2020 International Symposium on Nonlinear Theory and Its Applications,
NOLTA2020, Virtual, November 16-19, 2020

Evaluation of optical frequency dynamics in a semiconductor laser with time-delayed optical feedback

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Abstract – The optical frequency dynamics in chaotic laser outputs has been extracted from the laser intensity of low frequency fluctuations using optical heterodyne detection. We experimentally investigate the extraction of chaotic frequency dynamics at fast oscillations over GHz. We compare the characteristics of the optical frequency dynamics with that of the optical intensity dynamics. The optical frequency dynamics can be used to perform fast physical random number generation.

1. Introduction

Chaotic fluctuations of optical intensity outputs have been observed in a semiconductor laser with time-delayed optical feedback. These chaotic fluctuations have been used for engineering applications such as optical secure communication [1], reservoir computing [2], and fast physical random number generation [3,4].

Fast physical random number generation using chaotic lasers has been proposed in 2008 [3]. Various studies on fast physical random number generation using chaotic lasers have been conducted. Most of the studies are based on the generation of random numbers using chaotic fluctuations of the optical intensity. However, chaotic oscillations of the optical frequency can also be utilized. We expect that chaotic frequency dynamics may be useful for random number generation and the generation speed of random numbers could be enhanced by using both intensity and frequency dynamics in parallel.

The optical frequency of chaotic lasers cannot be detected directly because the frequency fluctuation is too fast in the order of several hundreds of THz. One of the detection schemes of the optical frequency of chaotic lasers is based on optical heterodyne detection and sliding fast Fourier transformation (FFT) for measuring the low frequency fluctuations (LFF) [5,6]. LFF shows slow power dropouts at frequencies from a few MHz to hundreds of MHz [7] while chaotic oscillation shows the oscillation frequency over GHz.

In the optical heterodyne detection method, the initial optical frequency detuning Δf_{ini} between a chaotic laser and a reference laser needs to be set to $\Delta f_{ini} \gg f_c$, where f_c is the frequency of the chaotic laser in the radio-frequency range. However, this condition for extracting chaotic frequency dynamics cannot be satisfied in experiment because of bandwidth limitation of experimental apparatus. Moreover, the random number generation using frequency dynamics extracted by the heterodyne scheme reduces the speed of random number generation, because Fast Fourier Transform (FFT) is time-consuming.

Instead of using the optical heterodyne detection, we use optical coherent detection for extracting the frequency dynamics and phase fluctuations [8]. Optical coherent detection has been mainly developed in the field of coherent optical communications. The complex electric field of lasers can be extracted by using optical coherent detection, therefore, the frequency and phase dynamics can also be extracted from the complex electric field. However, the construction of complex electric field that fluctuates over GHz has not been reported yet in chaotic semiconductor lasers with time-delayed optical feedback. It is necessary to observe the frequency dynamics of chaotic lasers for the applications of random number generation using the frequency dynamics.

In this study, we extract the temporal dynamics of optical frequency that fluctuates chaotically over GHz using the coherent detection scheme in a semiconductor laser with time-delayed optical feedback. We numerically investigate the conditions for observing frequency dynamics on the laser parameters. We also investigate the characteristics of optical frequency dynamics, compared with the intensity dynamics.

2. Numerical model of semiconductor laser with time-delayed optical feedback

Semiconductor lasers with time-delayed optical feedback can produce chaotic fluctuations over GHz. The dynamics of the semiconductor laser with time-delayed optical feedback is described by the Lang-Kobayashi equations [9], which are shown in the following equations,

$$\frac{dE(t)}{dt} = \frac{1+i\alpha}{2}\left[\frac{G_N[N(t)-N_0]}{1+\varepsilon|E(t)|^2} - \frac{1}{\tau_p}\right]E(t) + \kappa E(t-\tau)\exp(-i2\pi f_0 t), \quad (1)$$

$$\frac{dN(t)}{dt} = J - \frac{N(t)}{\tau_s} - \frac{G_N[N(t)-N_0]}{1+\varepsilon|E(t)|^2}|E(t)|^2, \quad (2)$$

where $E(t)$ is the complex electric-field amplitude, $N(t)$ is the carrier density, G_N is the differential gain, N_0 is the carrier number at transparency, f_0 is the solitary laser frequency, ε is the gain saturation coefficient, α is the linewidth enhancement factor, J is the injection current, τ

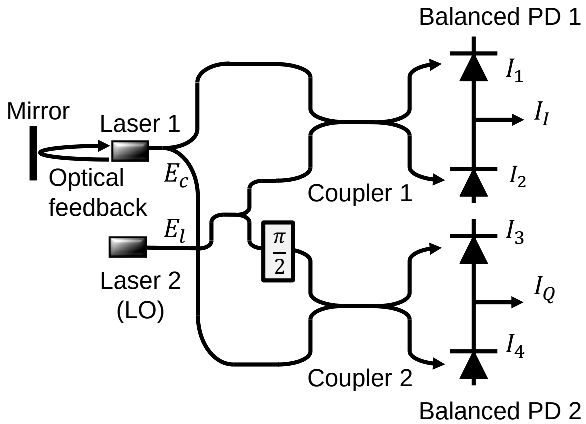

Fig. 1: Model for optical coherent detection scheme [8].

is the delay time in the external cavity for optical feedback, τ_p and τ_s are the photon and carrier life times, respectively, and κ is the feedback strength.

3. Extraction method of optical frequency dynamics using optical coherent detection

The optical coherent detection scheme can restore the complex electric field of the laser output, and extract the optical phase of the laser output. Figure 1 shows a model for the optical coherent detection scheme in a semiconductor laser with time-delayed optical feedback [8]. The laser 1 has optical feedback to generate chaotic fluctuations and the laser 2 is a frequency-stabilized laser without optical feedback. The laser 2 is called Local Oscillator (LO). The complex electric field amplitudes of the laser 1 (E_s) and the laser 2 (E_l) are represented as the following equations,

$$E_c(t) = A_c(t)\exp(i\omega_c t), \tag{3}$$

$$E_l(t) = A_l \exp(i\omega_l t), \tag{4}$$

where A_c and A_l are the complex amplitude of the laser 1 and 2, respectively, and ω_c and ω_l are the optical angular frequency of the laser 1 and 2, respectively. The complex amplitude A_l of the laser 2 is constant because the laser 2 has no optical feedback.

We can extract the frequency dynamics of the laser 1 by using an optical 90° hybrid which provides 90° phase shift. The output of LO is split into two paths. One output is interfered with the output from the laser 1 in the coupler 1, and the other output with 90° phase shift is interfered with the output of the laser 1 in the coupler 2. Note that these two optical couplers add 180° phase shift to the output from LO. The interfered optical outputs from these two couplers are detected at two balanced photodetectors (PD). The balanced PD can suppress the DC component and emphasize the beat between the laser 1 and LO. The optical intensities detected at the balanced PD 1 and 2 are given by the following equations;

$$I_1(t) = |E_c(t) + E_l(t)|^2/4, \tag{5}$$

$$I_2(t) = |E_c(t) - E_l(t)|^2/4, \tag{6}$$

$$I_3(t) = |E_c(t) + iE_l(t)|^2/4, \tag{7}$$

$$I_4(t) = |E_c(t) - iE_l(t)|^2/4. \tag{8}$$

The outputs from the balanced PDs are given by

$$I_I(t) = I_1(t) - I_2(t) = A_c(t)A_l\cos\{(\omega_c - \omega_l)t\}, \tag{9}$$

$$I_Q(t) = I_3(t) - I_4(t) = A_c(t)A_l\sin\{(\omega_c - \omega_l)t\}, \tag{10}$$

where $I_I(t)$ and $I_Q(t)$ indicate in-phase and quadrature components of the complex amplitude of the laser. Using Eqs. (9) and (10), we can restore the complex electric field of the laser 1, represented as,

$$I_e(t) = I_I(t) + iI_Q(t) = A_c(t)A_l\exp[i\{\Delta\omega t + \theta_s(t)\}], \tag{11}$$

where $\Delta\omega$ is the angular frequency detuning between the optical angular frequency of the laser 1 and 2 ($\Delta\omega = \omega_c - \omega_l$), and $\theta_s(\mathrm{t})$ is the phase of the laser 1. Equation (11) is equivalent to the complex electric field of the laser 1 $E_c(t)$ if the angular frequency detuning set to $\Delta\omega = 0$. Therefore, we can extract the phase of the laser 1 from the following equations,

$$\phi_e(t) = \arg(I_e(t)). \tag{12}$$

We calculate the optical frequency dynamics of the laser 1 using the extracted phase $\theta_e(t)$, represented as,

$$\Delta f_c(t) = \frac{1}{2\pi}\cdot\frac{\phi_e(t) - \phi_e(t-\tau_1)}{\tau_1}. \tag{13}$$

where τ_1 is the delay time for the frequency calculation.

To confirm the validity of the optical frequency dynamics calculated from the optical coherent detection scheme, we compare the frequency dynamics obtained from this scheme with its theoretical estimation. The theoretical estimation of the frequency dynamics can be calculated from the optical phase as follows,

$$\phi_t(t) = \arctan\left(\frac{E_{im}(t)}{E_{re}(t)}\right)., \tag{14}$$

where $E_{re}(t)$ and $E_{im}(\mathrm{t})$ are the real and imaginary parts of the complex electric field $E_c(t)$. The theoretical estimation of the optical phase can be calculated from the Lang-Kobayashi equations only in numerical simulation, but not in experiment. We can obtain the theoretical frequency dynamics $\Delta f_t(t)$ from $\phi_t(t)$ as follows,

$$\Delta f_t(t) = \frac{1}{2\pi}\cdot\frac{\phi_t(t) - \phi_t(t-\tau_1)}{\tau_1}. \tag{15}$$

The extraction of the frequency dynamics is successful if the frequency dynamics $\Delta f_c(t)$ obtained from the optical coherent detection scheme matches the theoretical estimation $\Delta f_t(t)$.

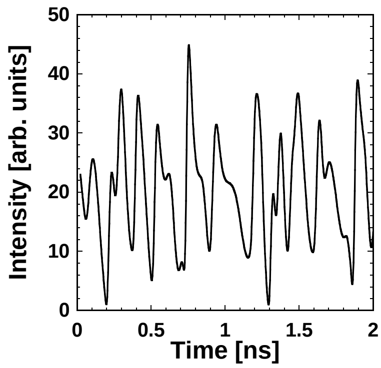

Fig. 2: Optical intensity dynamics of laser 1.

We calculate the cross-correlation value between $\Delta f_c(t)$ and $\Delta f_t(t)$ to evaluate the similarity quantitatively. The cross-correlation value is given by the following equation,

$$C = \frac{\langle[\Delta f_c(t) - \Delta\bar{f_c}][\Delta f_t(t) - \Delta\bar{f_t}]\rangle}{\sqrt{\langle[\Delta f_c(t) - \Delta\bar{f_c}]^2\rangle}\sqrt{\langle[\Delta f_t(t) - \Delta\bar{f_t}]^2\rangle}}. \quad (16)$$

where $\langle\cdot\rangle$ represents time averaging, $\Delta\bar{f_c}$ and $\Delta\bar{f_t}$ are time averages of $\Delta f_c(t)$ and $\Delta f_t(t)$, respectively.

4. Numerical results of extraction of optical frequency dynamics

We numerically extract the phase and frequency dynamics of the semiconductor laser with time-delayed optical feedback using the optical coherent detection scheme. Figure 2 shows the optical intensity dynamics of the laser 1. The optical intensity of the laser 1 fluctuates chaotically. We set the initial optical angular frequency detuning between the laser 1 and LO to $\Delta\omega = 0$ GHz.

Figure 3(a) shows the phase dynamics of the laser 1 obtained by the optical coherent detection scheme (the red curve). The phase dynamics is calculated from Eq. (12). The theoretical estimation of the phase dynamics is also shown in Fig. 3(a) (the black curve), which is calculated from Eq. (14). The cross-correlation value between the two curves is $C = 1.00$. Therefore, the extraction of phase dynamics of the chaotic laser is succeeded by using the optical coherent detection scheme in numerical simulation.

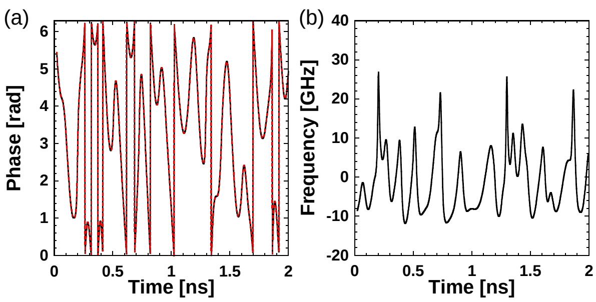

Fig. 3: (a) Phase dynamics of the laser 1 obtained by the optical coherent detection scheme (red), and theoretical estimation (black). (b) Frequency dynamics of laser 1 obtained by the optical coherent detection scheme.

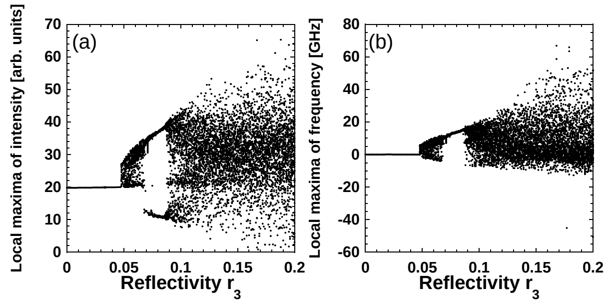

Fig. 4: Bifurcation diagrams of (a) optical intensity dynamics and (b) optical frequency dynamics as a function of the reflectivity r_3 of external cavity mirror (i.e., the feedback strength κ). Local maxima of temporal waveforms are plotted for different r_3.

We convert the phase dynamics to the frequency dynamics by using Eq. (13). Figure 3(b) shows the frequency dynamics calculated from Eq. (13) using $\tau_1 = 0.01$ ns. It can be seen that the frequency dynamics oscillates chaotically at frequencies of a few GHz.

5. Characteristic of frequency dynamics

We compare the characteristic of the frequency dynamics with that of the intensity dynamics. Figures 4(a) and 4(b) show the bifurcation diagrams of the intensity and frequency dynamics, respectively, as a function of the reflectivity r_3 of the external cavity mirror (i.e., the feedback strength κ). The feedback strength is calculated from $\kappa = (1 - r_2^2)r_3/r_2$, where r_2 is the facet reflectivity of the laser 1. From Fig. 4, both of the bifurcation diagrams show similar structures, and chaotic fluctuations are observed at the same r_3 for both the intensity and frequency dynamics. However, the frequency dynamics contains pulse-like oscillations at large r_3 in the bifurcation diagram of Fig. 4(b).

We also compare the frequency spectra between the optical intensity and frequency dynamics. Figures 5(a) and 5(b) show the Fourier spectra of the optical intensity dynamics and the optical frequency dynamics at $r_3 = 0.138$, respectively. In Fig. 5, the spectrum of the frequency dynamics looks flatter than that of the intensity dynamics. We evaluate the bandwidth of the chaotic spectra by using the effective bandwidth [10], which sums up large discrete

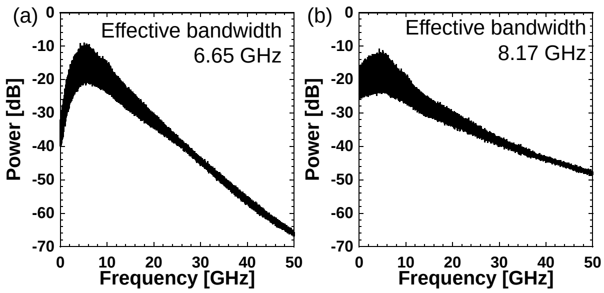

Fig. 5: Fourier spectra of (a) optical intensity dynamics and (b) optical frequency dynamics.

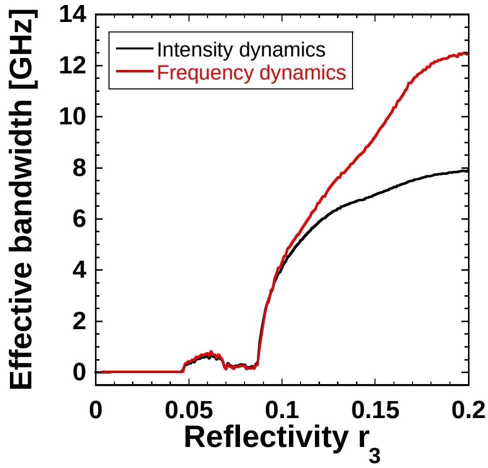

Fig. 6: Effective bandwidths of Fourier spectrum of the optical intensity dynamics (black) and the optical frequency dynamics (red) as a function of the reflectivity r_3 of the external mirror.

spectral segments of the power spectrum accounting for 80 % of the total power. The effective bandwidths of the intensity and frequency dynamics are 6.65 GHz and 8.17 GHz, respectively. Therefore, the bandwidth of the spectrum of the frequency dynamics is larger than that of the intensity dynamics.

We compare the effective bandwidth between the intensity and frequency dynamics. Figure 6 shows the effective bandwidths of the spectra of the intensity and frequency dynamics for different values of the reflectivity r_3. The black curve shows the bandwidth of the intensity dynamics, and the red curve shows the bandwidth of the frequency dynamics. From Fig. 6, the effective bandwidth of the frequency dynamics is larger than that of the intensity dynamics at the region of $r_3 > 0.1$, where both the frequency and intensity dynamics fluctuate chaotically. Chaotic fluctuations of the frequency dynamics with larger effective bandwidth could be useful for fast physical random number generation as an entropy source.

6. Conclusions

We numerically extracted the optical frequency dynamics of chaotic oscillations over a few GHz in a semiconductor laser with time-delayed optical feedback using the optical coherent detection scheme. We succeeded in extracting the phase and frequency dynamics of the chaotic laser. We compared the characteristic of the frequency dynamics with that of the intensity dynamics. Both of the frequency and intensity dynamics have similar bifurcation structure for different values of the reflectivity of the external mirror. We also compared the effective bandwidth of the Fourier spectra between the frequency and intensity dynamics. The effective bandwidth of the frequency dynamics is larger than that of the intensity dynamics when the frequency and intensity dynamics fluctuate chaotically. The frequency dynamics has larger bandwidth than that of the intensity dynamics and could be useful for fast physical random number generation.

We expect that it is possible to observe the optical frequency dynamics over GHz in experiment by using the optical coherent detection method. Furthermore, the speed of random number generation could be enhanced by using both the chaotic intensity and frequency dynamics in parallel.

Acknowledgments

This work was supported in part by JSPS KAKENHI (JP19H00868 and JP20K15185), JST CREST JPMJCR17N2, and The Telecommunications Advancement Foundation.

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