# Generation of Optimum DOA Estimation Algorithm by Using Genetic Algorithm

\*Shunsuke Yoshimura, Hiroshi Hirayama, Nobuyoshi Kikuma, Kunio Sakakibara Department of Computer Science and Engineering, Nagoya Institute of Technology Gokiso-cho, Showa-ku, Nagoya, 466-8555, Japan E-mail: hirayama@m.ieice.org

### Abstract

A novel method to generate the optimum direction of arrival (DOA) estimation algorithm using genetic algorithm (GA) is proposed. DOA estimation algorithms generally consist of parameters and operators. When parameters and operators are transformed into genes, DOA estimation algorithms are described as chromosomes. Chromosomes are optimized by GA.

#### **Keywords**:

Array Antenna DOA Estimation Genetic Algorithm Polish Notation Combinatorial Optimization

## **1. Introduction**

Recently, development of radio communication technology is remarkable. On the other hand, various radio waves make our radio environments much complicated. Thus, it is important to understand wave propagation structures. For the purpose, it is effective to estimate DOAs of individual incoming waves with array antennas and DOA estimation algorithms are researched [1]. Various kinds of DOA estimation algorithms have been proposed. Each algorithm has merits and demerits. Therefore, it is necessary to use algorithms properly according to each requirements. Better algorithm is desired, however, it requires trial-and-error investigation.

In this paper, we propose a novel method to generate optimum DOA estimation algorithm by using genetic algorithm (GA). GA is a powerful optimization algorithm especially for a combinatorial optimization problem [2]. When fitness can be calculated for an arbitrary bit string, GA finds the optimum bit string that gives minimum fitness. In a spectrum search type DOA estimation algorithm, angle spectrum function is denoted by combination of parameters and operators. By assigning bit strings to the parameters and operators, GA finds optimum angle spectrum function though a DOA estimation simulation in which bit string given by GA is transformed into an angle spectrum function. Root mean square error (RMSE) of DOA estimation simulation is used as fitness. We validate the proposed method through computer simulation.

## 2. Proposed Method

#### 2.1 Transformation of DOA Estimation Algorithms into Chromosomes

Table 1 shows the transformation of parameters and operators into bit strings (genes). In this table, one gene is constructed by three bits. Therefore, eight way of parameters and operators can be represented by one gene. Angle spectrum function is transformed into a chromosome, which consists of genes. Polish (prepositive) notation is used to represent plural arguments. For example,

 $\mathbf{R}_{xx}^{-1}$  is transformed into '02'. When chromosomes are '702', '072' and '027', they are equal to '02'. Considering the linear array shown in Fig.1, angle spectrum functions of beamformer and Capon methods are given by [1]

$$P_{BF}(\theta) = \frac{\boldsymbol{a}^{H}(\theta) \boldsymbol{R}_{xx} \boldsymbol{a}(\theta)}{\boldsymbol{a}^{H}(\theta) \boldsymbol{a}(\theta)}$$
(1)

Gene	Parameter or Operator		
0	$f(\mathbf{A}) = \mathbf{A}^{-1}$ : Inverse		
1	$f(\boldsymbol{A}, \boldsymbol{B}) = \boldsymbol{A}^{H} \boldsymbol{B} \boldsymbol{A}$ : Quadratic		
2	$\boldsymbol{R}_{xx}$ : Correlation Matrix		
3	$\boldsymbol{a}(\theta)$ : Mode Vector		
4	$f(\boldsymbol{A}) = \boldsymbol{A}^{H} \boldsymbol{A}$		
5	$f(\mathbf{A}, \mathbf{B}) = \mathbf{A}/\mathbf{B}$ : Quotient (B is scalar)		
6	1 : Unity		
7	Blank		

Table 1: Transformation of Parameters and Operators into Genes.





Figure1: *K*-element uniform linear array. (element spacing: *d*)

Figure.2: Flowchart of proposed method.

$$P_{CP}(\theta) = \frac{1}{\boldsymbol{a}^{H}(\theta) \boldsymbol{R}_{xx}^{-1} \boldsymbol{a}(\theta)}$$
(2)

They can be transformed into '513243' and '01302' or '561302', respectively. After transformation of DOA estimation algorithms, chromosome treated by GA is expressed as  $g = [g_1 \ g_2 \ \cdots \ g_6]$ . Here,  $g_i \ (i=1, 2, \cdots, 6)$  are genes from 0 to 7. Therefore, the range of g is from [000000] to [777777]. GA searches the optimum angle spectrum function in this range.

#### 2.2 Generating Optimum DOA Estimation Algorithm by Using GA

We propose a method to generate the optimum DOA estimation algorithm by using GA. Fig. 2 shows a flowchart of this method. First, DOA estimation algorithms are transformed into chromosomes with Polish notation. Next, generated algorithms (angle spectrum functions) are evaluated by computer simulation of DOA estimation. Average of RMSE is used as fitness. Optimization by GA is repeated until chromosomes (DOA estimation algorithms) converge. Finally, a optimum chromosome (DOA estimation algorithm) is generated.

## 3. Performance Analysis by Computer Simulation

Array configuration	Uniform linear array of isotropic elements
Element spacing	Half wavelength
Number of elements	8
Number of waves	2(uncorrelated, equal power)
Number of snapshots for DOA estimation	100
SNR	20dB

Table 2: Simulation conditions.

Table 3: Radio environment 1.			Table 4: Radio environment 2.		
DOA of the 1st wave	$\theta_1 = -80$ deg. to 65 deg.		DOA of the 1st wave	$\theta_1 = -5$ deg.	
DOA of the 2nd wave	$\theta_2 = \theta_1 + 15$ deg.		DOA of the 2nd wave	$\theta_2 = 10$ deg.	

Chromosome	Angle Spectrum function	Average of RMSE [deg.]
013042	$\frac{1}{\boldsymbol{a}^{H}(\boldsymbol{\theta})(\boldsymbol{R}_{xx}^{H}\boldsymbol{R}_{xx})^{-1}\boldsymbol{a}(\boldsymbol{\theta})}$	8.0281 × 10 <sup>-2</sup>
013702	$\frac{1}{\boldsymbol{a}^{H}(\boldsymbol{\theta})  \boldsymbol{R}_{xx}^{-1}  \boldsymbol{a}(\boldsymbol{\theta})}$	$1.4215 \times 10^{-1}$
013072		$1.4528 \times 10^{-1}$
013027		$1.5082 \times 10^{-1}$
113732	$(\boldsymbol{a}^{H}(\theta)\boldsymbol{a}(\theta)\boldsymbol{a}(\theta))^{H}\boldsymbol{R}_{xx}(\boldsymbol{a}^{H}(\theta)\boldsymbol{a}(\theta)\boldsymbol{a}(\theta))$	2.4290 × 10

Ta	ble 5:	Chromosomes	searched	by GA	(Top 5	5 fitness)	).
----	--------	-------------	----------	-------	--------	------------	----

Under conditions shown in Tables 2-4, the computer simulation is carried out to clarify the performance of the proposed method and the optimized DOA estimation algorithm. As the evaluation measure of DOA estimation, RMSE (root mean square error) is used.

First, the performance of the proposed method is examined under conditions shown in Table 2. The radio environment is Table 3. Here, fitness is calculated through two independent trials The convergences of genes by GA are shown in Fig.3. From this figure, it is found that g converge [013042]. When this chromosome is transformed into DOA estimation algorithm, we obtain a new angle spectrum,

$$P_{OPT}(\theta) = \frac{1}{\boldsymbol{a}^{H}(\theta)(\boldsymbol{R}_{xx}^{H}\boldsymbol{R}_{xx})^{-1}\boldsymbol{a}(\theta)}$$
(3)

5 chromosomes, which are searched by GA, of best fitness are shown in Table 5. The second best algorithm is Capon method.

Next, the performance of this angle spectrum is examined under same conditions and radio environment in Table 2 and 3. We compare this angle spectrum with beamformer and Capon methods. The performance is evaluated by RMSE calculated through 100 independent trials. The results of estimation are shown in Fig.4, in which Cramer-Rao bound (CRB) [3] is also shown. From this figure, it is found that  $P_{OPT}(\theta)$  shows the better performance than beamformer and Capon methods in these conditions.

Finally, each angle spectrum is compared under same conditions in Table 2. The radio environment is described in Table 4. The results of DOA estimation shows in Fig.5. From this figure, it is found that the new angle spectrum is higher resolution than beamformer and Capon methods in these conditions.

# 6. Conclusion

In this paper, we have proposed the method to generate the optimum DOA estimation algorithm by using GA. Through computer simulation, we have shown usefulness of the proposed method. The optimum angle spectrum generated by GA showed the best performance among beamformer and Capon methods. As the future work, we will increase kinds of genes and length of chromosome to generate better DOA estimation algorithms.

# References

- [1] N. Kikuma, Adaptive Antenna Technology (in Japanese), Ohmsha, Inc., 2003.
- [2] D. E. Goldberg, *Genetic Algorithms in Search*, *Optimization and Machine Learning*, Addison-Wesley, Reading MA, U.S.A., 1989
- [3] P. Stoica: "The Stochastic CRB for Array Processing: A Textbook Derivation," IEEE Signal Processing Letters, Vol.8, No.5, pp.148-150, May 2001.



Figure.3:Optimization of genes by GA.



Figure.4: DOA estimation evaluated by RMSE.

Figure.5: DOA estimation evaluated by angle spectrum.