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Exploring and Stabilizing a Desired Periodic Orbit in Simple Switched Dynamical Systems

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Abstract—This paper studies basic dynamics and control of simple switched dynamical systems. The dynamics is described by a piecewise constant equation, the trajectory is piecewise linear and the embedded return map is piecewise linear. First, using the return map, we analyze typical periodic/chaotic phenomena and related bifurcation. Second, combining the instantaneous state setting with the bi-section methods, we present a simple method for exploring and stabilizing a desire periodic orbit. The performance is investigated in basic numerical experiments.

1. Introduction

The switched dynamical systems (SDS) are defined by plural continuous-time subsystems connected by some discrete switching rules [1]. Depending on the rule, the SDSs can exhibit interesting periodic/chaotic orbits and related bifurcation phenomena. There exist various practical examples of the SDS, such as, the pwm signal generators in power electronics, switching power converters, and the sleep-waking-models [2]-[4]. Analysis of the SDS is important not only as basic nonlinear problem, but also as engineering applications.

This paper studies basic dynamics and control of a simple SDS. The SDS consists of subsystems having piecewise-constant (PWC) characteristics and switching rules depending on both time and state variable. The vector field is PWC, the orbits are piecewise linear (PWL) and the embedded return map is PWL. Using the return map, we analyze basic periodic and chaotic behavior. We then consider an exploring and stabilizing method (ESM) of a target periodic orbit (PEO). The desired PEO is given by the return map. When the target PEO is unknown, the ESM tries to find the PEO by successive instantaneous state setting based on the bi-section method. If the ESM is applied to a sub-region of state variable in which one target exists, we can explore

stabilize the target. The ESM operation is demonstrated by simple numerical experiment. Although there exist many works on analysis and control of the SDS, this is the first paper of the ESM: a systematic method that can realize both exploring and stabilizing simultaneously.

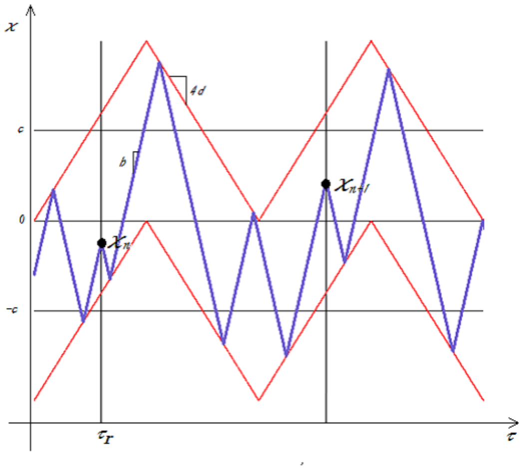

Figure 1: Switched dynamical system

2. Objective System and Return Map

The objective SDS is described by the following dimensionless equation:

$$\frac{dx}{d\tau} = \begin{cases} b & \text{for } x \le u(\tau) + c \equiv u_+(\tau) \\ -b & \text{for } \begin{array}{l} x \ge u(\tau) - c \equiv u_-(\tau) \\ \tau = nT + \tau_r \end{array} \end{cases}$$
$$u(\tau) = \begin{cases} 4d(\tau - 1/4) & \text{for } 0 < \tau \le 1/2 \\ 4d(\tau - 3/4) & \text{for } 1/2 < \tau \le 1 \end{cases} \quad (1)$$
$$u(\tau + 1) = u(\tau)$$

where τ is the dimensionless time, x is the dimensionless state variable and $0 < b$. $u_+(\tau)$ and $u_-(\tau)$ are the upper and lower threshold signals based on periodic triangular waveform with period 1. If the right-hand side is b then x rises. If x exceeds the u_+ then right-hand side is switched from b to $-b$ as shown in Fig 1. If the right-hand side is $-b$ then x decays. If x reaches the u_- then the right-hand side is switched from $-b$ to b. The state variable x can vibrate between u_+ and u_-. In addition to this threshold switching, this SDS has the phase switching: the right-hand side is switched compulsorily to $-b$ at time $n+\tau_r$. Figure 2 shows typical periodic/chaotic waveforms. Note that the phase switching can cause a variety of waveforms that are impossible to the SDS with the threshold switching only in previous works.

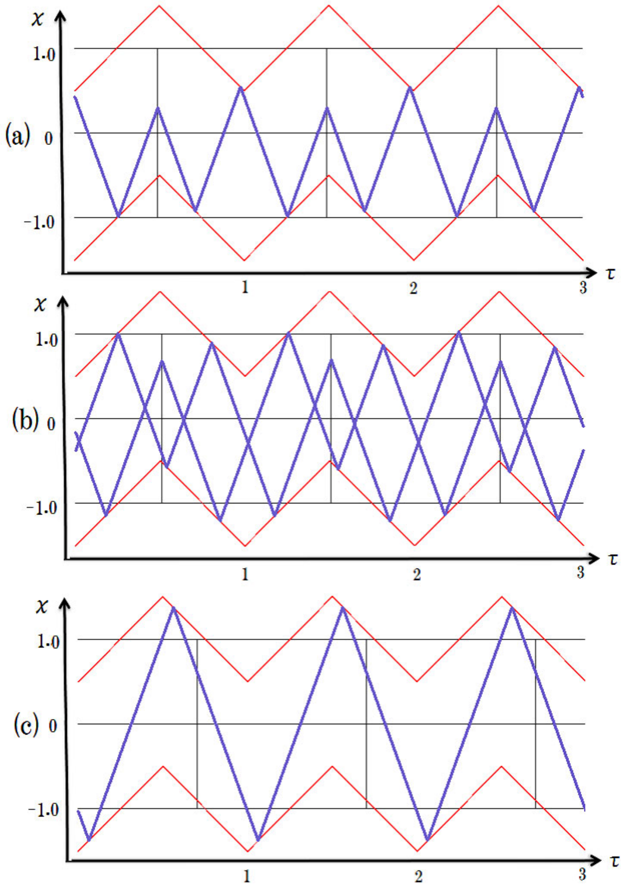

Figure 2: Typical waveforms for $b = 5.5, c = 1, d = 0.5$ (a) periodic waveform for $\tau_r = 0.49$, (b) chaotic waveform for $\tau_r = 0.51$ (overlapped drawing), (c) periodic waveform for $\tau_r = 0.7$.

In order to analyze the SDS dynamics, we derive the return map. Let x_n be the state variable at the phase switching at time $\tau_r + n$. Since x_{n+1} is determined by x_n, we can define the return map.

$$x_{n+1} = F(x_n) \tag{2}$$

Using the exact PWL solution, the return map is described by a PWL formula.

We give basic definition of the periodic orbit. A point p is said to be a periodic point with period k if $p = F^k(p)$ and $p \neq F^l(p)$ for $0 < l < k$ where F^k is the k-fold composition of F. A periodic point p with period 1 is referred to as a fixed point. A periodic point p is said to be stable and unstable if $|DF^k(p)| < 1$ and $|DF^k(p)| > 1$, respectively, where DF is the derivative of F. For simplicity, we select τ_r as a control parameter and fix other parameters $b = 5.5$, $c = 1$ and $d = 0.5$. Figure 3 shows return maps corresponding to wavefroms in Fig. 2. The map in Fig. 3 (a) has one stable and two unstable fixed points.

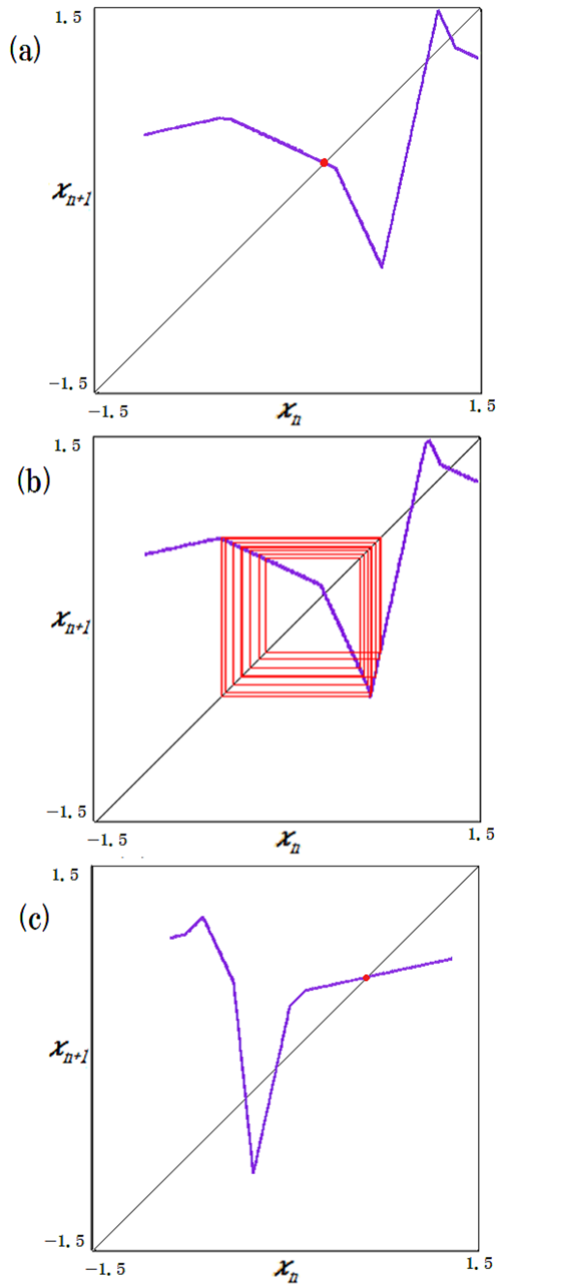

Figure 3: Typical return map for $b = 5.5, c = 1, d = 0.5$ (a) stable fixed point for $\tau_r = 0.49$, (b) chaos for $\tau_r = 0.51$, (c) stable fixed point for $\tau_r = 0.7$

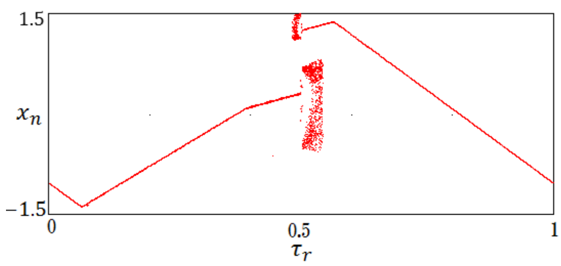

Figure 4: Bifurcation for τ_r

The map in Fig. 3 (b) has three unstable fixed points. This map exhibits chaotic behavior. The map in Fig. 3 (c) has one stable and two unstable fixed points. As the switching phase τ_r varies, the SDS exhibits bifurcation phenomena as shown in Fig. 4.

3. Exploring and Stabilizing Periodic Orbit

We present the ESM for a desired periodic waveform. For simplicity, we consider the case where the target periodic waveform corresponds to a fixed point of the return map. We use the unstable fixed point x_f for $\tau_r = 0.51$ in Fig. 3 (b) as an example.

$$\text{Target fixed point: } x_f = F(x_f)$$

We then assume that the target exists in some closed interval and F is monotonically decreasing

$$x_f \in [q, p] \equiv I_s, \;\; DF(x) < 0 \text{ for } x \in I_s$$

We define the ESM for the case $DF(x) < 0$ that can be translated easily to the case $DF(x) > 0$. Let l be the counter of the ESM.

Step 1. Let $l = 1$ and let $\tau_l = \tau_r + l$.

Step 2. Let $x_{M_l} = (p+q)/2$. Apply x_{M_l} as initial condition at $\tau = \tau_l$ to Eq. (1).

Step 3. Observe the x at time $\tau_l + 1$. If $|x(\tau_l + 1) - x(\tau_l)| < \epsilon$, then $x(\tau_l)$ is the approximate solution of the target and the ESM is terminated. Otherwise go to 4.

Step 4. If $x(\tau_l + 1) > x(\tau_l)$ then $x(\tau_l + 1) \to q$. If $x(\tau_l + 1) < x(\tau_l)$ then $x(\tau_l + 1) \to p$.

Step 5. $l + 1 \to l$, go to 2 and repeat until the maximum time limit τ_{max}.

Figure 5 shows the ESM in the return map where $x(\tau_l)$ and $x(\tau_l+1)$ correspond to x_n and $f(x_n)$, respectively. Figure 6 shows corresponding waveform where we can confirm the process of the ESM.

4. Conclusions

Basic dynamics of the simple SDS and the ESM for a desired periodic orbit is considered in this paper. The dynamics can be analyzed by the PWL return map and the ESM is realized by combination of the instantaneous state setting and the bi-section.

Future problems include exploring the existence region I_s (global search), laboratory experiment and application to practical systems.

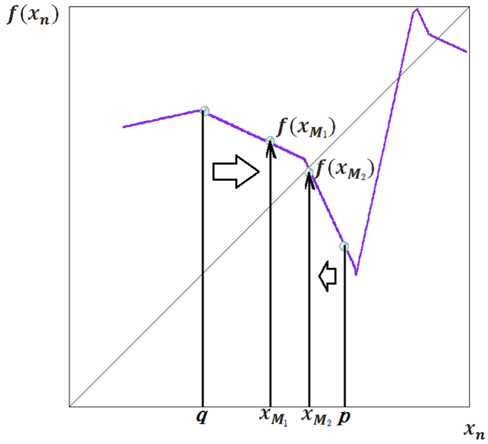

Figure 5: Exploring and stabilizing process in return map

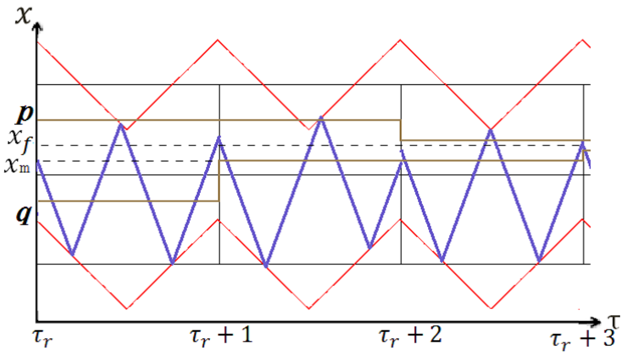

Figure 6: Exploring and stabilizing process in time-domain

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