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Bifurcation Structure of a Class 2 Silicon Nerve Membrane Integrated Circuit

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Abstract—We conducted experimental measurements of the silicon nerve membrane integrated circuit that was designed and fabricated through the TSMC 0.35 μm CMOS technology. The responses of the silicon nerve membrane, which possesses Class 2 excitability, to periodic pulse stimuli are investigated experimentally and the two-parameter bifurcation diagram is obtained in this paper. The bifurcation diagram shows the bifurcation structure of fundamental harmonic and sub-harmonic synchronization regions, and the emergence of period-adding bifurcations. It is also found that the fabricated Class 2 silicon nerve membrane exhibits nonperiodic behaviors subject to some periodic pulse stimuli.

1. Introduction

The neuromorphic engineering is a interdisciplinary field that is involved in biology, physics, mathematics, computer science, and engineering to design advanced information processing systems whose physical architecture or functional principles are based on those of the nervous system. Silicon neuron, which is one of the neuromorphic devices, is an electronic circuit that is designed to reproduce various activities in neurons.

There are some major approaches to model a silicon neuron. One is phenomenological modeling. This modeling is quite simplified and can lead to compact circuits, however, it realizes only limited properties of biological neurons such as in the integrate-and-fire models because it ignores the mechanisms under neuronal phenomena. Another is conductance-based modeling. This modeling develops various phenomena observed in the biological neurons by reproducing the ionic channel dynamics around neuronal cells, however, the circuit implementation tends to be complicated, and it is difficult to analyze its dynamics mathematically because of the intrinsic complexity. Recently, Kohno and Aihara proposed a new approach to design a silicon neuron which is named "Mathematical-structure-based design." They introduced mathematical analyses into modeling, and designed Class 2 silicon nerve membrane electric circuit by using the discrete MOSFETs [1, 2]. The modeling utilizes phase plane analysis and bifurcation analysis to reproduce the mathematical structures in neurons, and construct a biologically-plausible electronic nerve membrane.

In this paper, we report an integrated circuit (IC) that implements a silicon nerve membrane. The mathematical-structure-based design is utilized as the modeling approach. The prototype chip of our nerve membrane circuit was fabricated through the TSMC 0.35 μm CMOS technology. The MOSFETs are designated to operate in their subthreshold region, and therefore, the fabricated chip is intended to consume very low power. We conducted the experimental measurements and investigated responses of the Class 2 silicon nerve membrane to periodic pulse stimuli. The obtained two-parameter bifurcation diagram shows the bifurcation structure of fundamental harmonic and sub-harmonic synchronization regions, and the emergence of period-adding bifurcations.

It is well known that Class 2 nerve membrane, such as the Hodgkin-Huxley model, exhibits not only periodic but nonperiodic responses depending on the applied periodic stimuli [3, 4]. We also found that the fabricated silicon nerve membrane, which exhibits Class 2 excitability, shows nonperiodic behaviors subject to some periodic pulse stimuli.

2. Model of Silicon Nerve Membrane

Our silicon nerve membrane model is a two-dimensional system as follows:

$$\begin{aligned} C_v \frac{\mathrm{d}v}{\mathrm{d}t} &= f_m(v) - g(v) - n + I_a + I_{stim}, \\ \frac{\mathrm{d}n}{\mathrm{d}t} &= \frac{f_n(v) - n}{T_n}, \end{aligned} \qquad (1)$$

where v and n correspond to the membrane potential and the variable that represents activity of ionic channels, respectively. C_v represents the membrane capacitance and T_n is the time constant for variable n. I_a is an ionic current that is independent of the membrane potential, and I_{stim} is a stimulus current applied externally. Functions $f_x(v)$, where x is m or n, and $g(v)$ are sigmoid function expressed as follows:

$$\begin{aligned} f_x(v) &= M_x \frac{1}{1 + \exp\left(-\frac{\kappa}{U_T}(v - \delta_x)\right)}, \\ g(v) &= S\,\frac{1 - \exp\left(-\frac{\kappa}{2U_T}(v - \theta)\right)}{1 + \exp\left(-\frac{\kappa}{2U_T}(v - \theta)\right)}, \end{aligned} \qquad (2)$$

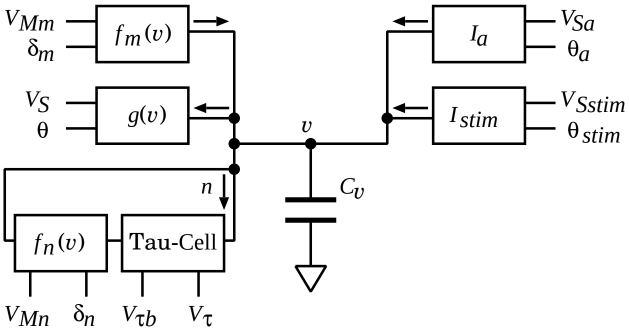

Figure 1: Block diagram of our silicon nerve membrane IC.

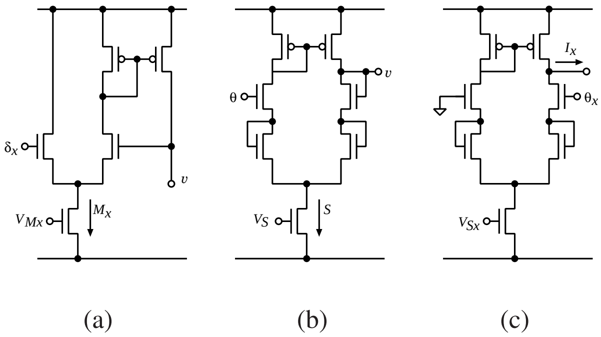

Figure 2: Schematic circuit of (a) $f_x(v)$, (b) $g(v)$, and (c) I_a or I_{stim} block.

where κ is the gate coupling coefficient that represents the coupling of the gate to the surface potential, and U_T is the thermal voltage.

Figure 1 shows the block diagram of the silicon membrane IC. It consists of a capacitor C_v, differential pair circuits shown in Fig. 2, and Tau-Cell [5] shown in Fig. 3 (a). The differential pair circuits realize the sigmoid functions of Eq. (2), and Tau-Cell integrates variable n. The dynamics of Tau-Cell is expressed as

$$\frac{dn}{dt} = \frac{I_\tau}{CU_T}(I_{in} - n), \qquad (3)$$

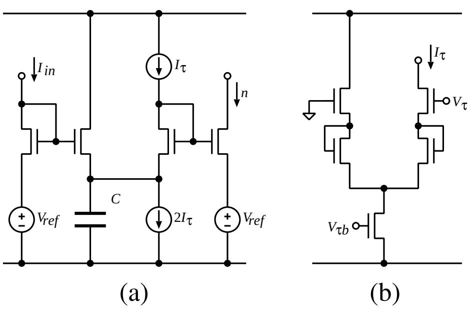

Figure 3: Schematic circuit of (a) Tau-Cell and (b) current source I_τ.

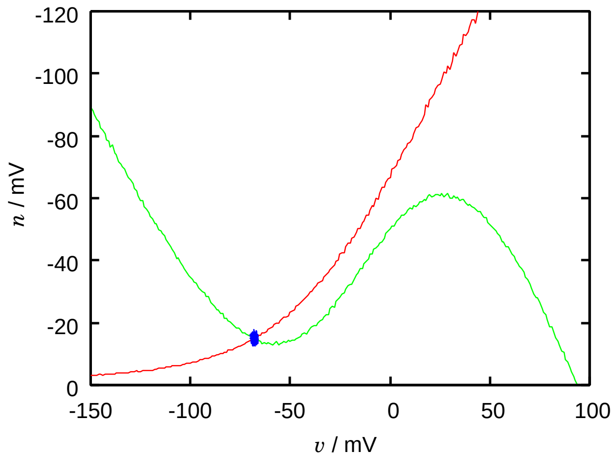

Figure 4: Measured v–n phase plane of our silicon nerve membrane IC. Red and green curves are n- and v-nullclines, respectively, and blue dots draw the attractor.

where C and U_T are capacitance shown in Fig. 3 (a) and the thermal voltage, respectively.

3. Experimental Measurement Results

Our chip includes a feedback subsystem that allows us to conduct voltage-clamp measurement and draw the nullclines on the v–n phase plane. We determined the operating parameter set of the silicon nerve membrane as shown in Table 1 utilizing this subsystem. Figure 4 shows the measured phase plane, where red and green curves are n- and v-nullclines, respectively. The current n is converted into voltage by means of an embedded highly-resistive circuit. The intersection of these nullclines is an equilibrium. We determined the physical parameters so that the equilibrium becomes a stable focus by bifurcation analysis. The blue dots draw the attractor, from which the resting membrane potential is estimated to be about −65 mV.

We examined the neural excitability by applying sustained stimulus. Figure 5 shows the firing frequency according to the sustained stimuli with various strengths. The silicon nerve membrane shows a sweep-direction-

Table 1: Parameter set

Parameter	Value	Parameter	Value
V_{DD}	1.650 V	δ_n	0.011 V
V_{SS}	−1.650 V	$V_{\tau b}$	−1.346 V
V_{Mm}	−1.303 V	V_τ	0 V
δ_m	−0.010 V	V_{Sa}	−1.309 V
V_S	−1.299 V	θ_a	0.196 V
θ	0.011 V	$V_{S\,stim}$	−1.345 V
V_{Mn}	−1.371 V	θ_{stim}	0 V

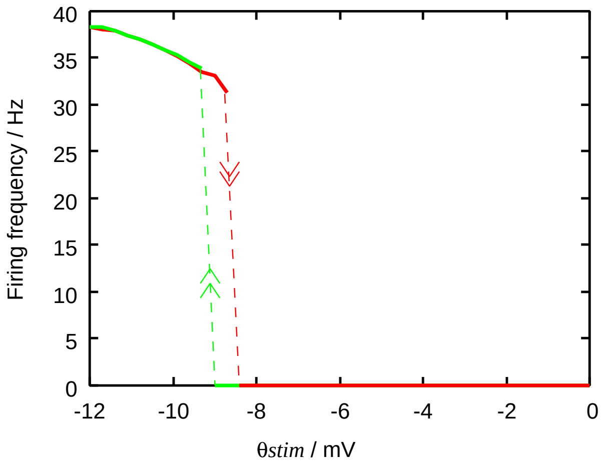

Figure 5: Measured firing frequency of our silicon nerve membrane according to sustained stimulus. Sweep-direction-dependent hysteresis is observed.

dependent hysteresis. It begins periodic firing with a nonzero frequency when a threshold is exceeded. Therefore, it is classified as Class 2. It is well known that the underlying mechanism of Class 2 neural excitability is due to a subcritical Andronov-Hopf bifurcation [6]. The firing frequency is about 35 Hz at the onset.

We investigated the responses to singlet pulse stimulus and examined that the silicon nerve membrane generates an overshoot, and has a threshold for its generation. It was also checked that the refractory period exists after the generation of an overshoot by applying doublet pulse stimulus.

Next, we investigated the responses of the silicon nerve membrane to periodic pulse stimulus. The responses to periodic pulse stimulus in the Hodgkin-Huxley model, which exhibits Class 2 excitability, have been studied in detail [3, 4]. It was reported that the response is not only periodic but nonperiodic depending on the strength and the interval of the applied periodic stimuli.

Figure 6 shows time series of membrane potential of the silicon nerve membrane in response to periodic pulse stimulus. Here, the pulse width and the interval are fixed at 1 ms and 18 ms, respectively, and the strength of the periodic stimulus is varied. When the pulse strength is small, the silicon nerve membrane hardly fires and displays subthreshold oscillation as shown in Fig. 6 (a). As the pulse strength increases, nonperiodic response emerges (Fig. 6 (b)). A periodic response appears when the larger pulse strength is applied (Fig. 6 (c)). As the pulse strength becomes larger, bifurcations take place and periodic responses of longer period appear (Figs. 6 (d) and (e)). And the response becomes nonperiodic again (Fig. 6 (f)), finally, the response becomes periodic again (Fig. 6 (g)).

Figure 7 illustrates the two-parameter bifurcation diagram of our silicon nerve membrane under periodic pulse stimulus. The horizontal and vertical axes are the pulse

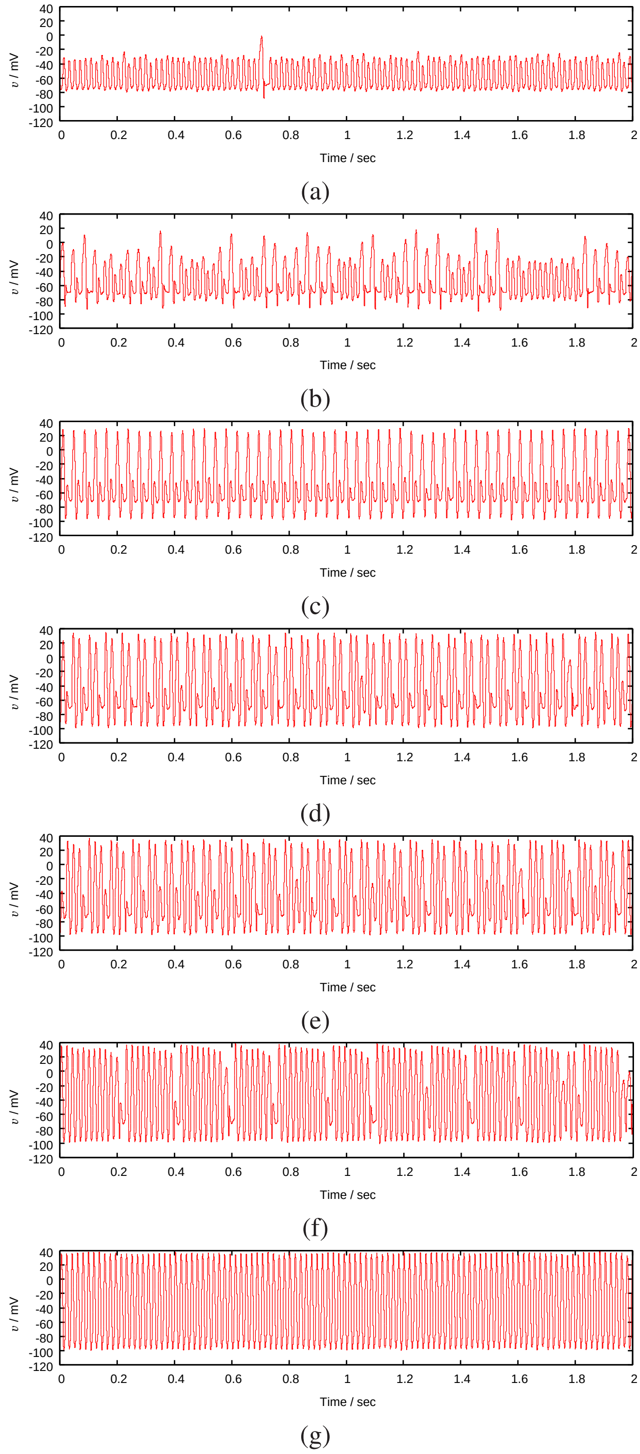

Figure 6: Responses of our silicon nerve membrane to periodic stimuli: (a) subthreshold oscillation (pulse strength is 56 mV); (b) non-periodic response (pulse strength is 62 mV); (c) periodic response (pulse strength is 72 mV); (d) periodic response (pulse strength is 86 mV); (e) periodic response (pulse strength is 92 mV); (f) nonperiodic response (pulse strength is 106 mV); (g) periodic response (pulse strength is 110 mV). Note that stimulus in voltage is converted to current by an integrated V–I converter.

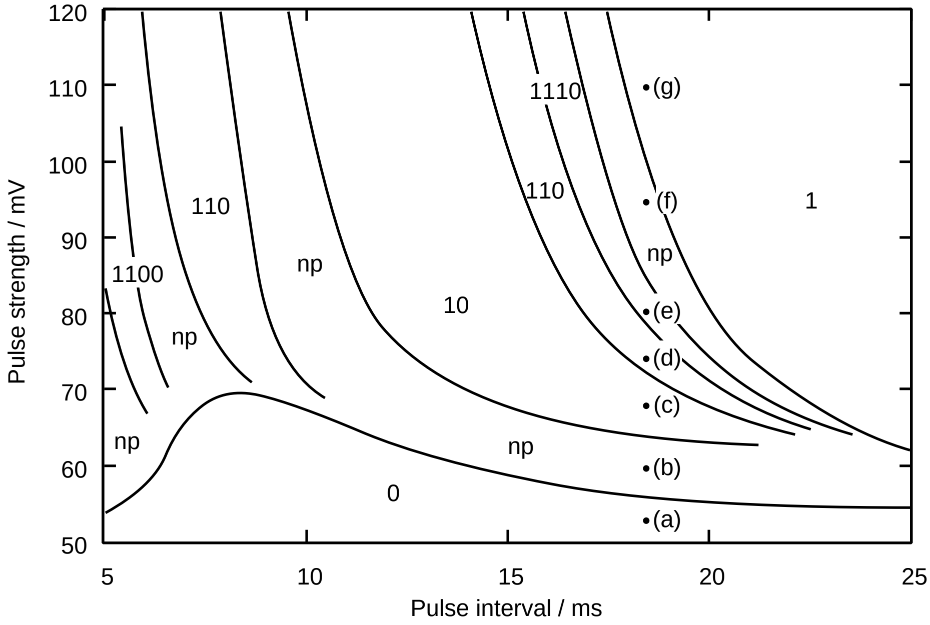

Figure 7: Two-parameter bifurcation diagram of our circuit in response to periodic pulse stimulus.

interval and the strength, respectively. We describe the firing patterns by sequences of '1' and '0' that represent firing and nonfiring, respectively. The nonfiring responses are observed when the pulse strength is sufficiently small. The firing pattern changes from '0' to nonperiodic responses (labeled 'np' in Fig. 7), in accordance with an increase of the pulse strength. Bifurcations from '10' to '110' or from '110' to '1110', namely, period-adding bifurcations take place as parameters are varied. Since the intrinsic firing frequency of our silicon nerve membrane is about 35 Hz, regions '1', '10', and '110' are considered as fundamental harmonic, 1/2 sub-harmonic, and 1/3 sub-harmonic synchronization region, respectively. Non-periodic responses exist between each synchronization region.

4. Conclusion

In this paper, we reported the experimental measurements of the fabricated chip of a silicon nerve membrane. We designed using the techniques of qualitative modeling of the neuronal activity. It was confirmed that our nerve membrane IC shows Class 2 neural excitability by applying sustained stimulus. The nerve membrane shows an excitable dynamics with generation of an overshoot in response to an external stimulus. We measured responses of our silicon nerve membrane to periodic pulse stimuli. It exhibits not only periodic but also nonperiodic responses. We constructed the two-parameter bifurcation diagram. It was clarified that the silicon nerve membrane under periodic pulse stimulus has a bifurcation structure of fundamental harmonic and sub-harmonic synchronization regions. The emergence of period-adding bifurcations were also observed.

Acknowledgments

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