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Global synchronization in networks with large coupling delays

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Abstract—The paper is devoted to study of synchronization in networks with large coupling delays. We start from studying of a pair of coupled oscillators and then go to larger ensembles. We show that full synchronization is possible for the delays which are even several times larger than the oscillators period. The common feature is the periodical structure of synchronization zones in the parameter space.

1. Introduction

Mutual synchronization of interacting oscillators is a fundamental effect observed in all fields of physics. Particularly, there is lot of evidence that this effect plays crucial role in many aspects of brain functioning. For example, there is an important link between synchronization of distributed neural activity and complex informational processing in cortical networks. A number of experiments in visual cortex and other cortical areas of animals show that discharges of different areas often demonstrate precise and context dependent temporal relations [1, 2].

This synchronization is shown to be provided by reciprocal cortical-cortical connections. A fact of great interest is that systematic phase lags may be established even between the discharges of neurons from distant areas. In many cases precise zero-lag synchronization between distant areas is observed. This effect may be very important because it is hypothesized to provide a mechanism for the large-scale integration of distributed brain activity. Different aspects of the integral cognitive process occur in different areas, and the temporal synchronization between these areas ensures binding of all these subprocesses [3, 4].

In the present work we consider synchronization in networks of pulse oscillators with time-delayed couplings developing the approach introduced in our previous papers [5, 6]. We are mainly concentrated on how oscillators may synchronize with zero lag in the presence of large delays which are of order and even larger than the period of the oscillations. We start from a pair of coupled oscillators, then continue with a small ensemble of four and finally go to large networks.

2. Model

Our model is based on the phase oscillator with phase $\varphi \in [0; 1]$ growing uniformly with the velocity $d\varphi/dt = \omega$. For $\varphi = 1$, the oscillator reaches threshold, emits a pulse, and resets its phase to zero. We consider an ensemble of N non-identical neurons with frequencies ω_j interacting with a time lags. It is described by the following system:

$$\frac{d\varphi_j(t)}{dt} = \omega_j + \sum_{k=1}^{N} \mu_{jk} f(\varphi_j(t))\delta(\varphi_k(t - \tau_{jk})). \qquad (1)$$

We use the technique of phase response curves (PRCs) to describe interaction between the neurons. When k-th neuron emits a pulse, it is received by j-th neuron with some delay τ_{jk}. When j-th neuron receives a pulse its phase instantly changes on value $\Delta\varphi = \mu_{jk} f(\varphi_j)$. The function $F(\varphi)$ is the so-called phase response curve. Further we use function $f(\varphi) = -\sin 2\pi\varphi$. Such form of PRC belongs to the so-called second class, which means that incoming pulses may either delay or advance neuron excitation. For the analysis of the model dynamics we use the map-based approach developed in [5, 6]. The main idea of this reduction is that system dynamics is governed by a series of discrete events which occur when the neurons emit or receive pulses. During these events (the so-called G-events) the neurons phases are perturbed: if the neuron emits a pulse, its phase resets to zero, and if it receives a pulse, its phase undergoes some shift. Between the $G-$events the phases of the neurons grow uniformly. We construct the map that describes how the system state changes between sequential G-events (the so-called G-map), and the further study of the system dynamics is based on this map.

3. Synchronization of two oscillators

Let us begin from studying of the full long-range synchronization of a pair of oscillators. We call the synchronization full if it occurs with the zero phase shift, i.e. the units undergoing zero-lag synchronization fire strictly simultaneously. And we call it "long-range" meaning that the coupling delay is large enough in respect to the period of intrinsic spiking. In our previous works we have proved, that synchronization of pulse oscillators is possible for arbitrary large coupling delays. For a pair of neurons with symmetric delayed coupling ($N = 2$, $\mu_{12} = \mu_{21} = \mu$, $\tau_{12} = \tau_{21} = \tau$) we have proved that synchronization occurs for the frequency mismatches limited by

$$\omega_2 - \omega_1 \le \zeta_0 = \frac{2\mu\omega_1}{1 - \mu}. \qquad (2)$$

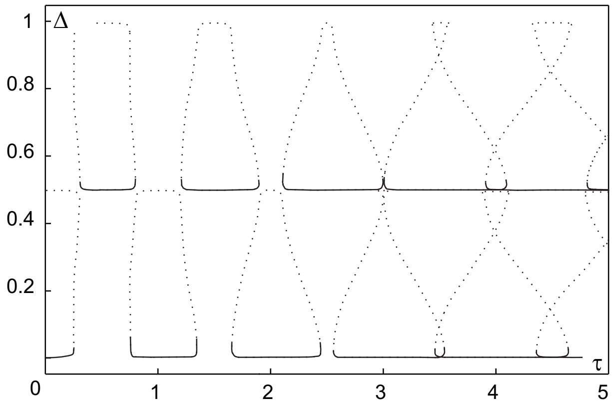

Figure 1: Synchronous regimes in a pair of oscillators with simmering coupling: phase lag versus coupling delay. Solid lines are for stable periodical solutions of the system, dashed lines are for unstable ones.

Synchronous regimes are observed in the so-called "synchronization intervals" of coupling delay τ (Fig. 1). For the case of close frequencies $\omega_1 \approx \omega_2$ these intervals cover almost all values of τ. The phase lag Δ between instants of neurons firing depends on τ and strongly changes from interval to interval. The intervals with in-phase and antiphase synchronization alternate while τ grows. So full synchronization is observed in certain intervals of the system parameter which form the periodical structure.

4. Synchronization of small ensemble

Let us pass to larger ensembles, for example consider a network of four globally coupled neurons. Let them be located in corners of a square, and let the delays between neighboring neurons equal $\tau/2$, while the delays between diagonal neurons equal τ. Coupling strength between each pair of neurons equals $\mu = 0.1$. Frequencies ω_j have Gaussian distribution with median value $\omega = 1$ and dispersion $\sigma = 0.01$. We set them not identical to study the influence of possible parameters varying and make sure that the regimes we obtain are not sensitive to them.

Studying of the network showed that depending on the delay parameter τ it may produce a number of various rhythmic patterns. We mark these patterns with sets of four numbers $(\varphi_1, \varphi_2, \varphi_3, \varphi_4)$, where φ_j means the phase lag between the first and the j-th neurons firing. The most typical are the following patterns: i) The pattern (0,0,0,0) of global synchronization, when all the units fire simultaneously in the same phase. ii) The pattern (0,0.5,0,0.5) or partial pairwise synchronization, when the first unit fires in phase with the third one, and the second unit fires sin phase with the fourth one, while these pairs fire in antiphase. iii) The pattern (0,0.25,0.5,0.75) of sequential firing, when all the units fire one after another with the quarter-period lag.

Each of these three patterns exists in definite interval of

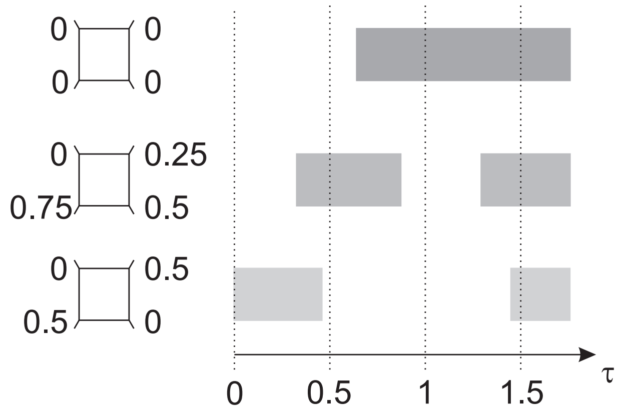

Figure 2: Possible output rhythmic patterns and τ intervals inside which they exist.

delay coefficient τ. These intervals are depicted in Fig. 2. One can see that the key features of the system dynamics are similar as in the case of two units. And similarly with the case of two units the regime of global synchronization exists in certain intervals of the system parameters. It is observable even for large coupling delays.

5. Synchronization of large ensembles

We finally consider a large network of $N \gg 1$ oscillators which are identical and globally coupled: $\omega_i = 1$, $\mu_{ij} = \mu$, $\tau_{ij} = \tau$. Regime of global synchronization corresponds to simultaneous firing of all the oscillators with some common period T. Let us find the corresponding solution of the system (1). For this we should find the summary phase shift which for each neuron is caused by all pulses which it receives during one period. For small $\tau < 1$ and weak coupling this summary shift be estimated as $\Delta\varphi = \mu N f(\tau)$, since the neuron receives N pulses when its phase equals $\varphi = \omega\tau = \tau$. Since the total phase change of each neuron during the period equals unity, the common period equals $T = 1 - \mu N f(\tau)$. Similarly this period can be found for larger τ when several more pulses are produced while each pulse passes from one unit to others. In this case the period be approximately calculated as $T = 1 - \mu N f(\tau \bmod 1)$.

The periodical solution corresponding to global synchronization is present in the system for all values of the coupling delay, but it may be not always stable. The stability of this solution means that the network returns to it after slight perturbation. Let this perturbation consist in the fact that in the initial moment the phases are not strictly zero but are distributed in some finite interval: $\varphi_j(t = 0) \in [0; \delta]$ $\delta \ll 1$. Let us study how this distribution is changed during one period T starting from the case of small delays $\tau < 1$. The timing of receiving of pulses from j-th unit will be identical for all other units and equal $\tau - \varphi_j$. The phase of the k-th neuron in the moment of receiving the pulse from the j-th neuron equals $\varphi = \tau + \varphi_k - \varphi_j$, so the phase shift cused

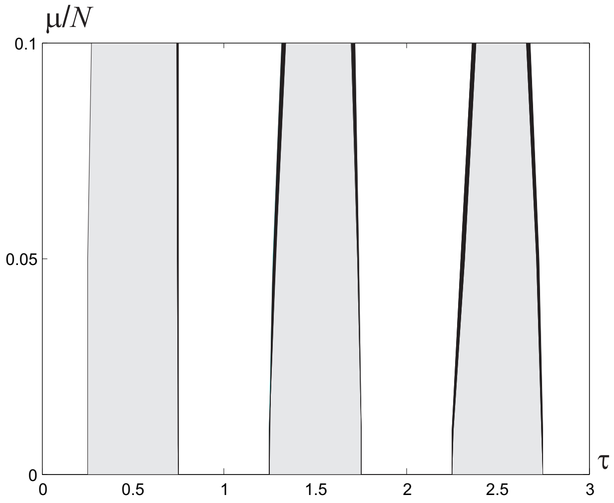

Figure 3: Dynamical regimes of the network of $N = 10$ oscillators.

by this pulse equals $\Delta\varphi = \mu f(\varphi) \approx \mu f(\tau) + \mu \frac{df(\tau)}{d\tau}(\varphi_k - \varphi_j)$. The total phase shift of k-th oscillator under the action of all these pulses equals

$$\Delta\varphi_k \approx \mu N f(\tau) + \mu \frac{df(\tau)}{d\tau} \sum_j (\varphi_k - \varphi_j), \quad (3)$$

which gives for the new value of this phase after the period

$$\bar{\varphi_k} = \varphi_k + \mu N f'(\tau)\varphi_k + \mu f'(\tau) \sum_j \varphi_j. \quad (4)$$

The sum in the last term can be proved not to change with time, and for the sake of contingency it can be set equal zero. This brings the equation () to the form of 1D map $\bar{\varphi} = \varphi + \mu N f'(\tau)\varphi$ which is stable for $f'(\tau) < 0$. This inequality is the sufficient condition for the stability of the whole network.

For large values of coupling delays the stability analysis is more complex but leads to the similar results. The network turns to be stable under fulfilment of the inequality

$$f'(\tau \bmod 1) < 0, \quad (5)$$

which consequently is the sufficient condition for stability of the global synchronization for arbitrary values of τ. For the given form of coupling function $F(\varphi) = -\sin 2\pi\varphi$ this condition fulfils in the sequence of intervals of τ: $\tau \in [m - 1/4; m + 1/4]$, $m \in Z$.

The dynamics of the network was studied numerically as well, and the map of its dynamical regimes is depicted in Fig. 3 for $N = 10$ oscillators. White corresponds to regions where global synchronization is stable, while in gray regions it is not observable. Black strips are narrow regions of bistability in which global synchronization is possible but occurs not for all initial conditions. For weak coupling the theoretical and numerical findings agree. For stronger coupling the areas of the stability of the synchronization enlarge. This means that fulfilment of the inequality (5) is sufficient but not necessary condition for the stability of the regime of global synchronization.

6. Conclusions

We have studied influence of coupling delay on dynamics of networks of coupled oscillators. We have shown that even large coupling delay does not prevent global synchronization which may explain long-range synchronization observed in numerous experiments. Global synchronization is observed in certain regions of the system parameters which form fine periodical structure in the parameter space.

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