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Adaptive Control of Transient Dynamics to Periodic Orbits

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Abstract—We propose an adaptive delayed feedback control technique of nonlinear maps by using a parameter adjustment together with resetting a variable. This technique can convert transient processes into either stable or unstable periodic orbits. Moreover, the controlled orbit includes a pre-determined point in the phase space, which is related to the resetting value of the variable. We consider the control process as a kind of memory processes, i.e. the resetting value is stored in the dynamical system after control. To accomplish the storing function of memory, we investigate the system whether it can be a closed-loop system by turning off the resetting.

1. Introduction

Chaotic dynamics is known as its unpredictability in the long-term behavior. This property is due to the instability of chaotic behavior, namely the sensitive dependency on initial conditions. Therefore, it had been believed that chaotic irregular behavior should be avoided when a engineering system is designed. However, chaotic dynamics is not completely random but it has a kind of hidden structure in its irregular motion. By using this structure, the concept of controlling chaos has been proposed in 1990 by Ott, Grebogi, and Yorke [1]. They showed that a chaotic irregular trajectory in a low dimensional system is able to be controlled to one of unstable periodic orbits embedded within a chaotic attractor by adding a small perturbation to the system parameter. Surprisingly, they did not avoid but exploited the above undesired properties of chaotic dynamics for controlling it.

Following the seminal work by Ott *et. al.*, a lot of researches on controlling chaos has been conducted both theoretically and experimentally [2]. Along the research line, the authors have proposed an adaptive delayed feedback control method that does not require any detailed information on a controlled system and only requires tuning of integer parameters [3, 4]. This control method has been also successfully implemented into an analog electric circuit experimentally [5].

In this presentation, we modify the adaptive control method to convert transient processes to periodic orbits by using resetting mechanism. More precisely, we reset the state variable to a fixed value every certain time interval, then the system dynamics is controlled to a periodic orbit which includes the reset value of the variable. In addition, we can vary the reset value to some extent, which implies that periodic orbits passing arbitrary point in a limited phase space can be controlled by the modified method. We interpret this control process as a kind of memory processes, where the strength of an external input related to the reset value is stored as the controlled periodic orbit. Finally, we briefly mention how to maintain the stored memory, where the reset input is turned off. Specifically, we consider the effect of noise to maintaining the stored memory.

The following is the organization of the paper. In Section 2, we explain the adaptive delayed feedback control in the Logistic map. Then, we modify the control method by adding a resetting mechanism and illustrate its controlling process by using a nonlinear map in Section 3. In Section 4, we demonstrate numerical simulations for the modified control method and for variation of the reseting point in the phase space. We explain the change of a stability of controlled periodic orbits as a bifurcation in the nonlinear map. Section 5 focuses on the application of the control process for an analog memory system. We investigate the stability of controlled states when noise exists. Finally, we summarize the results in Section 6.

2. The adaptive delayed feedback control

Let us introduce the adaptive delayed feedback control [3] in the Logistic map: $x_{n+1} = f(x_n, a) = ax_n(1 - x_n)$, where $x_n \in [0, 1]$ is a state variable and $a \in [0, 4]$ is a parameter. The adaptive delayed feedback from the state variable $x_{n-\tau}$ is applied to a with a time dependent function as $a_{n+1} = g_n(x_{n-\tau}, a_n)$. Note that a is not constant but time dependent, denoted by a_n. The time scale of a_n is determined by a integer parameter T such that $\tau \leq T$. The feedback function g_n is as follows:

$$g_n(x_{n-\tau}, a_n) = \begin{cases} \phi^{-1}(x_{n-\tau}) & (n = iT) \\ a_n & (otherwise), \end{cases} \tag{1}$$

where $\phi(a) = ax^*(1 - x^*) \equiv a/4$ with the critical point x^*, and $x_{n-\tau} = \max\{x_j\}_{n-T<j\leq n}$ $(n = iT)$.

The process of the control is as follows. First, we set the parameter $a_0 = 4$ for an initial value which generates fully developed chaos for the Logistic map. Then, we apply the feedback function g_n to the parameter a_n with an appropriate value of T. The value of a_n decreases monotonically by the control. Regarding the bifurcation diagram with respect to a, once the value of a_n is in the region of a periodic window with the period less than T, $x_{n-\tau}$ corresponds to the largest value of the periodic orbit. In the bifurcation diagram, such a periodic orbit as a function of a has a tangent point with the line $x = a/4$. This tangent point is a local attractor for the two dimensional system with respect to x_n and a_n. Therefore, once the a_n comes close to such a local attractor, it converges to the tangent point which corresponds to a superstable periodic orbit for the dynamics of x_n. This is the end of the control. Although a chaotic orbit is controlled to a superstable periodic orbit by the method, the period is not known in advance. We have proposed how to control the period by adding a further condition [4].

3. Resetting inputs

Concerning the dynamics of a_n for the control method introduced in the previous section, we cannot follow the dynamics of a_n outside of the periodic windows, i.e. chaotic region in the bifurcation diagram. This is because $x_{n-\tau}$ sensitively depends on both x_{n-T} for $n = iT$ and a_n due to chaotic dynamics. In order to overcome this difficulty, we add one condition to the control method. In the original method, we change a_n every T steps and use the value of x_{iT} for calculating x_{iT+1}. Instead of this, we use the same pre-determined value for all the calculation of x_{iT+1}, $i = 1, 2, \ldots$, denoted by x_r. By this resetting of x_n every T steps, we can follow the behavior of a_n outside of the periodic windows.

In Figure 1, we show the trajectory of a_n when we add the resetting condition when $T = 4$. As can be seen, the dynamics of a_n is clearly observed in the region from a_0 to the finally controlled state. The piece-wise nonlinear map represents the modified control method, i.e. $x_{n-\tau}$ for the map $x_{n+1} = f(x_n, a)$ with $x_0 = x_r$ as a function of a. We allow for the initial value of $x_{T-\tau}$ of the control process to be taken as an arbitrary value in $[0, 1]$. This initial value corresponds to the x coordinate of the controlling trajectory at $a_0 = 4$ in Figure 1. Note that by introducing the resetting mechanism, we can see the controlling process from any initial conditions to the controlled state (the tangent points) by using the piece-wise nonlinear map. Therefore, mathematical analysis of the control method might be possible more or less. However, such analysis is out of scope of the current paper and will be done somewhere else.

Finally, let us note that the condition of the controlled state is described as

$$f^j(x_r, a_n) = \tilde{x}, \qquad (2)$$

where $j \leq T$. Since x_r is defined as $x_r \equiv f^2(\tilde{x}, a_n)$, controlled state of x_n should be periodic. Moreover, if $\tilde{x} = x^*$, then controlled periodic orbits are superstable, which implies that controlled points in the $(a,\ x)$ plane corresponds to those of the original method. Furthermore, the controlled a_n provides a periodic orbit including x^* that is related to the resetting value.

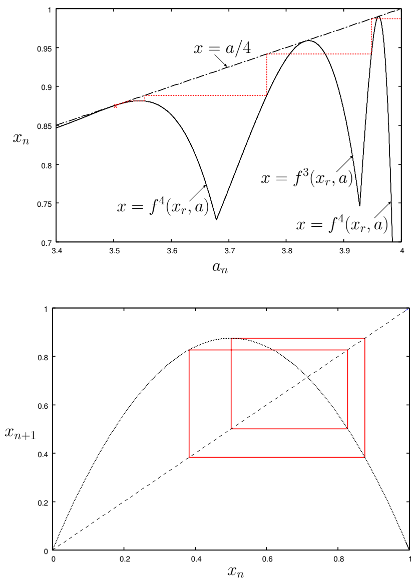

Figure 1: (Top) Trajectory of a_n (the dotted line) in the $(a,\ x)$ space when the resetting control is applied. The solid curve is a piece-wise nonlinear map representing the resetting control. The number of iterations of f lead x_r to the largest value of $\{x_i\}_{0 \leq i \leq T}$ with respect to a. The dashed-dotted line corresponds to the diagonal line for a standard map in (x_n, x_{n+1}). The trajectory are controlled to the tangent point between the nonlinear map and the line corresponding to superstable periodic point. $T = 4$. (Bottom) The controlled periodic orbit for the dynamics of x_n corresponding to the controlled a_n in the top panel. The periodic orbit include the critical point x^*.

4. Numerical simulations

In this section, we show numerical simulations of the proposed control method for different initial conditions of $x_{T-\tau}$. Then, we investigate the controlled periodic orbits when $\tilde{x}$ is varied from the critical point x^*.

Figure 2 shows controlled attractors in the $(a,\ x)$ space for $\tilde{x} = x^*$ and $T = 6$. The initial conditions are $a_0 = 4$ and 10000 different values of $x_{T-\tau}$ taken from the interval $[0.995 : 1]$. As can be seen, all the existing attractors are controlled. Note that the period of superstable periodic orbit for the dynamics of x_n is less than T.

Next, we are interested in varying the value of $\tilde{x}$ from x^*. As can be seen in the top panel of Figure 3, slightly changing the value of $\tilde{x}$ from x^* make the diagonal line cross with the piece-wise nonlinear map. This is because $f(\tilde{x}, a)$ is less than the maximum of the map $x_{n+1} = f(x_n, a)$. According to this change, the dynamics of a_n is no more monotonically decreasing but possibly increasing. The inset in the top panel shows that the trajectory of a_n increases around the fixed point. Another remarked thing by changing $\tilde{x}$

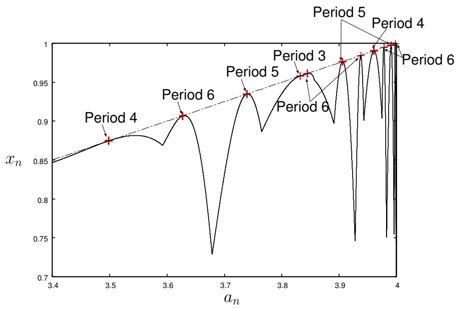

Figure 2: All the controlled attractors ($a_n > 3.4$) in the $(a,\ x)$ space when $T = 6$, indicated by +. Initial conditions are $a_0 = 4$ and 10000 different values of $x_{T-\tau}$ such that $x_{T-\tau} \in [0.995, 1]$. The indicated period corresponds to the period of a superstable periodic orbits in the dynamic of x_n for each attractor.

from x^* is that the corresponding periodic orbit for the dynamics of x_n can be unstable. In the bottom panel of Figure 3, the periodic orbit of x_n is shown together with a chaotic attractor which is generated by the value of the controlled a_n without resetting. This implies that the resetting input x_r or $\tilde{x}$ keeps the periodicity of x_n in spite of its instability. In other words, the controlled state in the whole system of a_n and x_n is an attractor due to the stability of a_n. It should be noted that the periodic orbit of x_n includes the resetting input as $1 - \tilde{x}$. This is because we use x_r as $f^2(\tilde{x}, a_n)$.

Now, what does happen when we vary $\tilde{x}$ more? Figure 4 answers this question. First, the stability of periodic orbit of x_n changes from superstable to "normally" stable, and to unstable, as $\tilde{x}$ varies. This change of stability is shown in the top left panel of Figure 4 that shows bifurcation of a fixed point of a_n when $\tilde{x}$ varies from x^* to 0.47. The black circles represent that the corresponding periodic orbits of x_n are stable for those fixed points of a_n. The fixed points of a_n not covered by the circles provide unstable periodic orbits for the dynamics of x_n. Next, the fixed point of a_n becomes unstable via the period doubling bifurcation at around $\tilde{x} \approx 0.482$. Note that the periodic orbit of x_n no longer exists as an attractor after the bifurcation. The period doubling bifurcation can be seen from the panel (i) and (ii) of Figure 4, where the nonlinear map and the diagonal line shift and the stability of the fixed point changes. When the relation between the nonlinear map and the diagonal line changes further, we can observe chaotic attractors in the $(a,\ x)$ space as shown in the panel (iii) of Figure 4. Finally, a boundary crisis occurs in the $(a,\ x)$ space when $\tilde{x} \approx 0.474$, the stable attractor of a_n disappears.

5. As an analog memory

In the previous section, we observe that there exist multiple stable attractors for the dynamics of a_n, even if the corresponding orbit of x_n is stably or unstably periodic, or not periodic. In this section, we interpret those attrac-

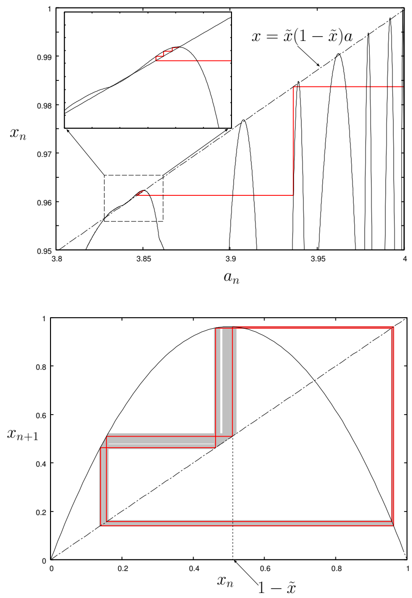

Figure 3: (Top) Control map when $\tilde{x} = 0.49$. $T = 6$. $x_{T-\tau} = 0.9837$. The inset is magnification around the controlled point. The piece-wise nonlinear map has cross points with the line $x = \tilde{x}(1 - \tilde{x})a$. (Bottom) Periodic orbit for the controlled $a_n(\approx 3.851)$ with the resetting input. The gray orbit shows the chaotic trajectory for stationary state with regards to the value of a_n without resetting. The controlled periodic orbit include $1 - \tilde{x}$ instead of $\tilde{x}$.

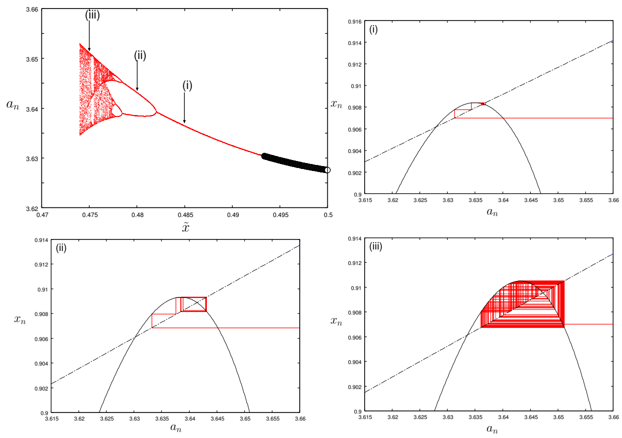

Figure 4: (Left top) Bifurcation behavior of one of the controlled fixed points of a_n with regards to $\tilde{x}$. The circles correspond to stable periodic orbits for the dynamics of x_n. The fixed points of a_n not covered by the circles generate unstable periodic orbits for the dynamics of x_n. The labels (i), (ii), and (iii) in the panel correspond to the labels in the other panels. (Panel (i)) The trajectory of a_n around a fixed point for $\tilde{x} = 0.485$. (Panel (ii)) The periodic trajectory of a_n with period 2 for $\tilde{x} = 0.48$. (Panel (iii)) The chaotic trajectory of a_n for $\tilde{x} = 0.475$.

tors as analog memory states and the control process as a memory process, where the system stores the strength of the resetting input $x_r \equiv f^2(\tilde{x}, a_n)$ in terms of the controlled parameter, denoted by a_f. Specifically, a_f is related to the periodic orbits of x_n and the resetting input such that $x_{n-\tau} = \tilde{x}(1-\tilde{x})a_f$, where $x_{n-\tau}$ is the largest value of the periodic orbit. Due to this relation, a_f can be maintained as long as the periodic orbits is controlled with adding the reseting input. However, there is no meaning in the memory system with adding the resetting input practically. Therefore, we turn off the resetting input once the storing, i.e. the control, is finished. Moreover, we take into account the effect of noise to the system after storing.

The following is the algorithm how to keep the value of a_f without the resetting input and with noise. The final value of a_f with the resetting input is denoted by a'_f and the final value of $\tilde{x}$ determined by $x_{n-\tau-1}$ is denoted by $\tilde{x}'$. First, we iterate $x_{n+1} = f(x_n, a'_f)$ for T steps from $x_0 = f(\tilde{x}', a'_f)$. Then, set the largest value of the T-step time series from x_1 to x_T as $\hat{x}$. Now, the value of a'_f is updated as $a'_f = \hat{x}/\tilde{x}'(1-\tilde{x}')$. Finally, we update $\tilde{x}'$ as $\tilde{x}' + \delta$, where δ follows $N(0, D)$. We repeat this procedure with adding different noise every updating.

We do numerical simulations for maintaining controlled a_f with noise by the above algorithm. Figure 5 shows the behavior of a'_f when $T = 6$ and $D = 0.0001$. In the top panel of Figure 5, we can confirm that the controlled a_f is maintained even if there exist noise and the resetting input is turned off. The inset shows the fluctuation of a'_f around the controlled fixed point a_f. However, it is not possible to maintain the state around a_f for a long time, since noise is added every update and there is a basin boundary of the attractor of a_f. In the bottom panel of Figure 5, the time series of $\hat{x}$ is shown. Although the state of $a'_f \sim \hat{x}$ fluctuates around the controlled state for a while, it suddenly goes down to the other attractor and fluctuates around it. The variation of the fluctuation of $\hat{x}$ depends on attractors. This result implies that it is not possible to maintain the memory state for an arbitrary long time without resetting and with some amount of noise. However, if the noise is not so strong and the time for maintaining a memory state is not so long, the proposed system can be used as an analog memory system.

6. Summary

In this presentation, we have modified the adaptive delayed feedback control method by adding resetting mechanism, and showed its effectiveness by numerical simulations. By the modification, we can control transient processes to periodic orbits including a pre-determined point in the phase space. Furthermore, we have investigated the behavior of the controlled states by varying the resetting input as a bifurcation parameter. By observing the bifurcation, we have made the system more robust than the original system in terms of control, namely the dynamics of the original state variable is stabilized by introducing dynamics to the system parameter. We have also discussed the possible application of the control system for a temporal analog memory system if the noise is not so strong.

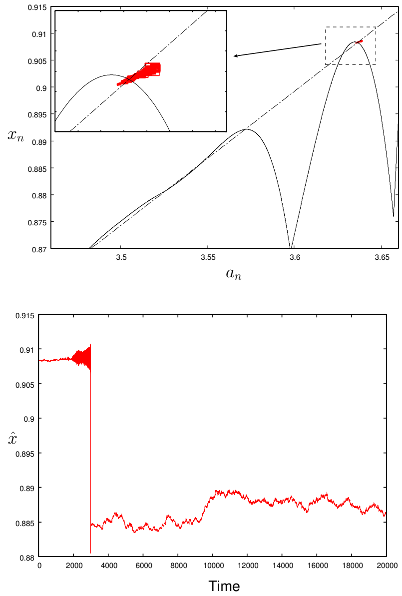

Figure 5: (Top) Maintained a'_f in the $(a,\ x)$ space for $T = 6$ with the noise of $D = 0.0001$. The length of time for the maintaining is 2800 updates. (Bottom) The time series of of $\hat{x}$ for a longer time of updates than 2800 updates. After sudden transition, a'_f stays around a different attractor.

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