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# Switching of periodic orbits in dynamic binary neural networks

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**Abstract**—This paper studies hardware implementation and applications of dynamic binary neural networks. The network is characterized by ternary connection parameters and the signum activation function. Depending on the parameters, the network can generate various binary periodic orbits. A periodic orbit corresponds to a control signal of central pattern generator. We present a simple FPGA-based hardware of the six-dimensional network. As an application of the hardware, we demonstrate a central pattern generator for typical insect gaits patterns.

## 1. Introduction

A dynamic binary neural network (DBNN) is an nonlinear dynamical system characterized by signum activation function and ternary binary connection [1]-[4]. A delayed feedback is applied and the DBNN can generate various binary periodic orbits. The DBNN has advantages in precise analysis of the dynamics and FPGA based low power hardware implementation. Storage of target binary periodic orbits (TBPOs) and stability analysis of the stored TBPO are important in study of nonlinear dynamics. Real/potential engineering applications include associative memories [5] [6], control of switching power converters [7] [8], and central pattern generators in robotics [9] [10]. The DBNN is an important study object from both fundamental and application viewpoints.

This paper studies storage and switching of multiple TBPOs in the DBNN in a typical example: switching of two TBPOs related to insect gaits patterns [9]. First, we consider condition for storage and switching of two TBPOs (TBPO1 and TBPO2) based on two sets of connection parameters. In the first set, TBPO1 is stored into the DBNN and all the elements of TBPO2 fall directly into the TBPO1. In the second set, TBPO2 is stored into the DBNN and all the elements of TBPO1 fall directly into the TBPO2. Second, we give examples of sparse connection parameters that realizes storage and switching of the two TBPOs. Presenting an FPGA-based hardware, the storage and switching of the TBPOs are confirmed experimentally. Applying a PWM signal, the TBPOs based control of a hexapod walking robot is demonstrated.

## 2. Dynamic Binary Neural Networks

The DBNN is characterized by ternary connection parameters and signum activation function as shown in Fig. 1. The dynamics is described by

$$x_i^{t+1} = F\left(\sum_{j=1}^{N} w_{ij}x_j^t - T_i\right)$$
$$F(x) = \begin{cases} +1 & \text{if } x \geq 0 \\ -1 & \text{if } x < 0 \end{cases}$$
$$w_{ij} \in \{-1, 0, +1\},\ T_i \in \{0, \pm 1, \pm 2, \cdots, \pm N+1\} \quad (1)$$

where $\boldsymbol{x}^t \equiv (x_1^t, \cdots, x_N^t)^\top$ is an $N$-dimensional binary state vector at discrete time $t$ and $x_i^t \in \{-1, +1\}$ is the $i$-th element, $i = 1 \sim N$.

For convenience, we introduce the vector form:

$$\boldsymbol{x}^{t+1} = \boldsymbol{F}(\boldsymbol{W}\boldsymbol{x}^t - \boldsymbol{T})$$
$$\boldsymbol{W} \equiv \begin{pmatrix} w_{11} & \cdots & w_{1N} \\ \vdots & \ddots & \vdots \\ w_{N1} & \cdots & w_{NN} \end{pmatrix},\ \boldsymbol{T} \equiv \begin{pmatrix} T_1 \\ \vdots \\ T_N \end{pmatrix}. \quad (2)$$

where $\boldsymbol{W}$ and $\boldsymbol{T}$ are referred to as connection matrix and threshold vector, respectively. Given an initial binary vector $\boldsymbol{x}^1$, the DBNN can generate various periodic/transient binary sequences.

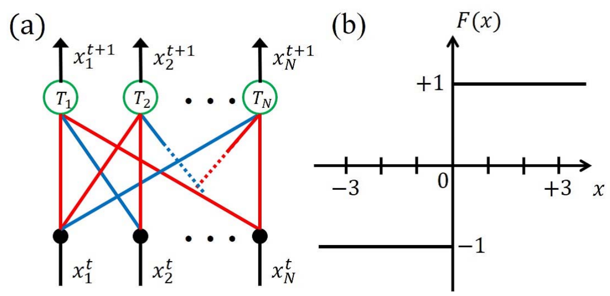


Figure 1: DBNN and signum activation function. Red and blue segments denote $w_{ij} = +1$ and $w_{ij} = -1$, respectively.

## 3. Switching of Target Binary Periodic Orbits

We consider switching of two target binary periodic orbits:

$$
\begin{array}{l}
\text{TBPO1 with period } p_a\text{: } a^1, a^2, a^3, \cdots \\
\left\{ \begin{array}{ll} a^s = a^u & \text{if } |s-u| = np_a \\ a^s \neq a^u & \text{otherwise} \end{array} \right. \\
\text{TBPO2 with period } p_b\text{: } b^1, b^2, b^3, \cdots \\
\left\{ \begin{array}{ll} b^s = b^u & \text{if } |s-u| = np_b \\ b^s \neq b^u & \text{otherwise} \end{array} \right.
\end{array} \tag{3}
$$

where $n$ denotes positive integers. Our objective is to set parameters $(w_{ij},\ T_i)$ to realize the following two phases.

Phase 1: TBPO1 is stored and all the elements in TBPO2 fall directly into the TBPO1.

$$
\begin{array}{ll}
a^{\tau+1} = \boldsymbol{F}(\boldsymbol{W}a^\tau - \boldsymbol{T}) & \text{for } \tau \in \{1, \cdots, p_a\} \\
a^k = \boldsymbol{F}(\boldsymbol{W}b^\tau - \boldsymbol{T}) & \text{for } \tau \in \{1, \cdots, p_b\}
\end{array} \tag{4}
$$

where $a^k$ is an element of TBPO1.

Phase 2: TBPO2 is stored and all the elements in TBPO1 fall directly into the TBPO2.

$$
\begin{array}{ll}
b^{\tau+1} = \boldsymbol{F}(\boldsymbol{W}b^\tau - \boldsymbol{T}) & \text{for } \tau \in \{1, \cdots, p_b\} \\
b^k = \boldsymbol{F}(\boldsymbol{W}a^\tau - \boldsymbol{T}) & \text{for } \tau \in \{1, \cdots, p_a\}
\end{array} \tag{5}
$$

where $b^k$ is an element of TBPO1.

Fig. 2 shows two TBPO examples corresponding to typical insect gaits patterns. TBPO examples corresponding to insect gaits.

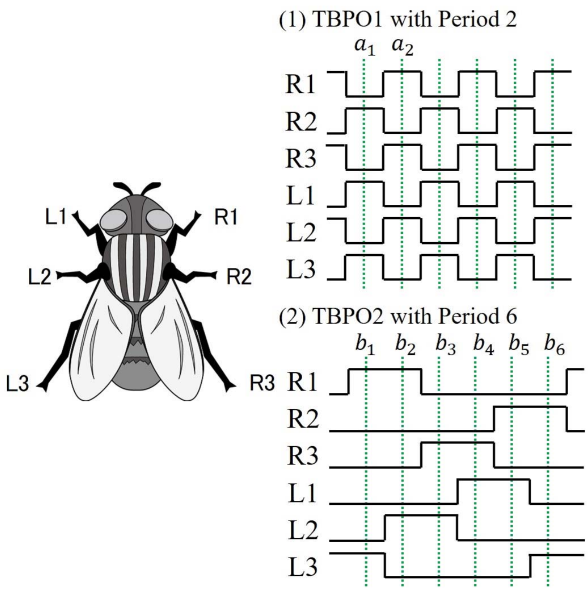


Figure 2: Two patterns of insect gaits.

$$
\begin{array}{l}
\text{TBPO1 with period 2:} \\
a^1 = (-1, +1. -1, +1, -1, +1) \\
a^2 = (+1, -1. +1, -1, +1, -1) \\
\text{TBPO2 with period 6:} \\
b^1 = (+1, -1. -1, -1, -1, +1) \\
b^2 = (+1, -1. -1, -1, +1, -1) \\
b^3 = (-1, -1. +1, -1, +1, -1) \\
b^4 = (-1, -1. +1, +1, -1, -1) \\
b^5 = (-1, +1. -1, +1, -1, -1) \\
b^6 = (-1, +1. -1, -1, -1, +1)
\end{array} \tag{6}
$$

In the TBPOs, symbol "+1" means movement and symbol "-1" mean no movement, respectively. Fig. 3 shows switching of TBPO1 and TBPO2.

After trial-and-errors, we have obtained two sets of parameters that realize the two phases of the switching TBPOs. The phase 1 is realize by

$$
W_2 = \begin{pmatrix} 0 & 0 & 0 & -1 & -1 & +1 \\ -1 & 0 & 0 & 0 & +1 & -1 \\ 0 & 0 & 0 & -1 & -1 & +1 \\ -1 & -1 & 0 & 0 & 0 & -1 \\ +1 & 0 & 0 & 0 & -1 & +1 \\ 0 & -1 & 0 & +1 & 0 & -1 \end{pmatrix} \quad T_2 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{7}
$$

The phase 2 is realize by

$$
W_6 = \begin{pmatrix} 0 & 0 & 0 & -1 & -1 & +1 \\ -1 & 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & -1 & +1 & -1 \\ -1 & -1 & 0 & 0 & 0 & -1 \\ +1 & 0 & 0 & 0 & +1 & +1 \\ 0 & +1 & 0 & +1 & 0 & +1 \end{pmatrix} \quad T_6 = \begin{pmatrix} 0 \\ 2 \\ 0 \\ 2 \\ 0 \\ 0 \end{pmatrix} \tag{8}
$$

These parameters correspond to network configurations are shown in Fig. 3

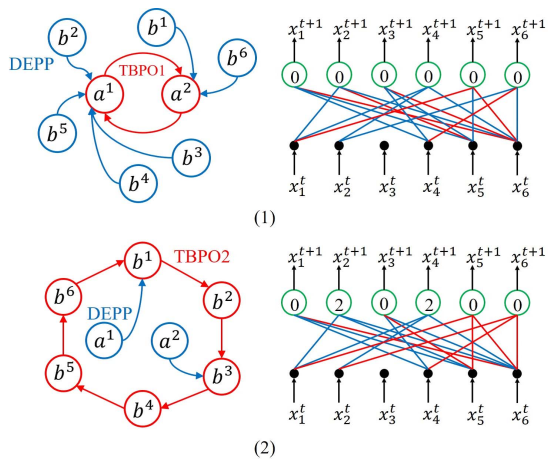


Figure 3: Switching of TBPOs and DBNN. (1) Phase 1: Switching to TBPO1 with period 2. (2) Phase 2: Switching to TBPO2 with period 6.

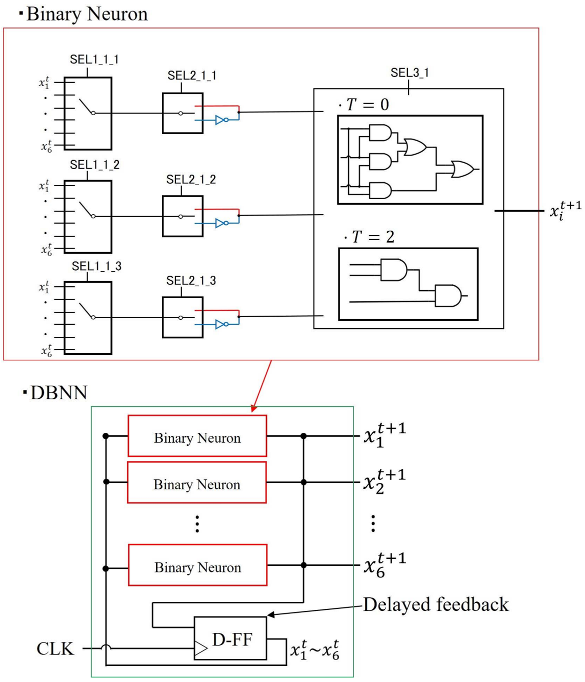


Figure 4: DBNN circuit design

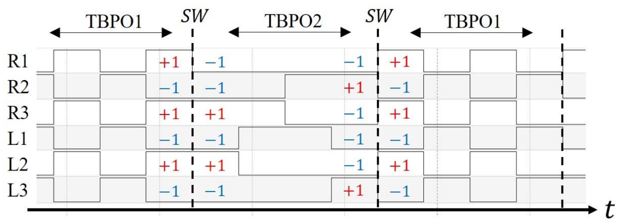


Figure 5: Measured waveform in an FPGA board

## 4. FPGA and Hexapod robot Implementation

Using Verilog, we have implemented an FPGA based hardware circuit as shown in Fig. 4. Using the circuit, we have performed laboratory experiments with the following tools:

- FPGA board: BASYS3 (Xilinx Artix-7 XC7A35T-ICPG236C)
- Clock frequency: 6 [Hz] [1]
- Measuring instrument: ANALOG DISCOVERY2.
- Multi-instrument software: Waveforms 2015.
- Verilog version: vivado 2018.2 platform (Xilinx).

The switching of TBPO1 and TBPO2 is confirmed experimentally as shown in Fig. 5.

[1]the default clock frequency 100 [MHz] is divided for stable measurements

In order to control a servomotors in a hexapod walking robot, we transform the TBPOs into pulse-width modulation (PWM) signals. Fig. 6 shows the method of generating the signal to move the servomotors. Applying PWM to the divided clock, the PWM signal is generated by a binary signal of a TBPO from the DBNN. Fig. 7 shows a binary DBNN signal and the pulse-width modulation signal in the yaw axes and roll axes. In the binary DBNN signals, symbol "+1" is designed to move the robot's legs forward and symbol "-1" is designed to move the robot's legs backward, respectively. Using the hexapod robot in Fig 8, we have confirmed waling patterns of TBPO1 and TBPOs experimentally.

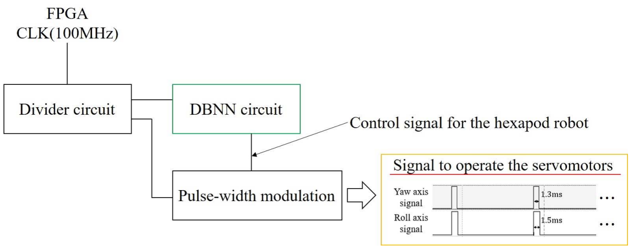


Figure 6: Signal to operate the servomotors

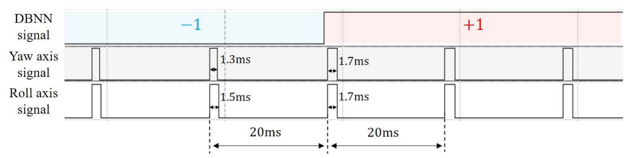


Figure 7: PWM control singnal for a hexapod walking robot.

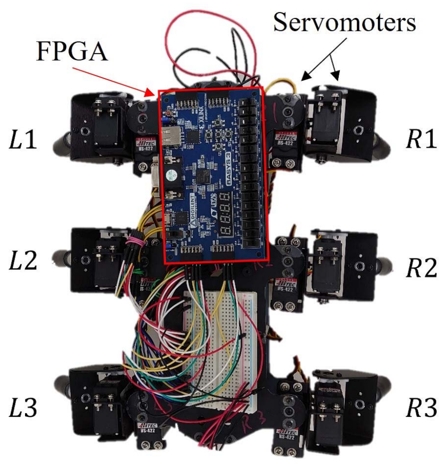


Figure 8: In the experiment, we used the hexapod robot. We have used the robot body and the servomotors of Lynxmotion's MH2 hexapod robot.

## 5. Conclusions

Storage and switching of TBPOs in DBNN are considered in this paper. We have given two sets of parameters that realize the storage and switching of two TBPOs related to insect gaits patterns. Presenting an FPGA based hardware, switching of the two TBPOs is confirmed experimentally. Future problems include detailed stability analysis of various TBPOs and learning algorithm for storage and switching of various TBPOs.

## References


[1] S. Koyama and T. Saito, Guaranteed storage and stabilization of desired binary periodic orbits in three-layer dynamic binary neural networks, Neurocomputing, in press.

[2] T. Suzuki, S. Anzai, S. Koyama and T. Saito, Dynamic binary neural networks for central pattern generators, Proc. KJCCS, E3, 2020.

[3] S. Anzai, S. Koyama, S. Aoki and T. Saito, Sparse dynamic binary neural networks for storage and switching of binary periodic orbits, T. Gedeon et al. (Eds.): ICONIP 2019, LNCS 11954, pp. 536-542, 2019.

[4] S. Aoki, S. Koyama and T. Saito, Theoretical analysis of dynamic binary neural networks with simple sparse connection, Neurocomputing 341, pp. 149-155, 2019.

[5] J. J. Hopfield, Neural Networks and Physical Systems with Emergent Collective Computation Abilities, Proc. of the Nat. Acad. Sci., 79, 1982.

[6] J. Hao and J. Vandewalle, A new model of associative memories, Proc. IJCNN II, pp. 166-171, 1992.

[7] B. K. Bose, Neural Network Applications in Power Electronics and Motor Drives - an Introduction and Perspective, IEEE Trans. Ind. Electron., 54, 1 (2007) 14-33.

[8] J. Rodriguez, M. Rivera, J. W. Kolar, and P. W. Wheeler, A Review of Control and Modulation Methods for Matrix Converters, IEEE Trans. Ind. Electron., 59, 1 (2012) 58-70.

[9] P. Ramdya, R. Thandiackal, R. Cherney, T. Asselborn, A. J. Ljspeert and D. Floreano, Climbing favours the tripod gait over alternative faster insect gaits, Nat. Commun. 8, 14494, 2017.

[10] A. Lozano, M. Rodriguez, R. Roberto Barrio, Control strategies of 3-cell central pattern generator via global stimuli, Sci. Rep. 6, 23622, 2016.