Lumped Circuit Model Analysis of Peano Line Antennas

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Abstract

Peano line antennas are used in applications where compactness and miniaturization are key objectives. Till date, no simple analytical model, barring only few complicated numerical techniques is available for its analysis. This paper presents a simple analytical model to calculate the resonant frequency of the antenna employing a lumped equivalent circuit representation.
Keywords : Peano line antenna , circuit model , lumped equivalent , resonant frequency

1. Introduction

With increasing demands for miniaturized circuitry in today's world, the antenna designers need to design compact antennas with performances similar to the more conventional designs. Recently, various techniques have been proposed to reduce the size of microstrip antennas [1-8]. Use of an edge shorted patch or a shorting pin are well known techniques for size reduction [1-3]. Inserting suitable slots in the radiating patch is also a common technique in reducing the dimensions of the patch antenna. The slots introduce parasitic capacitances, which tend to reduce the resonant frequency of the antenna [4-5]. However, an important technique for size reduction is the bending of the patch of a microstrip antenna to trace a particular space-filling curve. One such antenna is the Peano line antenna [6-7], where the patch traces a Peano space-filling curve. Convoluting the patch to trace a Peano curve essentially increases the path over which the surface current flows, which eventually results in lowering of the resonant frequency There have been a few attempts at analyzing the Peano line antenna [8], yet a simple circuit model is yet to be presented. This paper presents a simple circuit model of a microstrip Peano line antenna. Circuit response of the antenna by applying our model has been compared with simulated antenna response using Method of Moment based simulator IE3D with good agreement, thereby validating our approach.

2. Antenna Modeling Procedure

To model the Peano line antenna in terms of lumped circuit elements we first need to view the antenna as a cascade connection of several individual line segments. It may also be noted that, each right-angled bend needs to be considered separately. The segmented representation of the Peano line antenna is shown in Fig. 1. The line segments are represented by odd numbers, while the right-angled bends are denoted by even numbers. Each segment is then replaced by its equivalent lumped circuit model and the entire circuit is terminated by a suitable capacitive termination to account for the fringing fields from the open-ended line.

2.1 Modeling of Line Segments

The odd numbered segments shown in Fig. 1 represent line segments that can be considered as distributed transmission lines of particular lengths. The lumped-parameter equivalent for a particular length of transmission line is then employed [9-10] for each of the line segments. Fig. 2 shows the T-equivalent network for a two port distributed transmission line.

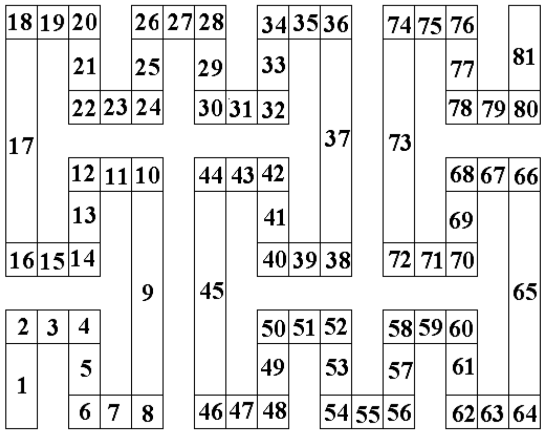

Fig. 1 Segmented representation of Peano line antenna

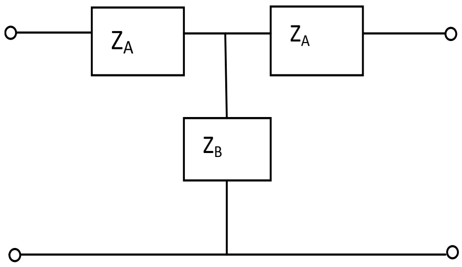

Fig. 2 T-equivalent network for transmission line

In fig. 2,

$Z_A = Z_0 \tanh(\gamma l/2)$ and $Z_B = Z_0 / \sinh\gamma l$ (1)

where, l is the length of the line segment and Z_0 is its characteristic impedance.

Considering the strip to be loss-free, the expressions above are modified to:

$Z_A = j\ Z_0 \tan(\beta l/2)$ and $Z_B = -\ j\ Z_0 / \sin\beta l$ (2)

When the relation $l \ll \lambda_g/4$ is satisfied, (2) gets modified to:

$Z_A = j\ Z_0\ \beta l / 2$ and $Z_B = -\ j\ Z_0 / \beta l$ (3)

Z_A and Z_B from (3) can now be expressed in the form of:

$Z_A = j\ \omega L l /2$ and $Z_B = 1 / j\ \omega C l$ (4)

where, L is inductance per unit length, C is capacitance per unit length, and ω is the operating angular frequency.

Comparing Z_A from (3) & (4), we get

$j\ Z_0\ \beta l / 2 = j\ \omega L l /2.$ (5)

Substituting

$\beta = 2\pi / \lambda_{eff}$, $\lambda_{eff} = \lambda_0 / \sqrt{\varepsilon_{eff}}$, $\lambda_0 = c/f$, $\omega = 2\pi f$

where, λ_{eff} is the effective wavelength and λ_0 is the free space wavelength, f is the operating frequency, and c is speed of light in free space,

ε_{eff} is the effective permittivity given by [11]

$$\varepsilon_{eff} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2} \times \frac{1}{\sqrt{1 + 12\frac{h}{w}}} \quad (6)$$

where, ε_r is the relative permittivity of the substrate, h is the height of the substrate and w is the width of the strip.

Thus from (5) we have,

$L = Z_0 \sqrt{\varepsilon_{eff}} / c$ (7)

Similarly, comparing Z_B from (3) & (4), we get

$C = \sqrt{\varepsilon_{eff}} / c\ Z_0$ (8)

The lumped equivalent of the T-network is shown in Fig. 3.

2.2 Modeling of right-angled bends

The right-angled bends, can also be modeled as equivalent lumped T-network [9]. The equivalent circuit for right-angled bend is shown in Fig. 4. Gupta *et al.* [12] have given closed-form formulae for the evaluation of bend capacitance and inductances as listed next.

$$\frac{C_b}{W}(pF/m) = \begin{cases} \frac{(14\varepsilon_r + 12.5)W/h - (1.83\varepsilon_r - 2.25)}{\sqrt{W/h}} + \frac{0.02\varepsilon_r}{W/h} & \text{For } w/h < 1 \\ (9.5\varepsilon_r + 1.25)W/h + 5.2\varepsilon_r + 7.0 & \text{For } w/h \geq 1 \end{cases} \quad (9)$$

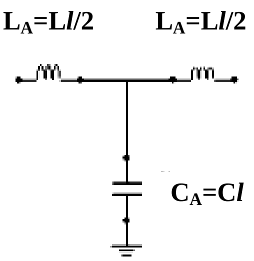

Fig. 3 Lumped equivalent model of the T-network

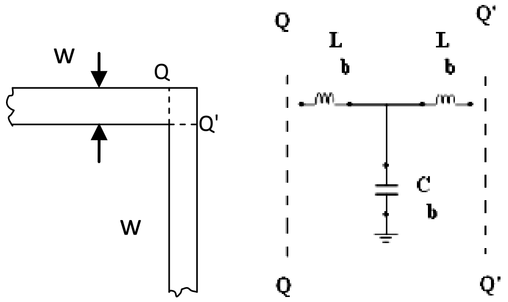

Fig. 4 Equivalent model of a right angled bend

$$\frac{L_b}{h}\left({}^{nH}\!/_{m}\right) = 100\left(4\sqrt{W/h} - 4.21\right) \tag{10}$$

where C_b is bend capacitance, L_b is bend inductance, W being the width of the strip, h the height of the substrate with permittivity ε_r.

2.3 Modeling of the open end

Due to fringing fields, the open end effect can be modeled as some equivalent capacitance connected at that end [9]. The expression for the open circuit capacitance C_{oc} has been formulated by Silvester and Benedek [13], which is listed below.

$$\frac{C_{oc}}{W}(pF/m) = \exp\left\{2.3026\sum_{i=1}^{5} C_i(\varepsilon_r)\left[\log\frac{W}{h}\right]^{i-1}\right\} \tag{11}$$

2.4 Cascading of the equivalent circuit

Having derived the lumped circuit equivalent for each segment, they are duly cascaded in proper order and terminated by the open circuit capacitance C_{oc}. The whole circuit looks like as it is shown in fig. 5. This circuit can be solved using circuit simulators to obtain the input reactance response with respect to frequency.

3. Results

In this article we present a microstrip Peano line antenna, with an antenna footprint of 25.5 X 25.5 mm^2. The strip width W is chosen as 1.5 mm. The height of the substrate h is taken to be 1mm and relative permittivity of the substrate ε_r as 4.3. The antenna is simulated using Method of Moments (MoM) based IE3D simulator from Zeland Software Inc., USA. The simulated antenna structure is shown in fig. 6. The antenna structure when segmented yields 81 segments. There are 2 line segments each 6mm long, 13 line segments each 4.5mm long, 6 line segments each 13.5mm long and 20 smaller line segments, each 1.5mm long. Apart from the line segments there are 40 right angled bends each of dimension 1.5mm X 1.5mm.

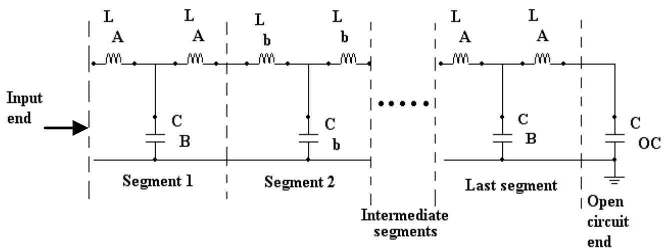

Fig. 5 Circuit representation of the antenna using lumped models

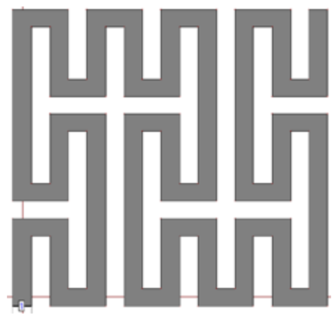

Fig. 6 Simulated antenna structure

3.1 Calculations

ε_{eff} = 3.2, Z_0 = 58.14 Ω, for 13.5mm line segments L_A = 2.34nH and C_B = 1.384pF, for 6mm line segments L_A = 1.04nH and C_B = 0.6153pF, for 4.5mm line segments L_A = 0.78nH and C_B = 0.4615pF, for 1.5mm line segments L_A = 0.26nH and C_B = 0.1538pF, for right angled bends L_b = 0.0688nH and C_b = 0.1387pF, open circuit capacitance termination C_{oc} = 0.0329pF.

3.2 Results

Fig. 7 shows the input reactance (X_{in}) versus frequency plots of the antenna using IE3D and our circuit model in the frequency range 0 to 1 GHz. The first resonant frequency for the antenna as simulated by IE3D is at 207.534 MHz. The corresponding value from circuit model is 193.836 MHz showing a difference of 6.6%. The second resonant frequency of the antenna is at 606.164 MHz, while the corresponding value from circuit model is 581.507 MHz (difference of 4.06%). The third resonance occurs at 996.575 MHz, while the corresponding value is 967.808 MHz for circuit model simulation (difference only 2.88%). The anti-resonant frequency points as observed from circuit simulation curves are also quite close to the anti-resonant points observed from IE3D simulation.

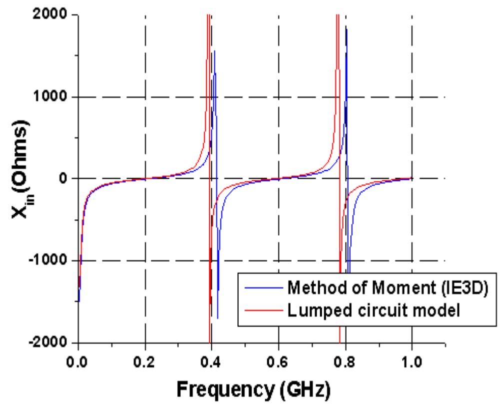

Fig. 7 Input Reactance versus frequency

4. Conclusion

This paper presents a simple circuit model analysis of a Peano line antenna to compute its resonant frequency. The lumped circuit equivalent of the antenna has been derived and validated against a standard electromagnetic simulator. The computed results reveal a close match with an error percentage of less than 7%. The marginal percentage of error (< 7%) in the resonant frequency can be attributed to electromagnetic coupling between the parallel and adjacently placed lines, which has been ignored in our analysis.

References

[1] N. Fayyaz, E. Shin, S. Safavi-Naeini, "A novel dual-band patch antenna for GSM band", Antennas and Propagation for Wireless Communications, pp. 156–159, 1998.

[2] R. Waterhouse, "Small microstrip patch antenna", Electron. Lett. 31, 604–605, April 13, 1995.

[3] S. Dey and R. Mittra, "Compact microstrip patch antenna", Microwave Opt. Technol. Lett. 13, 12–14, Sept. 1996.

[4] G. Kosiavas, A. Papiernik, J.P. Boisset, M. Sauvan, ''The C-Patch: A Small Microstrip Element'', Electronics Letters, Vol. 25, pp. 253–254, 1989 .

[5] K. L. Wong and K. P. Yang, "Small dual-frequency microstrip antenna with cross slot", Electron. Lett. Vol. 33, 1916–1917, Nov, 1997.

[6] J. Mc Vay and A. Hoorfar, "Miniaturization of top-loaded monopole antennas using Peano-curves", Radio and Wireless Symposium,IEEE,pp. 253-256, 2007

[7] T. Fukusako, T. Terada, K. Iwata, "Design and comparative study on planar small antennas using meander and peano line structure" International Symposium on Antennas and Propagation Society, pp. 2451-2454, 2007.

[8] X. Chen, Y. Liu, S. Safavi-Naeini, "Printed Plane-Filling Fractal Antennas for UHF Band", International Symposium on Antennas and Propagation Society, vol. 4, pp. 3425-3428, 2004

[9] T. C. Edwards, *Foundations for Microstrip Circuit Design* , 2nd Edition, John Wiley And Sons, 1991

[10] John. D. Ryder, *Networks, Lines and Fields*, 2nd Edition, Prentice Hall India, 1959

[11]Constantine A. Balanis. *Antenna Theory: Analysis and Design*, 3rd Edition, John Wiley and Sons Inc., 2005

[12] K.C. Gupta, R. Garg, I. Bahl, P. Bhartia, *Microstrip Lines and Slot Lines*, 2nd Edition, Artech House Publishers, 1996.

[13] P. Silvester, P. Benedek, " Equivalent capacitances of micostrip open circuits", IEEE Microwave Theory and Techniques, Vol. 20, No. 8, pp. 511 – 516, 1972