



The State of the Art of Verified Numerical Computation –A Tool for Computer Assisted Proofs of Nonlinear Problems–

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Abstract

Now, it is well-known that the verified numerical computation is one of most useful tool for computer assisted proofs of nonlinear problems. For instance, the existence of Lorenz's attractor is proved using verified numerical computation. In this talk, the state of the art of the verified numerical computations are outlined. More concretely, answers to the following questions are presented:

1. At present, what method is fastest to prove the existence of solutions for huge, say one hundred million, dimensional linear simultaneous equations?
2. How about for the ill-conditioned problems?
3. How about eigenvalue or singular value problems?
4. Is it possible to make the machine interval arithmetic independent of machine, *i.e.*, portable?
5. What kind of methods are suited for proving nonlinear problems?

Of course, answers to these problems are different by authors. In this talk, one of the stresses is on the use of error free transformations of floating point arithmetic. Recent remarkable progress of verified numerical computations is to make some numerical computations information-preserving processes without losing speed of computations. Since every numerical problem has its own difficulty, or condition number, fast numerical methods with adoptively changing accuracy are needed for solving it. The error free transformation is one of most useful tools for this.