Transmission Area through the Small Aperture Backed by Lossy Cavity

Jong Hwan Lee[1], Hyuk Woo Son[1], Ji Whan Ko[2], #Young Ki Cho[1]
[1] School of Electrical Engineering and Computer Science, Kyungpook National University
1370 Sangyuk-Dong, Buk-Gu, Daegu, Korea,
btlink08@naver.com, jacklywu486@nate.com, ykcho@ee.knu.ac.kr
[2] School of Electronic Engineering, Kumoh National Institute of Technology
1, Yangho-Dong, Gumi, Gyeongbuk, Korea, kojh@kumoh.ac.kr

1. Introduction

The various problems of electromagnetic coupling from one region to another through a aperture have been studied in the electromagnetic community. The work by Harrington [1] on a long rectangular aperture with capacitor installed in the middle of the aperture showed that effective transmission area(transmitted power normalized to incident power density) is $3\lambda^2/4\pi$ [m^2]. Recently several differently shaped apertures[2,3] have been introduced to demonstrate that effective transmission area can be also $3\lambda^2/4\pi$ [m^2] under resonant condition.

When an aperture is backed by a lossy cavity, what the maximum transmission area through the small coupling aperture will be is a natural curiosity. In order to solve the curiosity, a rectangular cavity with a small coupling rectangular aperture is investigated in this paper. In 1982, Liang[4] studied on the similar problem to the present one but he did not obtain the maximum transmission area. So we are going to focus our attention to investigating the condition for the maximum transmission through the small coupling aperture backed by lossy cavity. In particular the effect of the end wall conductivity on the internal field inside the cavity is examined by use of the approximate perturbation method.

2. Transmission line approach and equivalent circuit

Let's consider the structure of rectangular cavity with a small coupling rectangular aperture shown in Fig. 1. $\vec{E}^i$ and $\vec{H}^i = \hat{x}H_0 e^{-jk_0 z} = x\frac{E_0}{\eta_0}e^{-jk_0 z}$ denote the electric and magnetic field vectors, respectively, of the wave which is incident upon the aperture normally. The medium constants (ε_0 and μ_0) inside the cavity are assumed to be the same as those of the free space, L_x and L_y are width and height of aperture, respectively, and d is the length of cavity. Width a and height b of a rectangular cavity were chosen so that only dominant mode propagates. The geometrical parameters and the end wall conductivity of the cavity are assumed to be chosen for maximum absorption.

The equivalence principle allows the use of an equivalent magnetic current $\vec{M} = \hat{z}\times\vec{E} = V_0\vec{M}_0$ over the aperture region. Here, $\hat{z}$ is the unit vector in the direction of propagation of the incident wave, $\vec{E}$ is the electric field intensity at the aperture, and V_0 is an unknown coefficient to be determined. To determine the coefficient V_0, use is made of the requirement that the tangential component of the magnetic field is continuous across the aperture. This leads to an integral equation which can be converted into a single scalar equation representing the generalized Ohm's law. Using $\vec{M}_0$ as the testing function (Galerkin's method), we obtain $\left(Y^a + Y^b\right)V_0 = I$ where $Y^a = -2\iint_{aperture} \vec{M}_0\cdot\vec{H}_0\left(\vec{M}_0\right)ds$ and $Y^b = \iint_{aperture} \vec{M}_0\cdot\vec{H}_t\left(-\vec{M}_0\right)ds$ are generalized admittances at the aperture, respectively, of the half-space and of the cavity, and I is the generalized current source $I = 2 \iint_{aperture} \vec{M}_0 \cdot \vec{H}_0{}^i ds$ where $\vec{H}_0{}^i$ is the incident magnetic field over the aperture in free space [5].

We treat a rectangular cavity as a waveguide shorted with length d and express it as every transmission line which supports dominant and all high-order modes. Electric and magnetic fields are expressed as sum of incident field produced by equivalent magnetic current and reflected field in each transmission line terminated by surface impedance Z_s of the end wall whose conductivity is given by σ. Note that Electric field and magnetic field distribution inside cavity is assumed to be that for perfect conductor consisting of waveguide (perturbation method).

Fig. 2 shows the equivalent circuit for the present scattering problem. Admittances for the transmission lines of all high-order mode are summed to be jB. Y_1, β_1 and α_1 are characteristic admittance, phase constant and attenuation constant for transmission line for dominant mode, respectively. A turn ratio A is included in order to represent the dependence of dominant mode field amplitude on amplitude of the equivalent magnetic current $\vec{M}$.

3. Aperture-Cavity Resonances and Maximum Transmission Area

In Fig. 2, for the resonance condition to be met, the imaginary part of the total admittance (seen by the current source I) should be zero. Fig. 3 shows the equivalent circuit when the cavity length d is chosen such that the resonance condition may be satisfied. There are two values of the cavity length under the resonance condition. These two values of the cavity length have been found under the assumption that all the inside walls of the cavity be made of the copper except the inputside wall including the coupling aperture.

One resonant cavity length is $d_1 = 0.4937348\lambda_g$, where λ_g means the waveguide wavelength. Fig. 4 illustrates the electric field distribution for both cases that $d_1 = 0.4937348\lambda_g$ and $d_2 = 0.49999917\lambda_g$. It is seen that standing wave patterns are set up for both cases, but, the standing wave field for $d_1 = 0.4937348\lambda_g$ is much stronger than that for $d_2 = 0.49999917\lambda_g$. The resonance for d_1 is more important than that for d_2 because the resonance case for d_1 can be used to make $G^a = G^b$

When $d_1 = 0.4937348\lambda_g$, the $G^b = 2.1\times10^{-8}$ [℧] is obtained for the case that the inside wall of the cavity is made of copper as mentioned earlier. This value of $G^b = 2.1\times10^{-8}$ [℧] is, however, much smaller than $G^a = 9.8418\times10^{-7}$ [℧]. So in order for the maximum transmission condition($G^a = G^b$) via aperture as well as the above resonance condition(cancellation of the imaginary part of the total admittance) to be met, we need to investigate the conductivities of various materials which can give the values of G^b comparable to G^a.

In Table 1. important quantities such as R_s, d_1, G^b, and effective transmission area versus the various conductivities are listed. From this table it is seen that for $\sigma = 2.7\times10^4$ $[S/m]$, the conductance G^b can be made to be almost the same as the conductance G^a. Besides, for $\sigma = 2.7\times10^4$ $[S/m]$ the effective transmission area is seen to amount to $0.7815\ \lambda^2/\pi\ [m^2]$, which corresponds almost to $3\lambda^2/4\pi\ [m^2]$. This discrepancy is thought to be mainly due to the approximate perturbation method which has been used in calculation procedure of internal field inside the cavity.

4. Discussions and Conclusion

In order for the maximum transmission through the aperture to occur, two conditions should be satisfied. One condition is that standing wave should be established. As this result the imaginary part of the total admittance vanishes to zero. The other one is that the real part G^a of the Y^a (admittance looking into the left half space) should be equal to the real part of G^b of the Y^b (admittance looking into the lossy cavity). The transmission area is found to be $3\lambda^2/4\pi$ when the above two conditions are met. It is interesting to note that this transmission area is the same as that for the transmission area for the resonant aperture[2].

Acknowledgments

This research was supported by Basic Science Research Program through the National Research Foundation of Korea(NRF) funded by the Ministry of Education, Science and Technology(2010-0024647)

References

[1] R. F. Harrington, "Resonant behavior of a small aperture backed by a conducting body," IEEE Trans. Antennas & Propagat., vol, AP-30, no.2 pp. 205-212, March 1982.

[2] Ji-whan ko, Junho Yeo, Jong-eon Park, Sam-yul Choi, Young-ki Cho, "Resonant Transmission of a Class of Sub-wavelength Apertures in Thin Conducting Screen," APMC, Singapore, vol. 1, 2009.

[3] J. E. Park, J. I. Lee, J. W. Ko, Y. K. Cho, "Physics in Resonant Transmission on the Small Aperture Coupling," in Proceedings ISAP'09, pp. 887-890,.

[4] Chang-Hong Liang, David K. Cheng, "Electromagnetic fields coupled into a cavity with a slot-aperture under resonant conditions," IEEE Trans. Antennas Propagat. vol. Ap-30, no.4, pp. 664-672, July , 1982.

[5] R. F. Harrington and J. R Mautz, "A generalized network formulation for aperture problems," IEEE Trans. Antennas Propagat., vol. AP-24, pp.870-872, Nov, 1976.

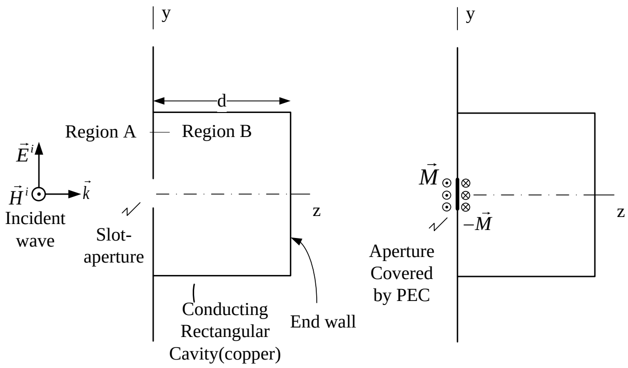

Fig. 1(a): One rectangular cavity with a small coupling rectangular aperture and that applying equivalence principle

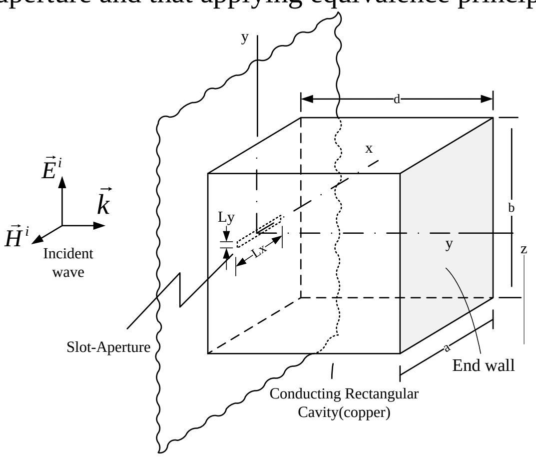

Fig. 1(b): Dimineisons (λ=3 [m], a=0.765λ, b=0.339λ, Lx=0.25λ, Ly=0.02λ)

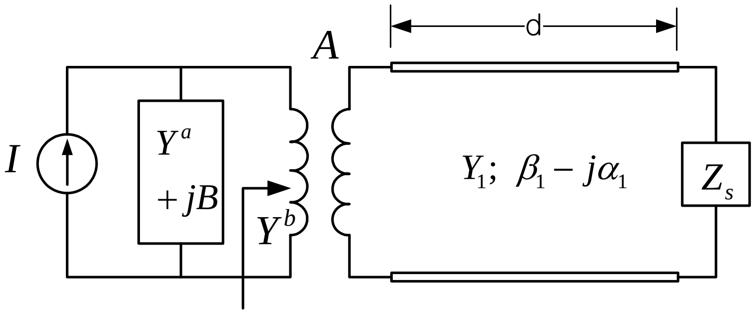

Fig. 2: Equivalent circuit representation

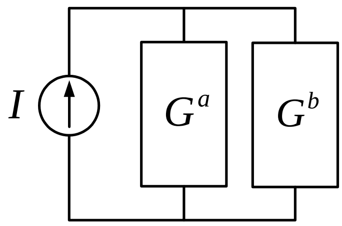

Fig. 3: Equivalent circuit for resonance

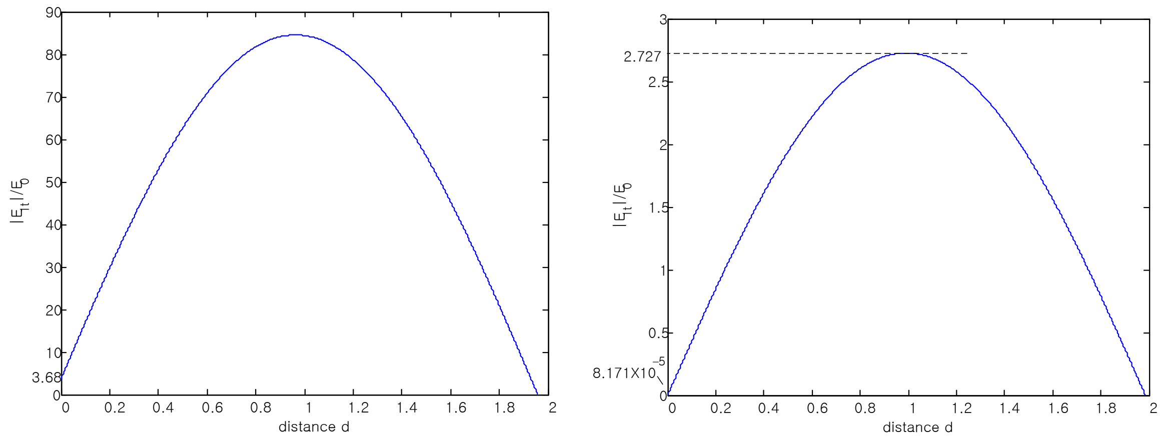

Fig. 4 the electric field distribution for cases that $d_1 = 0.4937348\lambda_g$ and $d_2 = 0.49999917\lambda_g$.

Table 1. R_s, d_1, G^b and effective transmission area versus the various conductivities

	σ (conductivity) [S/m]	R_s [Ω]	d_1 [m]	G^b [℧]	Effective transmission area [m^2]
Copper	5.8×10^{7}	0.0026	1.9605	2.1×10^{-8}	0.0649 λ^2/π
Bronze	10^{7}	0.0063	〃	5.1×10^{-8}	0.1475 λ^2/π
	10^{6}	0.0199	〃	1.62×10^{-7}	0.3804 λ^2/π
	10^{5}	0.0628	〃	5.11×10^{-7}	0.7050 λ^2/π
Graphite	7×10^{4}	0.0751	〃	6.1×10^{-7}	0.7401 λ^2/π
	3×10^{4}	0.1147	〃	9.31×10^{-7}	0.7812 λ^2/π
	2.7×10^{4}	0.1209	〃	9.84×10^{-7}	0.7815 λ^2/π
	2×10^{4}	0.1405	〃	1.12×10^{-6}	0.7765 λ^2/π
	10^{4}	0.1987	〃	1.62×10^{-6}	0.7327 λ^2/π
	10^{3}	0.6283	〃	5×10^{-6}	0.4093 λ^2/π