_{mn}$ are constants. It means that to analyze array characteristics (including noise parameters) we can use the standard radiation pattern technique and the considered in the paper method is applicable for transmission as well as for receiving. Therefore further we will consider the receiving antenna meaning that the transmission characteristics are the same. Notice, the combined equations (2) is always consistent due to problem formulation.

Let us consider the radiation pattern in the general linear form:

$$D(\varphi, \vartheta) = c_1 G_1(\varphi, \vartheta) + c_2 G_2(\varphi, \vartheta) + \dots + c_N G_N(\varphi, \vartheta) . \quad (4)$$

If the condition (3) is true the above mentioned requirement of exact signal $A_i$ restoration is equivalent to the condition $D(\varphi, \vartheta) = 0$ in the points $\varphi = \varphi_m$ $(m \neq i)$ and in such case $D(\varphi_i, \vartheta_i) = A_i K_i$. Of course for different $i$ the coefficients $c_1, \dots, c_N$ (and so functions $D(\varphi, \vartheta)$) are different too. We will mark the radiation patterns by subindex: $D_i(\varphi_i, \vartheta_i) = A_i K_i$. It is simple to show that in such case $D_m(\varphi, \vartheta) = C_{m1}G_1(\varphi, \vartheta) + C_{m2}G_2(\varphi, \vartheta) + \dots + C_{mN}G_N(\varphi, \vartheta)$. Thus the problem of signal restoration is the problem of finding the constants $C_{mn}$ corresponding to the above mentioned condition.

The expressions $\exp(j r \cos(\varphi - \beta_n))$, $n$=1...$N$ are linearly independent functions of argument $\varphi$. So the functions $G_n(\varphi, \vartheta)$ of argument $\varphi$ are linearly independent too except special cases that we do not consider in this paper. It means that for any $\{\varphi_1, \varphi_2, \dots, \varphi_{i-1}, \varphi_{i+1}, \dots, \varphi_N\}$ we can always find $\{C_{i1}, C_{i2}, \dots, C_{iN}\}$ so that $D_i(\varphi, \vartheta) = 0$ in these points and $\sum_{n=1}^{N} |c_n| \neq 0$. The constants $C_{mn}$ can be calculated by the formula:

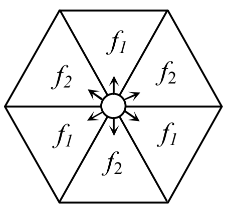


Figure 2: The pattern of 6-sectors antenna using frequency reuse factor 1/2.

$$\begin{bmatrix} C_{i1} \\ C_{i2} \\ . \\ . \\ . \\ . \\ C_{i,N-1} \end{bmatrix} = -c_N \begin{bmatrix} G_1(\varphi_1) & G_2(\varphi_1) & \dots & G_{N-1}(\varphi_1) \\ G_1(\varphi_2) & G_2(\varphi_2) & \dots & G_{N-1}(\varphi_2) \\ . & . & . & . \\ G_1(\varphi_{i-1}) & G_2(\varphi_{i-1}) & \dots & G_{N-1}(\varphi_{i-1}) \\ G_1(\varphi_{i+1}) & G_2(\varphi_{i+1}) & \dots & G_{N-1}(\varphi_{i+1}) \\ . & . & . & . \\ G_1(\varphi_N) & G_2(\varphi_N) & \dots & G_{N-1}(\varphi_N) \end{bmatrix}^{-1} \begin{bmatrix} G_N(\varphi_1) \\ G_N(\varphi_2) \\ \dots \\ G_N(\varphi_{i-1}) \\ G_N(\varphi_{i+1}) \\ \dots \\ G_N(\varphi_N) \end{bmatrix} . \quad (5)$$

The paper length does not allow to expound all the mathematical manipulations and discuss all the practically interesting characteristics. In this paper we will consider only the most specific properties and compare them with the properties of other types of antennas. The comparison with multibeam cylindrical array is shown in Table 1. Note that the most part of the data presented in this table are rough estimation and can be used only for qualitative comparison. The exactly calculated data for the 7 elements array are presented in the next section.

Table 1: The comparison of multibeam and beam-free cylindrical arrays.

| | Multibeam array [1] | Beam-free array |
|---|---|---|
| Minimal number of frequency channels needed to provide continuous coverage area for 1 base station | 2 | 1 |
| Maximal number of users in case of using 2 frequency channels for multibeam antenna and 1 channel for beam-free antenna | $N/2$ - $N$ [2] | $N$ |
| Maximal number of users ($N_f$ – number of frequency channels, $R_f$ – frequency reuse factor, $R_f \le 1$) | $\frac{N \cdot N_f \cdot R_f}{2}$ | $N \cdot N_f$ |
| Minimal distance between users operating at the same frequency determined by the beam (sector) width | 360°/$N$ | 0° [3] |
| Minimal distance between users operating at the same frequency determined by 13dB level of the signal-to-interference ratio | 180°/$N$ | 0° [3] |

$N$ is a number of array elements.

[1] according to antenna construction considered in the paper [1]

[2] depends on frequency reuse factor

[3] In theory the beam-free method can provide any required signal-to-noise ratio (SNR) for any users' position because the method assumes an exact signal restoration (i.e. the signal-to-interference ration (SIR) is equal to infinity). But Eqs. (5) and (4) yield that in the limit of small $\Delta\varphi$ ( $\Delta\varphi$ is an angle between users) the directive gain is proportional to $\Delta\varphi^2$ . So to provide required SNR the signal source power has to be inversely proportional to $\Delta\varphi^2$ . Refer to the notes on Table 2 for the SIR and directive gain definitions used here.

## 2. Comparison with the Standard 6-Sector Antenna for Base Station

Let us compare a beam-free cylindrical array with a typical 6-sector antenna using frequency reuse factor 1/2 (Figure 2). For simplicity, we will consider the problem in a 2-dimensional formulation. The analysis of Eq. (3) shows that a beam-free equispaced array consisting of 6 elements is able to select signals not for every users' position. (The example is shown in the Figure 3.) To avoid this problem the array has either to be not equispaced or to consist of 7 elements. We will consider the second case, the equispaced cylindrical array consisting of 7 elements with axialsymmetric radiation patterns so that $\beta_n = 360^\circ(n-1)/N$ . The results of comparison are presented in the Table 2.

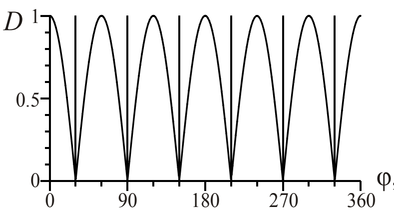


Figure 3: The example of users' position (vertical lines) and corresponding radiation pattern when a beam-free equispaced array consisting of 6 elements with axialsymmetric radiation patterns cannot serve the users.

(If $N$ is an even number and the antenna is equispaced the condition (3) is not true for the users' position $\varphi_m$=180°/N+360°(m-1)/N. The radiation pattern calculated by (5) and (4) is equal to zero over the all $\varphi_m$.)

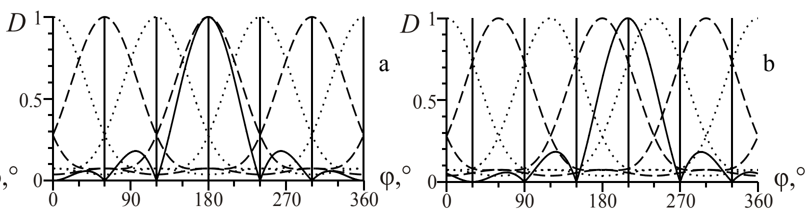


Figure 4: Radiation patterns of a typical 6-sector antenna and the considered 7-elements array.

dash line – radiation pattern of the sectors operating at the first frequency

dot line – radiation pattern of the sectors operating at the second frequency

solid line – radiation pattern of the considered array

vertical lines – users' positions

Table 2: The comparison of the beam-free cylindrical array consisting of 7 elements with a typical 6-sector antenna.

| Number of users | 7 | | 6 | | 6 | | 2 | | 2 | | 2 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Users' position | See [1] | | Figure 4a | | Figure 4b | | $\Delta\varphi = 60^\circ$ [2] | | $\Delta\varphi = 36^\circ$ [3] | | $\Delta\varphi = 20^\circ$ | |
| Characteristic | SIR, dB | DG, dB | SIR, dB | DG, dB | SIR, dB | DG, dB | SIR, dB | DG, dB | SIR, dB | DG, dB | SIR, dB | DG, dB |
| Typical 6-sector antenna (2 frequencies) | n/a | n/a | 19.6 | 7.2 | 17.5 | 4.6 | 19.1 | 4.6 | 13 | 1.9 | n/a | n/a |
| Beam-free array, $r$=3.58 [4] (1 frequency) | ∞ | 5.8 | ∞ | 7.3 | ∞ | 6.7 | ∞ | 6.7 | ∞ | 5.9 | ∞ | 3.5 |
| Beam-free array, $r$=1 [5] (1 frequency) | ∞ | 4.7 | ∞ | 8.1 | ∞ | 8.1 | ∞ | 8.1 | ∞ | 5.7 | ∞ | 0.4 |

SIR is signal-to-interference ratio. SIR $= D^2(\varphi_i) / \sum_{m \neq i} D^2(\varphi_m)$, where the summation includes all the sources operating on the frequency of wanted signal. In the case of the multisector antenna $D(\varphi)$ is a radiation pattern of the sector where user $\varphi_i$ operates. In the case of the beam-free array $D(\varphi)$ means $D_i(\varphi)$. $\varphi_i$ is position of wanted signal. See also the note [3] to the Table 1.

DG is the directive gain in the $\varphi_i$ direction. DG=$360° D^2(\varphi_i) / \int_0^{360^\circ} D^2(\varphi) d\varphi$; definitions for $D(\varphi)$ and $\varphi_i$ are as above.

n/a means that antenna cannot serve such number or such position of users.

[1] 7 randomly placed users: $\varphi_1$=7°, $\varphi_2$=76°, $\varphi_3$=143°, $\varphi_4$=186°, $\varphi_5$=249° (wanted signal), $\varphi_6$=273°, $\varphi_7$=332°.

[2] $\Delta\varphi = 60^\circ$ is a minimal distance between users operating at the same frequency determined for the 6-sector antenna by the sector width.

[3] $\Delta\varphi = 36^\circ$ is a minimal distance between users operating at the same frequency determined for a typical 6-sector antenna by 13dB level of signal-to-interference ratio.

[4] $r$=3.58 is an optimal value calculated with the method obtained in [1].

[5] The analysis yields that if $r \leq 2$ the direction pattern of beam-free cylindrical array weakly depends on $r$ and so the case $r$=1 shows the antenna properties for small $r$.

## 3. Conclusion: the Main Properties of a Beam-Free Cylindrical Array

1. The capacity (maximal number of users operating on the same frequency) is equal to the number of the antenna elements.
2. The comparison of antennas consisting of the same number of elements shows that the capacity of beam-free antenna is up to 2 times higher than the capacity of a standard 6-sector antenna using frequency reuse factor 1/2 and up to 3 times higher than the capacity of 3-sector antenna using frequency reuse factor 1/3.
3. The detailed comparison with the 6-sector antenna shows that in the most cases such characteristics of the beam-free array as SNR, SIR, directive gain, etc., are higher.
4. Theoretically in case of the beam-free array the minimal distance between the users operating on the same frequency is equal to 0. Practically this distance is limited by the maximal transmitter power. In any case for the same transmitter power this parameter is significantly smaller for the beam-free array than for the multisector antenna.
5. Beam-free array is easily flexible. To increase the capacity it is just sufficient to add the antenna elements without construction changing.
6. The presented antenna increases capacity of CDMA networks as well as FDMA.
7. The beam-free method needs special procedure to determine users' position and more precision amplitude-phase manipulations.

## References

[1] I. Yu. Sergeev, G. A. Karpunin, "Final Report on Project "Torus", GEOSCAN, Moscow, 2007.

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