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Turning the Hopf Cochlea into a Listener

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Abstract—The Hopf Cochlea is a hard- and software implemented model of the mammalian cochlea that is constructed from a series of feedforward coupled nonlinear Hopf system amplifier sections. All salient nonlinear aspects of hearing can be traced back to the physical properties of the Hopf oscillators. At each location along the cochlea, the amplification strength is effectively governed by a single real parameter characterizing the distance of the Hopf oscillator from the Hopf-bifurcation point. Using these parameters, given a mixture of input signals (e.g., a set of musical instruments) it should be possible to tune the cochlea towards a single sound component. Introducing an autocorrelation-based tuning measure, we demonstrate the tunability of the Hopf Cochlea on recorded real-life instruments of different timbres and pitches. Despite the strongly nonlinear and therefore interaction-prone nature of the device, strong and simple tuning patterns permit an easy tuning to sounds of varying pitch.

1. Introduction

As early as 1863 H.L.F. Helmholtz proposed a place-frequency mapping along the basilar membrane (tonotopic principle), enabling the cochlea to perform a Fourier decomposition of an incoming stimulus. In 1948 T. Gold conjectured (see [4]) that an active amplification process compensates for the damping of wave energy in the liquid filled cochlea ducts, explaining inconsistent results obtained earlier on dead cadavers [1]. Subsequent measurements performed in living animals (cf. Ruggero [13]) finally revealed sharp frequency tuning of the basilar membrane response at a specific location, and a nonlinear compressive amplification of the stimulus. These effects emerge naturally and metrically verifiably if the mechanical amplification process originates from coupled dynamical systems each operating in a regime just below the Hopf bifurcation point [8, 3]. This then led to the implementation of these observations in a Hopf cochlea device that reproduces all the salient measurements emerging from the mammalian cochlea [9, 12, 15, 11]. Using this device that offers in contrast to the biological example excellent experimental accessibility and reproducibility, all salient nonlinear hearing phenomena discovered by de Boer [2] could be explained and be metrically verified [11]. The necessity of nonlinear amplification (much for faint sounds, lesser for louder sounds) accounts thus for the ability of the mam-

malian hearing to obtain a coherent picture of an incoming complex sound by attributing to it a pitch quality [11]. Less obviously emerging from the Hopf property is our ability to tune in on one particular sound component in a sound mix, say on a particular instrument within an orchestra. This tuning is initiated by the cortex via its decision on what to focus. Therefore, it is implemented predominantly via efferent connections to the auditory brainstem nuclei and to the outer hair cells that implement the Hopf amplification profile. These connections were well-known to the physiologists, but their purpose has not been recognized until quite recently ([5] for a review, [7]). From the Hopf cochlea concept it directly follows that these connections naturally define the distance of the Hopf amplifiers to the bifurcation point. As such, they also can be used in experiments to determine the efficacy and the rules adherent to this tuning. In what follows, we explore these rules. The present investigation can serve as a guiding example of the coherent tuning of a complex interacting network towards a coherent reaction.

A system exhibiting a Hopf bifurcation is generically described by a complex inhomogeneous Hopf differential equation [8]:

$$\frac{d}{dt}z = (\mu + i)\omega_{ch}z - \omega_{ch}|z|^2z - \omega_{ch}F(t),$$

where $F(t), z(t) \in \mathbb{C}$, where ω_{ch} is the characteristic oscillation frequency of the system and where $\mu \in \mathbb{R}$ is the bifurcation control parameter and $F(t)$ is the forcing stimulus (we consider here a by ω_{ch} rescaled Hopf equation). Writing $z(t) = R(t)e^{i\theta(t)}$ and separating real and imaginary parts, the free equation (i.e. $F(t) \equiv 0$) obtains the form:

$$\frac{d}{dt}R = R\mu\omega_{ch} - \omega_{ch}R^2, \quad \frac{d}{dt}\theta = \omega_{ch}.$$

The behavior of the free system is determined by the control parameter μ . For $\mu < 0$ the system has a stable fixed point at $R = 0$. For $\mu > 0$, the fixed point $R = 0$ becomes unstable and stable limit cycle solutions of the form $z(t) = \sqrt{\mu}e^{i\omega_{ch}t}$ emerge (so-called 'SOAE'). Assuming a 1 : 1 locking between system and stimulus [3] for a periodic forcing $F(t) = F e^{i\omega t}$ (i.e. $z(t) = R e^{i(\omega t + \theta)}$) removes the time dependence and yields an algebraic equation relating the forcing amplitude F to the response amplitude R . After taking the modulus squared (multiply the equation by the

complex conjugate equation), the resulting equation of 3'rd order in R^2 reads

$$F^2 = \gamma^2 R^6 - 2\gamma\mu R^4 + [\mu^2 + (1 - \phi^2)] R^2,$$

with $\phi \equiv \frac{\omega}{\omega_{ch}}$ and where $\gamma > 0$ has been included in the damping term $-\omega_{ch} |z|^2 z$ to model response latencies [9, 12]. To see how Hopf systems account for the hearing characteristics (sharp frequency tuning, compressed amplification) mentioned above it is instructive to study the Hopf equation in the different regimes of parameter space ($\Omega = span(\phi, \mu)$). At the bifurcation point ($\mu \simeq 0$) and close to resonance ($\phi \simeq 1$), a non-linear signal response $R \propto F^{\frac{1}{3}}$ relation emerges. For vanishing signal strength, the gain $G = R/F = F^{-2}$ increases towards infinity. Assuming $\mu < 0$ and $\phi \simeq 1$ one obtains the linear response $R = F/|\mu|$ for weak stimuli where the quadratic and cubic term in R^2 can be neglected. As F increases, the term $\gamma^2 R^6$ starts to contribute, and a compressive nonlinear regime of the system response is entered. Hence for weak stimuli, the response of the subcritical resonant Hopf system is almost linear, whereas for moderate stimuli $\frac{dF}{dR} < 0$ holds. Away from resonance ($\phi \neq 1$), the last term dominates, leading to a linear response $R = F/|1 - \phi|$.

In engineering terms, the cochlea can be considered as an array of feed-forward coupled amplifiers indexed by i , the dynamics of each being described by a Hopf equation of logarithmic decreasing characteristic frequencies ω_{ch}^i and local bifurcation parameters μ^i . Additionally, every amplifier is followed by a low pass filter to account for the viscous fluid damping. This model underlies the design of the biomorphic Hopf cochlea [9, 12, 15, 11].

2. Controlling a complex auditory system

A setting of the parameter μ of all oscillators equal to a value slightly below the bifurcation point (e.g. at $\mu_i = -0.1$ for all sections) results in an amplification profile that is very close to the one measured experimentally in animals [13, 9, 15, 11]. In contrast to this 'flat' tuning, the choice of different μ -values for individual sections might help the mammalian hearing system to effectively focus on desired sounds and to eliminate unwanted signals already at the very beginning of the auditory processing pathway. The purpose of our contribution is to evaluate the potential of this strategy offered to the mammalian hearing system by means of its efferent connections to the cochlea. To understand the relationship between this topic and complex networks, recall that the activation of already two sections will trigger the activation of a whole number of sections representing combination-tones [16, 17, 12, 15, 11], see Fig. 1.

It is therefore by no means clear, whether and if so to what extent the cochlear network can be tuned towards the extraction of a particular sound. If too many sections need

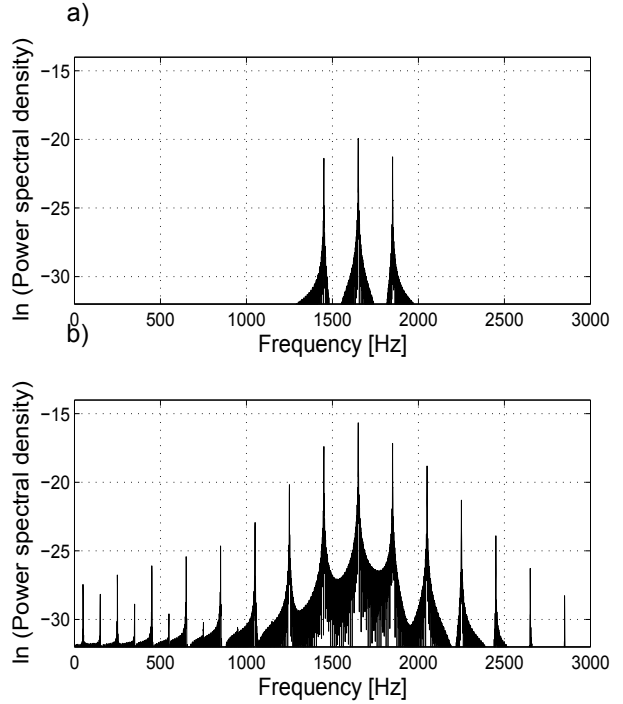


Figure 1: Combination tone generation. a) Power-spectrum of the input, an AM-sound with $A = 0.1$, $f_{car} = 1.65$ kHz, $f_{mod} = 0.2$ kHz consisting of three frequencies. b) Power-spectrum after the 5th section of the cochlea (of total 17) with characteristic frequency CF=1676 Hz. Strong combination tone generation is visible which would be absent in a classical, linear cochlea.

to be manipulated at the same time or if no simple coherent tuning strategies along the cochlear array can follow a signal of changing pitch, then an active tuning towards sources will be elusive. We will show that exactly due to the nonlinearity of interactions between the amplifiers, the Hopf cochlea is even easier tunable than a linear device.

We begin our presentation with the overall picture of the effects by tuning. Figure 2 shows the loss in gain in dB upon moving one specific section's μ -value μ_s from the bifurcation away to a more negative value. Physiological investigations [10, 5] indicate that we should primarily consider this case of tuning and not the situation where the cochlea's amplifiers are initially far from the bifurcation point and the tuning is accomplished by pushing a few sections towards the bifurcation point. In principle, both mechanisms appear possible and might even be used at the same time. For the following figure, we use a cochlea of 20 sections covering a range of 3 octaves from 2000 Hz to 16000 Hz to compare with physiological results obtained by [14]. Intermediate characteristic frequencies are logarithmically spaced according to the tonotopic organization of the mammalian cochlea. For Figure 2, the gain is measured at the 5'th of 20 sections (characteristic frequency: 10328 Hz), comparable

to measurements in [14] where a location 3.5mm from the oval window was chosen. Input sounds were pure tones with frequencies between 2000 Hz and 17448 Hz and five amplitude values (maximum of the wav-file coefficients) in the range of $\sqrt{10}^{-2}$ to $\sqrt{10}^{-10}$. The latter range corresponds to a spacing of 20dB. The observed tuning effect is strongest in the range of small amplitudes and small μ_5 -values, in accordance with the principle that the Hopf amplifier is most sensitive in the small-amplitude regime. When changing the value of μ_5 to -1.0 , the maximal loss measured at the lowest amplitude $\sqrt{10}^{-10}$ is roughly 19 dB (a value consistent with physiological data) and could still be increased when moving μ_5 to more negative values (e.g. -2.0).

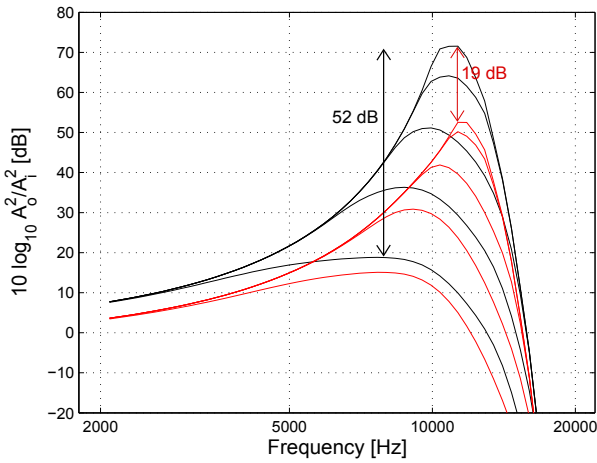


Figure 2: Loss in gain upon moving away one section from the bifurcation. Characteristic isointensity curves for the gain (spacing 20 dB) obtained at the 5th section for flat tuning $\mu = -0.05$ (black) and changed $\mu_5 = -1.0$ (red). The uppermost curves with highest gain correspond to a stimulus level of -100 dB, the lowermost curves to a level of -20 dB. The black arrow indicates a difference in peak gain for the -20 dB and the -100 dB curves of 52dB (flat tuning), which is comparable to values in [14] (38.5 and 59.5 dB). The red arrow shows the 19dB-decrease in gain for the lowest stimulus level -100 dB when μ_5 is changed to -1.0 .

To assess the tunability of this highly nonlinear device for complex sounds, we define a tuning measure TQ that characterizes the cochlea’s capability of extracting from a mixture of two sounds one desired sound. Motivated by the current models of pitch perception, this measure uses the *summary autocorrelation function (SACF)*, defined as the sum of normalized autocorrelations of each section’s output. TQ is obtained as the (e.g., Euclidean) distance of the cochlea’s SACF to the normalized autocorrelation (NACF) of the target signal x , divided by the Euclidean distance of the cochlea’s SACF to the NACF of the unwanted signal y .

Mathematically, this can be cast as

$$TQ(x, y) := \frac{\|\text{NACF}(x) - \sum_i \text{NACF}(f_i(x+y))/N\|_2}{\|\text{NACF}(y) - \sum_i \text{NACF}(f_i(x+y))/N\|_2}, \quad (1)$$

where the sum extends over the sections and where f_i denotes the output at the i ’th section. The value of the measure TQ is between 0 and ∞ : A small TQ close to zero denotes successful tuning whereas a large TQ indicates bad tuning. The tuning of the cochlea towards a complex sound may thus be formulated as a TQ -minimization problem in μ -parameter space. We approached this problem with various state-of-the-art numerical minimization algorithms, such as the evolutionary algorithm *CMA-ES*.

3. Results

For the tuning experiments on real sounds, a cochlea of 17 sections covering 4 octaves was chosen (characteristic frequencies ranging from 220 to 3520 Hz). The mixed sound that we start with was composed of sounds of two characteristic organ pipes, one being a wooden flute-like 8’ register (“Floete”) and the other being a metallic reed-stop (“Zinke”). Both data strings were normalized with respect to their total energy (the square-root of the sum of the squared wave-coefficients). The amplitudes were set to be at -110 dB. This falls into the low-amplitude regime where tuning effectivity is promising. Results of *CMA-ES* tuning for recorded organ sounds are shown in Figure 3.

Fig. 3a shows an emerging tuning pattern when the “Zinke” is the target sound while it sweeps from 220Hz to 440Hz in steps of one semitone (x-axis). The sound to suppress is the “Floete”, constant at a pitch of 220Hz. As the z -value of the contour plot, the logarithm of the bifurcation-parameter μ is displayed, where red codes for close to and blue for far away from the bifurcation point. In a second plot, the resulting tuning measure TQ (red) is compared to the tuning measure when flat tuning is applied (black). Fig. 3b shows the corresponding figures for the reverse case, when the sweeping “Floete” is the target signal and the constant 220-Hz “Zinke” is the undesired sound.

From the minimization procedure, surprisingly simple tuning patterns emerge. The patterns can be well-understood from the characteristic power spectra of the two sound sources: While the “Floete” sound is poor in higher harmonics and thus dominated by its fundamental frequency, the “Zinke” sound displays a wide spectrum of strong higher harmonics, with the third and the fifth harmonics standing out. When a sweeping “Zinke”-signal is the target, we thus find an enhancement of the 3rd and 5th harmonic (indicated by dashed black lines to guide the eye). Contrarily, in Fig. 3b, targeting the “Floete” reveals a simple pattern where the ground frequency is enhanced and all higher frequencies (that mainly originate from the “Zinke”) are suppressed. In both cases, the tuning measure of the tuned cochlea (red curves) are well below those obtained

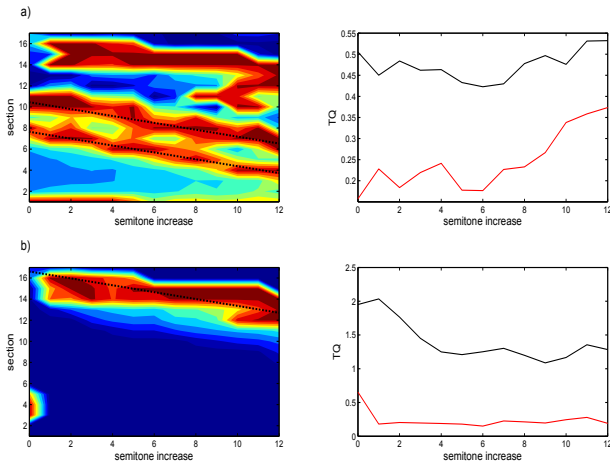


Figure 3: a) Tuning patterns for the “Zinke” as the target, b) for the “Floete” as the target. Red indicates close to bifurcation values (strong amplification), blue indicates large distance from bifurcation point (weak amplification). Tuning towards “Zinke” clearly requests the enhancement of the 3d and of the 5th harmonic (two parallel red stripes, indicated by black dashed lines). In the tuning towards the “Floete”, higher frequencies are suppressed and only the ground frequency is enhanced (single red stripe, black dashed line). Below we display the corresponding TQ of the tuned cochlea (red line) compared to a flat-tuned cochlea (black line) ($\mu = -0.1$; TQ well below 1 indicates successful tuning).

for flat tuning (black curves). The tuning measures of the tuned cochlea are consistently and significantly below the value 1, implying that the cochlea successfully enhanced the target signal, independent of the target’s pitch and of the to-be-suppressed signals. Regarding the amplitude-dependence of the tuning, we find relatively similar TQ-values for amplitudes from -110 to -40 dB. This refutes an impression from Fig. 2 that with higher amplitudes the tuning efficacy might worsen. We noticed, however, that in this case the tuning patterns wear out and become more difficult to interpret. The experiments demonstrate impressively that despite its highly complex and nonlinear nature, the Hopf cochlea is remarkably fine-tunable.

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