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Measurement Technique for Experimentally Estimating the Phase Sensitivity Function of an Oscillator

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Abstract—This paper describes an experimental procedure for extracting the phase-sensitivity of an oscillator to noise perturbations. The method relies on injection-locking the oscillator and measuring the widths of the ranges over which the oscillator synchronizes with an injected small-amplitude signal. The resulting sensitivity function can be employed to predict accurately how internal noise sources contribute to output phase-noise and jitter. The experimental estimation procedure is applied to a relaxation oscillator that exhibits strongly nonlinear behavior.

1. Introduction

There is great interest in the field of oscillator design in understanding the sensitivity of an oscillator to noise [1, 2]. One way of capturing this is the so-called Phase Sensitivity Function $\Gamma(t)$. If an accurate model of the oscillator is available, then Γ can be computed numerically.

Maffezzoni introduced a method for determining Γ which is based on measuring the widths of the locking regions when the oscillator is injection-locked to a synchronizing signal [3]. These widths are related ot the coefficients of the Fourier Serier (FS) expansion of Γ. Increasingly better approximations to Γ can be obtained by calculating more FS coefficients.

We have previously developed an efficient method for extracting the boundaries of Arnold tongues (locking regions) in injection-locked frequency dividers (ILFDs) [4]. A variant of this technique is described in this work. In particular, we calculate the widths of a number of tongues and thereby determine the FS coefficiencts of Γ.

Although we illustrate the method for the same circuit described by Maffezzoni, for which an analytical model exists, we emphasize that the technique works for oscillators where an accurate simulation model is not available.

2. Arnold Tongue Scenario

Nonlinear dynamics provides a paradigm for frequency entrainment (injection locking), namely the so-called *standard circle map* or *sine* map [5]. Consider a first (dependent) oscillator with unforced frequency f_0 that is driven by a second (independent) oscillator with frequency f_s. If the phase θ of the first oscillator is sampled at the frequency of the second oscillator, then samples are described by a discrete-time dynamical system of the form:

$$\theta[n+1] = \theta[n] + \Omega - \frac{k}{2\pi}\sin(2\pi\theta[n]), \qquad (1)$$

where $\Omega = f_0/f_s$ is the ratio of the unforced and injected signals and k is the strength of the coupling.

The behavior of this system is summarized in a two-dimensional bifurcation diagram, as shown in Fig. 1.

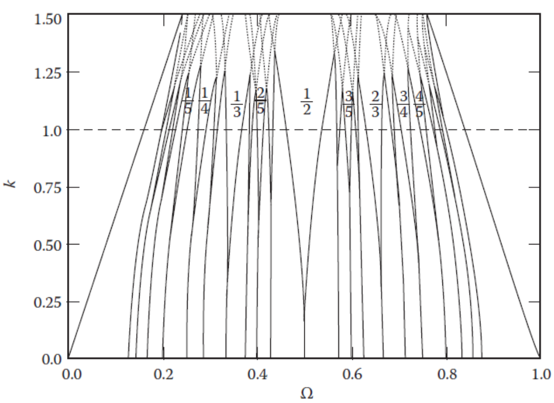

Figure 1: Bifurcation diagram summarizing typical locking behavior in an ILFD. The abscissa Ω denotes the relative forcing frequency; the ordinate k indicates the strength of the coupling. The so-called "divide-by-ρ"regions corresponding to constant values of the rotation number ρ are called Arnold tongues (adapted from [5]).

The abscissa (horizontal axis) Ω in Fig. 1 shows the relative frequency of the injected signal and the ordinate (vertical axis) k is the strength of the coupling between the input signal and the oscillator. The bifurcation diagram is organized into regions called Arnold tongues; the rotation number ρ is constant inside each tongue.

3. Phase Sensitivity Function

When a small-signal perturbation is injected into an oscillator, Maffezzoni [3] has shown that the externally injected signal is dominated by the phase-sensitivity function $\Gamma(t)$ for some certain types of perturbation [1, 2]. The waveform of the function is T_0-periodic and can be expanded in a Fourier series of the form:

$$\Gamma(t) = \Gamma_0 + \sum_{k=1}^{N_H} \Gamma_k \cos(k\omega_0 t + \phi_k) \tag{2}$$

where $\omega_0 = 2\pi/T_0$ is the free-running frequency of the oscillator, Γ_k and ϕ_k are rational numbers, and N_H is the number of significant harmonic components.

For harmonic oscillators, the phase-sensitivity function is dominated by its first harmonic component Γ_1; for non-harmonic oscillators, the function can be affected by a few significant harmonic components.

When the oscillator is in its locked state or, in other words, when 1:m synchronization behavior occurs (the so-called "divide-by-m" locking region), the lower and upper boundaries (ω_1 and ω_2, respectively) of the corresponding Arnold tongue are approximately

$$\omega_1 = m\omega_0 - \frac{m\omega_0 A_{in}\Gamma_m}{2}, \quad \omega_2 = m\omega_0 + \frac{m\omega_0 A_{in}\Gamma_m}{2}, \tag{3}$$

where A_{in} is the amplitude of the externally injected small-signal. Γ_m can be extracted by calculating

$$\Gamma_m = \frac{(\omega_2 - \omega_1)}{m\omega_0 A_{in}\cos(\theta)}, \tag{4}$$

where θ is an as yet unknown phase which can be obtained by applying

$$\theta = \frac{\pi - \Delta\theta}{2}, \tag{5}$$

where $\Delta\theta$ is the phase difference (or swing range) between the upper and lower locking region boundaries; thus, $\Delta\theta = \theta_{max} - \theta_{min}$.

The DC component of the phase-sensitivity function $\Gamma(t)$ is defined by

$$\Gamma_0 = \frac{(\omega - \omega_0)}{A_{in}\omega_0}, \tag{6}$$

where ω is the response frequency of the oscillator when a DC component is injected.

The final quantity we require is the phase constant ϕ_k,

$$\phi_k = -\frac{2\pi(t_k - t_1)}{T/k} \tag{7}$$

where t_1 and t_k are the times of a pair of zero-crossing events when the injected small-signal is at the centre of tongue k. t_1 corresponds to "divide-by-1" synchronization and t_k refers to "divide-by-k." For additional details concerning the extraction procedure for ϕ_k, see [3].

4. Experimental Estimation Procedure

4.1. Experimental Circuit

In the experiment, we have studied the relaxation oscillator described in [3]; this is shown in Fig. 2. All transistors are BS170 NMOS devices. In this circuit, the cross-coupled pair $Q1$ and $Q2$ produces negative resitance; $Q3$, $Q4$ and $Q5$ provide the bias currents. The component values are shown in Table 1.

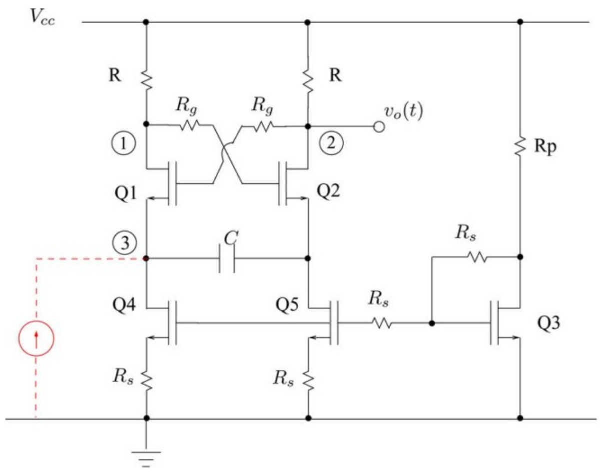

Figure 2: Relaxation oscillator (adapted from [3])

Parameter	Value
V_{cc}	7.0 V
R	200 Ω
R_g	1 kΩ
R_s	10 Ω
R_p	1 kΩ
C	1 nF

Table 1: Component values.

The input and output nodes are nodes 3 and 2 in Fig. 2, respectively.

In our experiment, an opamp-based Howland current source is used to inject the external small-signal perturbation; this is shown in Fig. 3. Terminal V_{IN^-} is grounded. The input signal is applied between V_{IN^+} and ground. The current injected into node 3 in Fig. 2 is defined by:

$$I_{OUT} = \frac{V_{IN^+}}{R}, \tag{8}$$

where $R = 100\ k\Omega$ in our example.

The experimental setup is shown in Fig. 4. The frequency generator is Agilent model 33250A. The frequency counter is Agilent model 53230A. In addition, an oscilloscope is connected to the output of the oscillator; the oscilloscope is Tektronix model TDS 3034B. The experiment

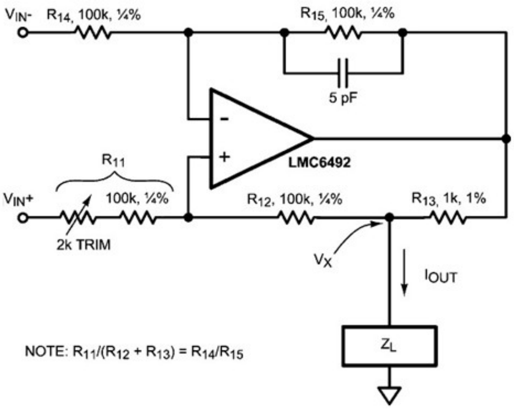

Figure 3: Howland current source

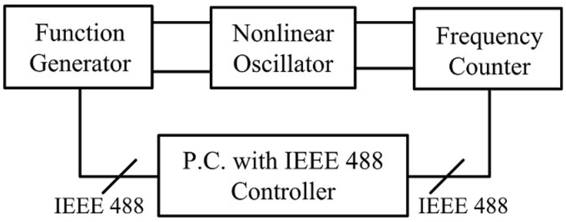

Figure 4: Experimental setup.

is controlled by Labview software. The measured data is analysed using Matlab.

4.2. $\Gamma(t)$ Estimation Procedure

With the experimental setup described in Sec. 4 and the parameter values in Tab. 1, the circuit oscillates with a free-running frequency $f_0 = 711$ kHz ($\omega_0 = 2\pi f_0$).

Maffezzoni showed that the DC component and the first three harmonic components are dominant. Therefore, in our experiment, we determine the widths of the first three Arnold tongues; these are shown in part in Fig. 5.

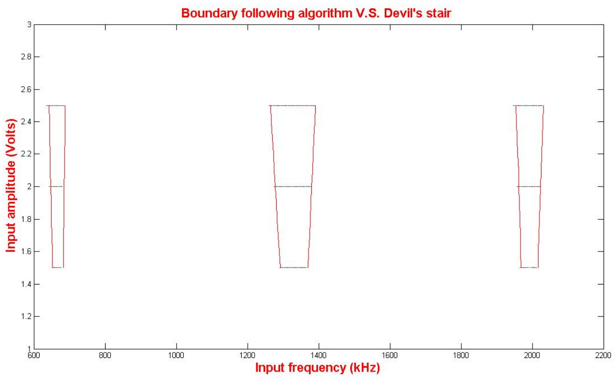

Figure 5: The red curves are measured by the boundary-following algorithm based in Labview software [4] and the horizontal curves are measured by devil's staircase method

The middle curves in Fig. 5 correspond to an injected signal amplitude of $A_{in} = 20\mu A$. We will use these cross-sections through the Arnold tongues to extract the FS coefficients of $\Gamma(t)$.

4.2.1. Γ_0

Firstly, by DC injection, we obtain the DC response $\omega = 2\pi \times 712.5$ kHz. Subsituting into Eq. (7), we obtain

$$\Gamma_0 = 106 \text{ A}^{-1}. \tag{9}$$

4.2.2. Γ_k *for* $k > 0$

We next extract Γ_k (for $k = 1, 2, 3$) in the Fourier series (2). The lower and upper (ω_1, ω_2) limits in (5) can be easily found from Fig. 5. The value of θ in the denominator can be calculated using (6). In the experiment, it is measured and recorded by the Agilent 53230A universal frequency counter.

The basis of our method is to sweep the frequency through the locking region, and to measure the standard deviation of the phase (the term phase here represents the phase-difference between the injected signal and the corresponding output response), as shown in Fig. 6.

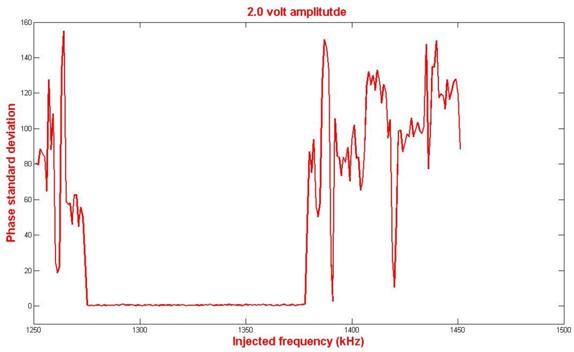

Figure 6: Phase standard deviation

When the standard deviation is close to zero, the oscillator is deemed to be in a locked region. Then, the phase is extracted, as shown in Fig. 7.

Using this method, we obtain:

$$\Gamma_1 = 1502 \text{ A}^{-1}, \ \Gamma_2 = 2606 \text{ A}^{-1}, \ \Gamma_3 = 3627 \text{ A}^{-1}. \tag{10}$$

4.2.3. ϕ_k *for* $k > 0$

Finally, the last term to be calculated is the phase constant ϕ_k. The way to measure it is explained in [3]. Injecting the centre frequency for each Arnold tongue, store the waveforms of all injected signals using the oscilloscope. Then plot two of them with the same time-reference. By measuring the time interval between two adjacent zero crossings of the injected signal waveforms and using (8), we obtain

$$\phi_1 = 0 \text{ rad}, \ \phi_2 = -1.0028 \text{ rad}, \ \phi_3 = -1.8802 \text{ rad}. \tag{11}$$

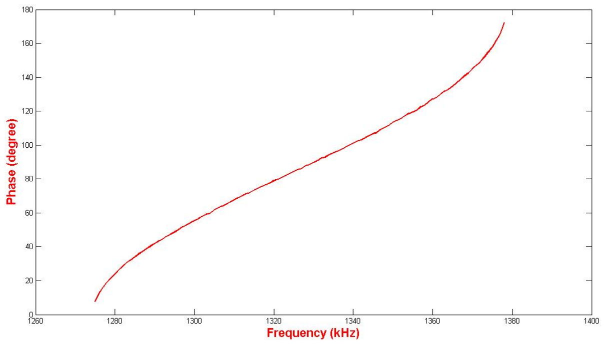

Figure 7: Phase of locking region

4.3. Experimentally Estimated $\Gamma(t)$

We conclude by plotting the phase sensitivity function $\Gamma_{(t)}$ using(2) and the coefficients determined above.

The experimentally estimated phase sensitivity function is shown in Fig. 7.

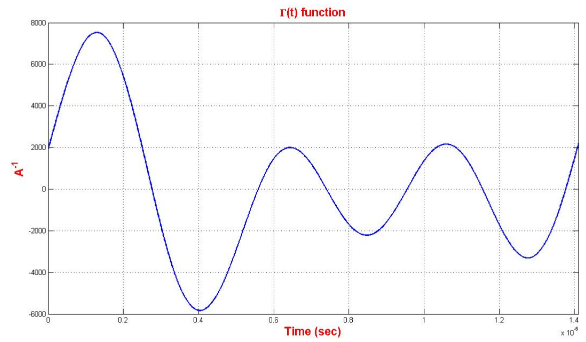

Figure 8: Experimentally determined phase sensitivity function $\Gamma_{(t)}$

Compare this with the estimate shown in Fig. 9 that was produced by Maffezzoni using the analytical model of the circuit. While our curve is qualitatively similar to that produced by simulation, further work is required to explain away the differences between the two curves.

While we used a Howland current source to inject the synchronization signal, Maffezzoni used a voltage source with a large series resistor. This could account for the scaling difference between the curves. Futhermore, there are clear discrepancies between the two circuits in terms of the free-running frequency f_0, indicating that the component values used are different. Finally, Maffezzoni used five FS coefficients while we calculated just four. We are carrying out further research to reconcile the differences between the two sets of results.

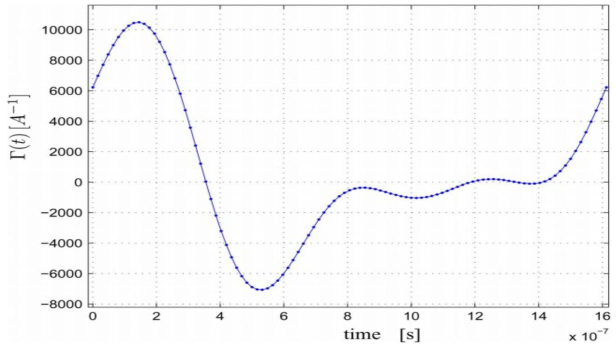

Figure 9: Simulated phase sensitivity function (reproduced from [3]).

5. Conclusion

We have presented an experimental method for estimating the FS coefficients of the phase sensitivity function $\Gamma(t)$ of an oscillator. We have confirmed the viability of the method in the case of a relaxation oscillator for which Maffezzoni has produced simulated results [3].

Acknowledgement

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