2-D DOA Estimation with Arbitrary Planar Antenna Array Using MS Technique in Combination with EM Algorithm

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Abstract

This paper proposes the 2-D DOA estimation method with an arbitrary planar antenna array using MS technique in combination with EM Algorithm, and shows good performance of the proposed method in estimation accuracy and computation time.

Keywords: 2-D DOA, Manifold Separation(MS), Root-MUSIC, planar array antenna, EM algorithm

1. Introduction

It is necessary to understand radio propagation structures and further consider signal recovering techniques in mobile communications. For the purpose, it is most effective to estimate DOAs (directions of arrival) of individual incoming waves with array antennas. Also, in radar systems, it is required to discriminate the desired signal from interference. Recently, DOA estimation using EM (expectation-maximization) algorithm [1] based on the maximum-likelihood (ML) approach receives much attention. It is because the EM algorithm remains stable in scenarios involving small numbers of snapshots, coherent signals and low SNR.

However, the ML approach generally has the high computational complexity caused by optimization of the likelihood function. As is often the case with 2-D DOA estimation, increased estimation parameters require too much computation time. Therefore, this paper deals with a solution which improves the estimation performance by using EM algorithm in combination with non-searching algorithm such as Root-MUSIC [2]. Since there is a restriction of array structure in using Root-MUSIC, we apply the 2-D MUSIC using Manifold Separation (MS) technique [3] [4] with an arbitrary planar antenna array to the EM algorithm. Through computer simulation, we show that 2-D MUSIC using MS technique provides improved performance in terms of computation time and estimation accuracy.

2. Signal Model and DOA Estimation

2.1 Signal Model

Consider that the array antenna used for 2-D DOA estimation is an N-element planar array shown in Figs.1 and 2, and also that it receives L ($L < N$) narrow-band waves whose respective DOAs are (θ_1, ϕ_1), $(\theta_2, \phi_2), \ldots, (\theta_L, \phi_L)$ and complex amplitudes are $s_1(t)$, $s_2(t), \ldots,$ $s_L(t)$. When the array response vector (mode vector) of the lth incoming wave is given by $\boldsymbol{a}(\theta_l, \phi_l)$ $(l = 1, 2, \ldots, L)$, the array input vector $\boldsymbol{x}(t)$ can be expressed as

$$\boldsymbol{x}(t) = \sum_{l=1}^{L} \boldsymbol{a}(\theta_l, \phi_l)\, s_l(t) + \boldsymbol{n}(t) = \boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{n}(t) \tag{1}$$

$$\boldsymbol{A} = [\boldsymbol{a}(\theta_1, \phi_1),\ \boldsymbol{a}(\theta_2, \phi_2), \ldots, \boldsymbol{a}(\theta_L, \phi_L)],\ \boldsymbol{s}(t) = [s_1(t),\ s_2(t), \ldots, s_L(t)]^T \tag{2}$$

where $\boldsymbol{A}$ and $\boldsymbol{s}(t)$ are called the array response matrix (mode matrix) and the signal vector, respectively, and $\boldsymbol{n}(t)$ is the internal additive noise vector.

2.2 2-D MUSIC Using MS Technique

The Manifold Separation (MS) technique is the method of modeling the array response vector as the product of a sampling matrix, which depends only on the sensor array configuration, and two Vandermonde structured vectors. Here, we approximate the array response vector by using 2-D DFT, which is modeled as

$$[\boldsymbol{a}(\theta,\phi)]_n = \sum_{m_a=-\frac{M_a-1}{2}}^{\frac{M_a-1}{2}} \sum_{m_e=-\frac{M_e-1}{2}}^{\frac{M_e-1}{2}} G_n(m_a, m_e)\, e^{jm_a\phi} e^{jm_e\theta} + \varepsilon(M_a, M_e) \tag{3}$$

$$= vec\{\boldsymbol{G}_n\}^T \boldsymbol{d}(\theta,\phi) + \varepsilon(M_a, M_e) \tag{4}$$

where $\boldsymbol{G}_n$ is 2-D Fourier coefficient, M_a and M_e are the number of Fourier coefficients considered in θ and ϕ, respectively, and $\varepsilon(M_a, M_e)$ is the modeling errors due to truncation of the 2-D Fourier series. Also, $vec\{\boldsymbol{G}_n\}$ stacks the matrix into a column vector and $\otimes$ represents the Kronecker product. Moreover, $\boldsymbol{d}(\theta,\phi)$ is composed of the following Vandermonde structured vectors

$$\boldsymbol{d}(\theta) = \left[e^{-j\frac{M_e-1}{2}\theta}, \ldots, 1, \ldots, e^{j\frac{M_e-1}{2}\theta}\right]^T \in \mathbb{C}^{M_e\times 1},\ \boldsymbol{d}(\phi) = \left[e^{-j\frac{M_a-1}{2}\phi}, \ldots, 1, \ldots, e^{j\frac{M_a-1}{2}\phi}\right]^T \in \mathbb{C}^{M_a\times 1} \tag{5}$$

$$\boldsymbol{d}(\theta,\phi) = \boldsymbol{d}(\phi) \otimes \boldsymbol{d}(\theta) \in \mathbb{C}^{M_e M_a \times 1} \tag{6}$$

As a result, the array response vector can be expressed as

$$\boldsymbol{a}(\theta,\phi) = \boldsymbol{\Gamma}\boldsymbol{d}(\theta,\phi),\ \boldsymbol{\Gamma} = [vec\{\boldsymbol{G}_1\}, \ldots, vec\{\boldsymbol{G}_N\}]^T \in \mathbb{C}^{N\times M_e M_a} \tag{7}$$

Thus, we can have the 2-D MUSIC cost function which is given by

$$S_{\text{MUSIC}}(\theta,\phi) = \left(\boldsymbol{d}^H(\theta,\phi)\boldsymbol{\Gamma}^H \boldsymbol{E}_n \boldsymbol{E}_n^H \boldsymbol{\Gamma}\boldsymbol{d}(\theta,\phi)\right)^{-1} \tag{8}$$

where $\boldsymbol{E}_n$ is the eigenvectors spanning the noise subspace. If estimated elevation and azimuth angles are given by $\hat{\theta}$ and $\hat{\phi}$, respectively, eq.(8) can be divided into the following two 1-D MUSIC cost functions:

$$S_{\text{MUSIC}}(\theta,\hat{\phi}) = \left[\boldsymbol{d}^H(\theta)\left\{\left(\boldsymbol{d}(\hat{\phi}) \otimes \boldsymbol{I}_{M_e}\right)^H \boldsymbol{\Gamma}^H \boldsymbol{E}_n \boldsymbol{E}_n^H \boldsymbol{\Gamma}\left(\boldsymbol{d}(\hat{\phi}) \otimes \boldsymbol{I}_{M_e}\right)\right\}\boldsymbol{d}(\theta)\right]^{-1} \tag{9}$$

$$S_{\text{MUSIC}}(\hat{\theta},\phi) = \left[\boldsymbol{d}^H(\phi)\left\{\left(\boldsymbol{I}_{M_a} \otimes \boldsymbol{d}(\hat{\theta})\right)^H \boldsymbol{\Gamma}^H \boldsymbol{E}_n \boldsymbol{E}_n^H \boldsymbol{\Gamma}\left(\boldsymbol{I}_{M_a} \otimes \boldsymbol{d}(\hat{\theta})\right)\right\}\boldsymbol{d}(\phi)\right]^{-1} \tag{10}$$

which allow us to employ 1-D root-MUSIC. However, eqs.(9) and (10) requires difficult pairing of roots when multiple waves are incoming. Therefore, we try to use this algorithm with the EM algorithm.

2.3 EM Algorithm

The EM algorithm is the method based on maximum-likelihood estimation [1]. In the EM algorithm, iterative calculation is carried out for getting DOAs from unobservable complete data rather than observed incomplete data $\boldsymbol{x}(t)$. Each iteration consists of two steps: E-step (expectation) which approximates the complete data by conditional expectation and M-step (maximization) which maximizes the likelihood of the complete data. The mth iteration of the EM algorithm proceeds as follows.

E-step : The maximum likelihood estimate of complete data $\boldsymbol{x}_l^{(m)}(t)$ is calculated by using the 2-D DOA estimate $\left(\theta_l^{(m)}, \phi_l^{(m)}\right)$ and the complex amplitude estimate $s_l^{(m)}(t)$ of the lth wave, which is given by

$$\boldsymbol{x}_l^{(m)}(t) = s_l^{(m)}(t)\boldsymbol{a}(\theta_l^{(m)}, \phi_l^{(m)}) + \beta\left[\boldsymbol{x}(t) - \boldsymbol{A}^{(m)}\boldsymbol{s}^{(m)}(t)\right] \quad (l = 1,\ 2,\ \cdots,\ L) \tag{11}$$

where $\boldsymbol{a}(\theta_l^{(m)}, \phi_l^{(m)})$ is the array response vector of the lth wave at the mth iteration, and $\boldsymbol{A}^{(m)}$ is the corresponding array response matrix. Also, β is a non-negative coefficient of noise term, and it affects the convergence characteristics.

M-step : The updated values $\theta_l^{(m+1)}$ and $\phi_l^{(m+1)}$ of the lth wave are obtained by using the covariance matrix of complete data: $\boldsymbol{C}_l^{(m)} = E\left[\boldsymbol{x}_l^{(m)}(t)\boldsymbol{x}_l^{(m)}(t)^H\right]$, as shown below.

$$\text{Method1} : \left(\theta_l^{(m+1)}, \phi_l^{(m+1)}\right) = \arg\max_{(\theta,\phi)} \boldsymbol{a}^H(\theta,\phi)\boldsymbol{C}_l^{(m)}\boldsymbol{a}(\theta,\phi) \tag{12}$$

$$\text{Method2} : \begin{pmatrix} \theta_l^{(m+1)} \\ \phi_l^{(m+1)} \end{pmatrix} = \begin{pmatrix} \arg\max_{\theta} \boldsymbol{a}^H(\theta, \phi_l^{(m)})\boldsymbol{C}_l^{(m)}\boldsymbol{a}(\theta, \phi_l^{(m)}) \\ \arg\max_{\phi} \boldsymbol{a}^H(\theta_l^{(m)}, \phi)\boldsymbol{C}_l^{(m)}\boldsymbol{a}(\theta_l^{(m)}, \phi) \end{pmatrix} \tag{13}$$

Both E-step and M-step above-mentioned are repeated until estimated parameters converge.

3. Performance Analysis by Computer Simulation

Under conditions shown in Fig.2, Tables 1 and 2, the computer simulation is carried out to clarify the performance of the proposed algorithm. In the DOA estimation, the EM algorithms using Fast [1] EM1 (using eq.(12)) and Fast EM2 (using eq.(13)), EM-Hybrid which estimates θ by searching and obtains ϕ by 1-D MS-root-MUSIC, and MS2D-EM-root-MUSIC which is the 2-D MS technique in combination with EM algorithm are compared. As the evaluation measure of estimated results, RMSEs (root mean square errors) of θ and ϕ are used, which is calculated through 200 independent trials. The number of incoming waves is assumed to be estimated exactly. Noise term coefficient β is $1/\sqrt{L}$.

First, the performance comparison of various EM algorithms is practiced when the incoming waves are coherent (Table 2). SNR is varied from -10 to 30 dB with a 5 dB step. The algorithm is terminated if the increment of the log-likelihood function is smaller than 10^{-6} or the number of iterations reaches the maximum value which is set to 30. The incoming waves are perfectly out of phase and completely correlated with each other. The estimation results are shown in Fig.3 along with Cramer-Rao bound (CRB) [5]. Around SNR = 0 dB, MS2D-EM-root-MUSIC shows a performance closer to the CRB in the elevation estimation than the other algorithms.

Next, the convergence characteristics of various EM algorithms are examined. The radio environment is described in Table 2. The number of iterations is varied from 1 to 30. Figure 4 shows that the convergence becomes rapid by employing 2-D MS technique in combination with EM algorithm.

Finally, averaged computation times of various algorithms are measured. It is calculated through 1800 trials. The results are shown in Table 3. As a consequence, MS2D-EM-root-MUSIC has the shortest computation time.

Table 1: Simulation conditions.

Array configuration	Planar array of isotropic elements
Number of elements	10
Number of waves	2
SNR	-10dB to 30dB

Table 2: Radio environment.

Number of waves	2 (coherent, equal power)
DOA(θ, ϕ)	(30°, −120°),(75°, 80°)
Initial value of EM	(25°, −125°),(80°, 85°)

Table 3: Average of computation time.

Fast EM1	44.5757	sec
Fast EM2	0.4842	sec
EM Hybrid	0.3906	sec
MS2D-EM-root-MUSIC	0.2616	sec

4. Conclusion

In this paper, we have proposed that methods of using MS technique in combination with EM algorithm with the arbitrary planar array configuration. In the 2-D angle estimation performance, we have shown that MS2D-EM-root-MUSIC almost completely attains the CRB in estimation error. Furthermore, we have shown that it has high speed of convergence and also that it is able to reduce computation time. As the future work, we will examine how to set the initial values of EM and the possibility of array antenna calibration using MS2D-EM-root-MUSIC.

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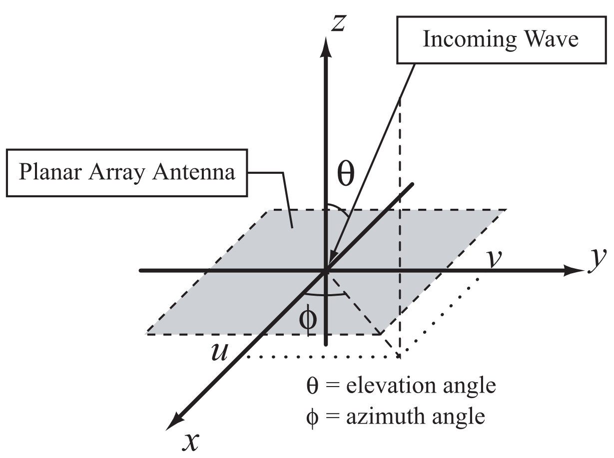


Figure 1: Planar antenna array and 2-D DOA of incoming wave.

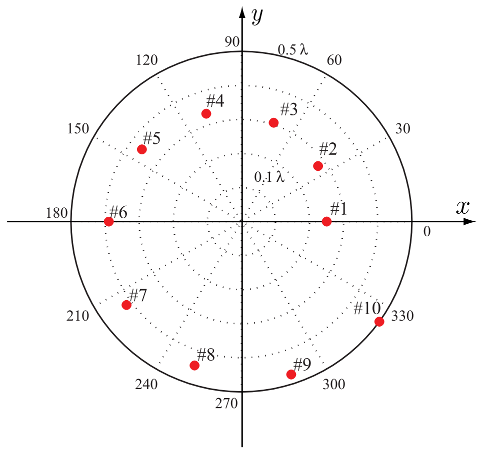
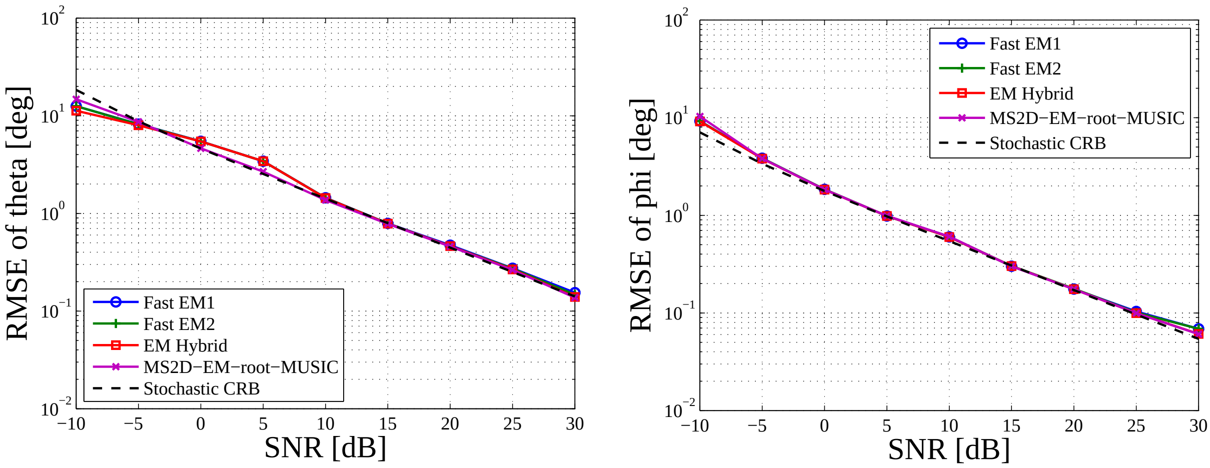
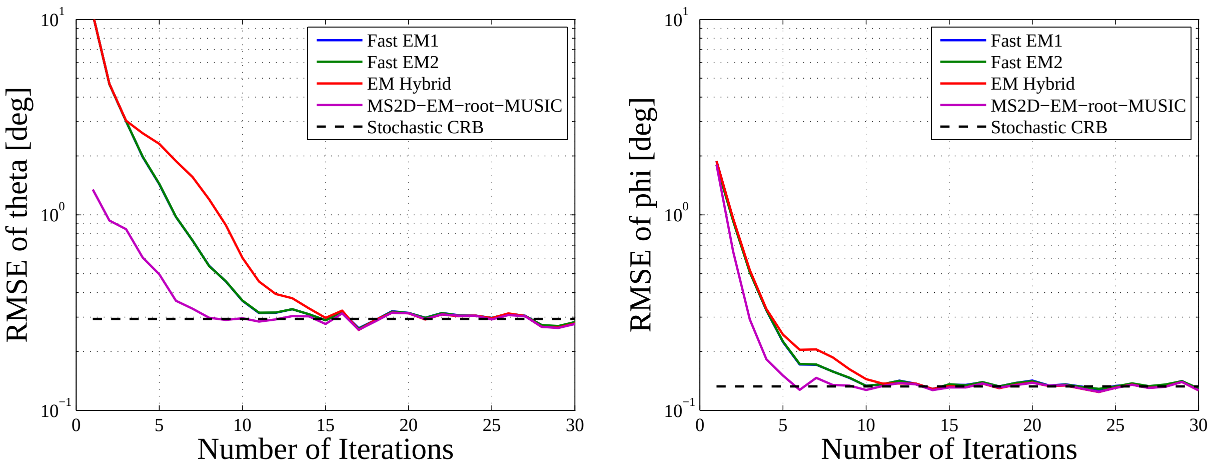


Figure 2: The geometry of the planar array elements considered in this paper.



(a) (b)

Figure 3: Performance comparison of various EM algorithms for (a) elevation and (b) azimuth estimation.



(a) (b)

Figure 4: Convergence of RMSE of (a) elevation and (b) azimuth estimation (SNR = 20dB).