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# Analysis of an Interrupted Electric Circuit with Non-Ideal Switching 

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#### Abstract

Spike noise and switching delay can affect the qualitative property of the converter circuit. In this paper, we assume that these non-ideal switching occur in an interrupted circuit. We study the effect of non-ideal switching in the circuit. First, we show the circuit and explain the behavior of the waveform. Then, we derive the return map for the following analysis. Finally, we study the dynamical effect of non-ideal switching based on the return map and bifurcation diagrams.


## 1. Introduction

The discrete map of the electric circuits containing the switch constructs a piecewise smooth map. The DC/DC converter circuits are the typical example of it. These systems exhibit many interesting phenomena due to the complex switching action. Also, it is important to analyze the complex behavior of the circuit for understanding the qualitative property. Thus, many papers have studied the nonlinear phenomena in the circuit [1][2].

The previous works assumed that the switching action behave ideally. On the other hand, Banerjee et al clarified the effect of spike noise and time delay based on the laboratory experiment [3]. However, there is no paper that studied the effect of spike noise or time delay switching except for Refs [3][4][5]. It is important to study the effect of nonideal switching for understanding the qualitative property of the switching circuit. Thus, we have proposed a simple interrupted circuit with spike noise or switching delay, which simulates the switching dynamics of the converters, and have studied the dynamical effect of switching delay and spike noise [6][7]. The effect of switching delay or spike noise have been studied in the previous work. Although there is a possibility that both of the switching delay and spike noise occur in the interrupted circuit, there is no paper which studied such situation. Therefore, we propose the circuit model with non-ideal switching in order to understand the qualitative property of the system.

In this paper, we focus on the situation that both of the switching delay and spike noise occur in the interrupted electric circuit. First, we show the circuit with switching delay and spike noise, and then we explain the behavior of the waveform. Next, we derive the return map in order to analyze the circuit. Finally, we explain the effect of non-
ideal switching based on the return map and bifurcation diagrams.

## 2. An interrupted electric circuit

### 2.1. Circuit description

Figure 1 shows an interrupted circuit with with non-ideal switching. The circuit equation is described as follows:

$$
\frac{d y}{d \tau}=-y+h(\tau, y), h(\tau, y)= \begin{cases}B & : \text { system a }  \tag{1}\\ 0 & : \text { system } \mathrm{b}\end{cases}
$$

Now, we use $\tau=t /(R C), y=v$ and $B=E$ as the dimensionless variables. In Eq. (1), $y$ is the state variable and $\tau$ is the normalized time variable.

Figure 2 (a) shows an example of the waveform with ideal switching. The system changes from a to b when the waveform reaches the reference value $y_{\mathrm{r}}$. After that, the system changes from $b$ to $a$ if the next clock pulse arrives.

Figure 2 (b) shows an example of the waveform in the system with non-ideal switching. The basic switching rule is the same as the system with ideal switching. But, if the waveform reaches the reference value $y_{\mathrm{r}}$ the system changes from a to b after the switching delay $\tau_{\mathrm{d}}$. In addition, the spike noise $h$ appears via every switching action. If the waveform at $\tau=k T$ or the peak of the spike noise $y_{k}+h$ is higher than the reference value $y_{\mathrm{r}}$, the system keeps state $b$ until the next clock pulse arrives.


Figure 1: Circuit model with non-ideal switching.

(b)System with non-ideal switching

Figure 2: Example of the waveform.

### 2.2. Return map

Next, We derive the return map to analyze the property of the system. Figure 3 shows the classified waveform with non-ideal switching. The border $D_{1}$ and $D_{2}$ are used for the classification of the waveform.

$$
\begin{equation*}
D_{1}=y_{\mathrm{r}}-h, D_{2}=\left(y_{\mathrm{r}}-B\right) e^{T-\tau_{\mathrm{d}}}+B, \tag{2}
\end{equation*}
$$

The system with non-ideal switching has three types of the possible waveforms during the clock interval. In particular, Fig. 3 (a) shows the waveform that keeps state a. Furthermore, The waveform that the system changes from a to b is shown in Fig. 3 (b). Figure 3 (c) expresses the waveform that keeps state $b$. Note that the waveform that keeps state $b$ is never seen in a system with ideal switching. Also, there are two types of the waveform in Fig. 3 (c). The return map is shown below.

$$
\begin{align*}
& y_{k+1}=F\left(y_{k}\right) \\
& =\left\{\begin{array}{c}
\left(y_{k}-B\right) e^{-T}+B, \quad y_{k} \leq D_{2} \\
\frac{y_{k}-B}{y_{\mathrm{r}}-B}\left(y_{\mathrm{r}}-B+B e^{\tau_{\mathrm{d}}}\right) e^{-T}, \\
\left(D_{1} \leq y_{k} \text { and } y_{k-1}<D_{2}\right) \\
\left(D_{2}<y_{k} \leq D_{1}\right) \\
y_{k} e^{-T}, \quad D_{1} \leq y_{k} \text { and } D_{2} \leq y_{k-1}
\end{array}\right. \tag{3}
\end{align*}
$$

$$
\begin{align*}
& \text { Figure 3: Classified waveforms during the clock interval } T \text {. } \\
& \text { The derived function is expressed as follows: } \\
& (k-1) T k T \quad D_{1} \leq y_{k} \text { and } D_{2} \leq y_{k-1} \\
& \frac{d F\left(y_{k}\right)}{d y_{k}}=\left\{\begin{array}{l}
e^{-T}, \quad y_{k} \leq D_{2} \\
\frac{y_{k}-B+B e^{\tau_{\mathrm{d}}}}{y_{\mathrm{r}}-B} e^{-T}, \\
\left(D_{1} \leq y_{k} \text { and } y_{k-1}<D_{2}\right) \\
\left(D_{2}<y_{k} \leq D_{1}\right)
\end{array}\right. \\
& \begin{array}{l}
e^{-T}, \quad D_{1} \leq y_{k} \text { and } D_{2} \leq y_{k-1}
\end{array} \tag{4}
\end{align*}
$$



It is clear from Eq. (4) that the slope of return map is a constant value. In contrast, we define the return map and the derived function in the system with ideal switching at $h=0$ and $\tau_{\mathrm{d}}=0$. Moreover, we define the return map and the derived function in the system with spike noise at $h=0.2$ and $\tau_{\mathrm{d}}=0$. In addition, we define the return map and the derived function in the system with switching delay at $h=0$ and $\tau_{\mathrm{d}}=0.2$.

## 3. Analytical result

Figure 4 shows an example of the return map in the system with ideal switching, spike noise, switching delay and non-ideal switching. The red regions in Fig. 4 shows invariant interval. The border $D_{3}$ and $D_{4}$ are used for the derivation of invariant interval.

$$
D_{3}=y_{\mathrm{r}}, D_{4}=\left(y_{\mathrm{r}}-B\right) e^{T}+B
$$

Here, the range of the invariant interval $I_{1}$ is given by

$$
\begin{equation*}
I_{1}=\left[F^{2}\left(D_{4}\right), F\left(D_{4}\right)\right] \tag{6}
\end{equation*}
$$

where

$$
\left\{\begin{array}{l}
F^{2}\left(D_{4}\right)=y_{\mathrm{r}} e^{-T}  \tag{7}\\
F\left(D_{4}\right)=y_{\mathrm{r}}
\end{array} .\right.
$$

Therefor, the invariant interval $I_{1}$ is

$$
\begin{equation*}
I_{1}=\left[y_{\mathrm{r}} e^{-T}, y_{\mathrm{r}}\right] \tag{8}
\end{equation*}
$$

Additionally, we express the range of the invariant interval $I_{2}$ as follows:

$$
\begin{equation*}
I_{2}=\left[F\left(D_{1}\right), F\left(D_{4}\right)\right], \tag{9}
\end{equation*}
$$

where

$$
\left\{\begin{array}{l}
F\left(D_{1}\right)=y_{\mathrm{r}} \mathrm{e}^{-T}-h e^{-T}  \tag{10}\\
F\left(D_{4}\right)=y_{\mathrm{r}}
\end{array}\right.
$$

Thus, the invariant interval $I_{2}$ is

$$
\begin{equation*}
I_{2}=\left[y_{\mathrm{r}} e^{-T}-h e^{-T}, y_{\mathrm{r}}\right] . \tag{11}
\end{equation*}
$$

Similarly, the range of the invariant interval $I_{3}$ is expressed as

$$
\begin{equation*}
I_{3}=\left[F\left(D_{3}\right), F\left(D_{2}\right)\right], \tag{12}
\end{equation*}
$$

where

$$
\left\{\begin{array}{l}
F\left(D_{3}\right)=y_{\mathrm{r}} e^{-T}  \tag{13}\\
F\left(D_{2}\right)=y_{\mathrm{r}}+\left(e^{-\tau_{\mathrm{d}}}-1\right)\left(y_{\mathrm{r}}-B\right)
\end{array} .\right.
$$

Therefore, the invariant interval $I_{3}$ is

$$
\begin{equation*}
I_{3}=\left[y_{\mathrm{r}} e^{-T}, y_{\mathrm{r}}+\left(e^{-\tau_{\mathrm{d}}}-1\right)\left(y_{\mathrm{r}}-B\right)\right] . \tag{14}
\end{equation*}
$$

On the other hand, the range of the invariant interval $I_{4}$ is shown as

$$
\begin{equation*}
I_{4}=\left[F\left(D_{1}\right), F\left(D_{2}\right)\right] \tag{15}
\end{equation*}
$$

where

$$
\left\{\begin{array}{l}
F\left(D_{1}\right)=y_{\mathrm{r}} e^{-T}-h e^{-T}  \tag{16}\\
F\left(D_{2}\right)=y_{\mathrm{r}}+\left(e^{-\tau_{\mathrm{d}}}-1\right)\left(y_{\mathrm{r}}-B\right)
\end{array} .\right.
$$

From these results, the invariant interval $I_{4}$ is

$$
\begin{equation*}
I_{4}=\left[y_{\mathrm{r}} e^{-T}-h e^{-T}, y_{\mathrm{r}}+\left(e^{-\tau_{\mathrm{d}}}-1\right)\left(y_{\mathrm{r}}-B\right)\right] . \tag{17}
\end{equation*}
$$

Here, we set the parameter $h>0, \tau_{\mathrm{d}}>0$ and $B>y_{\mathrm{r}}$. It is obvious that the invariant intervals of the system with spike noise, switching delay and non-ideal switching are larger than the system with ideal switching. Moreover, invariant interval of the system with non-ideal switching is the largest in Eqs. (10), (13), (16). We consider that expansion of the invariant interval affect a periodic solution.

Figure 5 shows the two parameter bifurcation diagrams in the system with ideal switching, spike noise, switching


Figure 4: Example of return map.
delay and non-ideal switching. Horizontal axis and vertical axis denote the reference value $y_{\mathrm{r}}$ and clock pulse $T$, respectively. Note that two types of the diagrams in the circuit with ideal switching, spike noise, switching delay and non-ideal switching are calculated by increasing and decreasing the parameters $\left(y_{\mathrm{r}}\right.$ and $\left.T\right)$. The figure of rightside and left-side are observed same figure in Fig. 5 (a). On the other hand, Fig. 5 (b)-(d) have coexistence region of the solution in the circle region. Here, it is clear that the coexistence region of the system with non-ideal switching is the largest in Fig. 5.

Next, we pay attention to structure of the return map. We find the new mapping region in the system with spike noise, switching delay and non-ideal switching (see the blue circle region in Fig. 4). This is the result from the waveform that occurs never seen in the system with ideal switching. In addition, we know that the new mapping region of the system with non-ideal switching is larger than the system with spike noise and switching delay. This is why there is new type of the waveform that keeps state b during the clock interval in the system with non-ideal switching. Specifically, the existence regions of the above new types of the waveform, in the circuit with spike noise $I_{\mathrm{s}}$, in the circuit with switching delay $I_{\mathrm{d}}$, in the circuit with spike noise and switching delay $I_{\mathrm{m}}$, are defined as follows:

$$
\begin{align*}
& I_{\mathrm{s}}=\left\{y_{k-1}, y_{k} \mid D_{1} \leq y_{k} \text { and } D_{4} \leq y_{k-1}\right\}  \tag{18}\\
& I_{\mathrm{d}}=\left\{y_{k-1}, y_{k} \mid D_{3} \leq y_{k} \text { and } D_{2} \leq y_{k-1}\right\}  \tag{19}\\
& I_{\mathrm{m}}=\left\{y_{k-1}, y_{k} \mid D_{1} \leq y_{k} \text { and } D_{2} \leq y_{k-1}\right\} \tag{20}
\end{align*}
$$


(a)System with ideal switching

(b)System with spike noise $(h=0.2)$

(c)System with switching delay $\left(\tau_{\mathrm{d}}=0.2\right)$


Figure 5: Two-parameter bifurcation diagrams ( $B=3.0$ ).

We mathematically conclude that if the spike noise and switching delay occur in the interrupted circuit, existence region of the new type of the waveform is enlarged. It will seriously affect in the qualitative property of this class of the circuit.

## 4. Conclusion

This paper has presented the analysis of an interrupted electric circuit with non-ideal switching. First, we showed the circuit model and explained the behavior of the waveform. Then, we derived the return map for the following analysis. Finally, we discussed the dynamical effect of nonideal switching based on the bifurcation diagrams. As the result, we found that if both of the spike noise and switching delay occur in the interrupted circuit, existence region of the new type of the waveform is enlarged. It will seriously affect in the qualitative property of this class of the circuit. The analytical results will be adapted to the practical DC/DC converters, because our circuit model simulates the dynamics of the current-model controlled DC/DC converters. In the future, we will consider the bifurcation analysis mathematically.

## References

[1] S. Banerjee and G.C. Verghese., "Nonlinear Phenomena in Power Electronics: Attractors, Bifurcations, Chaos, and Nonlinear Control," Piscataway, NJ: IEEE Press, 2001.
[2] C.K. Tse., "Complex Behavior of Switching Power Converters," Boca Raton: CRC Press, 2003.
[3] S. Banarjee, S. Parui, and A. Gupta. "Dynamical Effects of Missed Switching in Current-Model Controlled dc-dc Converters" IEEE Trans. Circuits and Systems II, vol. 51 (12), pp. 649-654, 2004.
[4] T. Maruyama, N. Inaba, Y. Nishio and S.Mori. "Chaos in an Auto Gain Controlled Oscillator Containing Time Delay." IEICE Transaction on Electronics, Information and Communication Engineers A 1989; 72(11): 18141820. (In Japanese)
[5] M. Kuboshima and T. Saito. "Bifurcation from a chaos generator including switched inductor with time delay." IEICE Transaction on Fundamentals of Electronics 1997; 80(9): 1567-1571.
[6] H. Asahara, T. Kousaka. "Qualitative Analysis of an Interrupted Electric Circuit with Spike Noise." International Journal of Circuit Theory and Applications, Vol. 39-11, pp. 1177-1187, 2011.
[7] A. Matsuo, H. Asahara, T. Kousaka. "Experimental Verification of Chaotic Attractor in an Interrupted Circuit with Spike Noise." IEE Japan, Vol.132-C, No.5, pp.1-2, 2012.(In Japanese)

