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Synchronization in coupled excitatory and inhibitory neurons with ladder structure

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Abstract—We investigate synchronization phenomenon in coupled excitatory and inhibitory neurons with ladder structure. As a result, from bifurcation analysis, the neurons have a wide parameter region in which stable synchronous firing is observed as the external DC current of hub neuron is lager than that of outer neurons. We obtain that the neurons achieve synchronization at the higher firing frequency than criteria frequencies of the neurons by comparing the firing period with the synaptic injection interval.

1. Introduction

In the real neuronal networks, complex networks such as scale-free or small-world networks have been observed [1, 2]. Synchronization phenomena observed in the neuronal network play a key role of information processing. On the other hand, they become one of the important factors of several neurological disease. Hence, investigation about synchronization in the complex network is very important and has attracted the attention of many researchers.

The complex network contains many sub-networks (motifs), and their behaviors are studied to estimate the phenomena in the whole network. In the neuronal networks, the motif coupled excitatory and inhibitory neurons is observed. In these networks, the excitatory and inhibitory neurons correspond to pyramidal and interneuron cells, respectively. Moreover, the high oscillation frequency such as gamma frequency which is found in the neocortical network consisting of this motif [3]. Hence, to clarify the generation of the high oscillation frequency, the analysis of synchronization in this motif is very available way. However, enough researches for synchronization in this motif have not been reported as far as we know.

In this study, we consider the system of mutually coupled three neurons in a line (called ladder) as a basic hub structure [4, 5]. To reproduce the above motif, we choose the excitatory neuron as a hub neuron and inhibitory neuron as outer neurons. We analyze bifurcation structure when the external DC current of the hub excitatory neuron.

As a result, we find that the neurons have wide parameter region in which 1:2 synchronous firing is observed as the external DC current of the hub neuron is larger than that of outer neurons. By comparing the firing period with the synaptic injection interval, the neurons acquire synchronization at the higher firing frequency than the criteria frequencies of neurons.

2. Neuron model

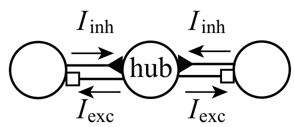


Figure 1: Schematic diagram of coupled neurons with ladder structure

We consider coupled neurons model shown as Fig. 1. In this figure, the hub and outer neurons are excitatory and inhibitory neurons, respectively. The hub neuron activates the outer neurons by excitatory synaptic current($I_{\rm exc}$), then the inhibitory synaptic current($I_{\rm inh}$) is injected to the hub neuron from the outer neurons(Fig. 1). The system equation is shown as follow:

$$C_{\rm M}\frac{dV_i}{dt} = F(V_i) + I_{{\rm ext}i} + I_{{\rm syn}i} \quad (i = 0, 1, 2),$$

where, $F(V_i)$ is single Morris-Lecar equation (see Appendix), and $I_{{\rm ext}i}$ is external DC current. In this study we consider mutual coupling, then the synaptic current $I_{{\rm syn},i}$ is described as follow:

$$I_{\rm syn0} \quad \text{(middle)} = g_{\rm syn}(V_{\rm syn} - V_0)(\alpha_1 + \alpha_2) \qquad (1)$$

$$I_{{\rm syn}i} \quad \text{(outers)} = g_{\rm syn}(V_{\rm syn} - V_i)\alpha_0 \qquad (i \neq 0), \quad (2)$$

where, $g_{\rm syn}$ and $V_{\rm syn}$ are the maximum synaptic coupling conductance and the reversal potential, respectively. The reversal potential $V_{\rm syn}$ is fixed as -65[mV] (resp. 0[mV]) for the inhibitory (resp. excitatory) synapse. The synaptic channel α_i is shown by,

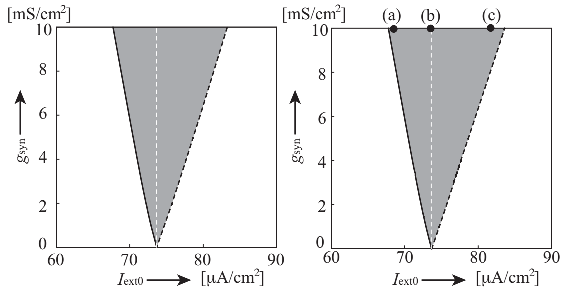
$$\frac{d\alpha_i}{dt} = \frac{\beta_i}{\tau_2} \qquad (3)$$

$$\frac{d\beta_i}{dt} = -\frac{\alpha_i}{\tau_1} - (\frac{1}{\tau_1} + \frac{1}{\tau_2})\beta_i \quad (4)$$
$$(i = 0, 1, 2).$$

The solution $\alpha_i(t)$ of Eqs. (3) and (4) with the initial condition $(\alpha_i, \beta_i) = (0, 1)$ at $t = 0$ is calculated as $\alpha_i(t) = \frac{\tau_1}{\tau_1 - \tau_2}(e^{-\frac{t}{\tau_1}} - e^{-\frac{t}{\tau_2}})$. τ_1 and τ_2 are, respectively, the raise and the decay time constants of the synapse assumed as (τ_1[msec], τ_2[msec])=(0.5, 7.0) (resp. (0.5, 2.0)) for inhibitory (resp. excitatory) synapse [6]. In this study, the synaptic delay is fixed as 1[msec].

2.1. Result

We investigate synchronization phenomena by changing the firing frequency of the hub neuron. Hence, in this study we change the firing frequency of the hub neuron by varying its external DC current. The values of external DC currents of outer neurons are fixed as 73.67 or 78.55 for class I or class II neurons. They correspond to the fundamental firing frequency f_f = 20[Hz].



(a) class I - class I (b) class I - class II

Figure 2: Bifurcation diagrams. Sub caption of figures indicate the type of membrane excitabilities of Hub - Outer neurons. The saddle node and pitchfork bifurcation are indicated by solid and dashed curves. The stable synchronous firing is observed in the shaded region.

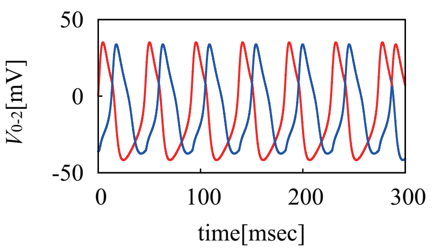


Figure 3: Waveforms observed in the shaded region. The red and blue curves indicate the membrane potentials of hub and outer neurons, respectively.

2.1.1. *Bifurcation diagram*

Figure 2 is bifurcation diagram obtained by using the algorithms in [7] and [8]. In the real neuronal networks the membrane excitability of most pyramidal cell is classified into the class I type. On the other hands, the excitability of the interneuron indicates the both class I and II types characteristics. Hence, we choose the class I neuron as the membrane excitability of the hub neuron.

From this bifurcation diagram, we can find the Arnold's tangue formed by saddle-node (indicated by solid curves) and pitchfork bifurcation (dashed curves). The pitchfork bifurcation is subcritical. In the shaded region, we can observe stable 1:2 cluster synchronization(the outer neurons synchronize in in-phase) shown as Fig. 3. We can see that this region expand asymmetrically for I_{ext0} = 73.67 when the coupling conductance increases. As g_{syn} increases, the neurons can easily achieve synchronization at lager external DC current of hub neuron than 73.67. The criterion firing frequency of the hub neuron becomes high as external DC current increases. From this result, we suggest the neurons acquire synchronization at higher firing frequency than f_f.

From Fig. 2, we can find the same phenomenon in their bifurcation structures. Hence, there is few difference by the class of outer neurons in this case.

2.1.2. *Firing period and synaptic injection interval*

Next, we analyze the relationship between the firing period and the synaptic injection interval of the neurons. Figure 4 explains definition of the firing period P and the synaptic injection interval ts. ts indicates the interval of the synaptic current injection from the firing of the neuron.

Figures 5(a)-5(c) correspond to the points(a)-(c) in the Fig. 2(b). In the waveforms(upper figures), red and blue curves indicate the membrane potentials of the hub and outer neurons, respectively. The middle shows the firing period P, and the lower indicate the synaptic injection interval ts. In the middle figures, the open circles and squares indicate the criteria firing periods of neurons(uncoupled), on the other hand, the filled symbols denote that of the coupled neurons with synaptic connection.

From Fig. 5(b), the uncoupled neurons have same firing period (=50 [msec]). Moreover the timing of firing is also same, thus neurons synchronize in in-phase. However, by synaptic curent injection, the firing timing of the hub neuron becomes faster than that of the outers. The difference of P on the time series correspond to the phase difference. Thus, there is a phase difference between the firing of the hub and outer neurons and these nurons synchronize in out-of-phase.

From time series variation of ts and P in Fig. 5(b), for the firing period of the hub neuron, we can find the "phase delay" following the inhibitory synaptic current injection. In this time, ts becomes longer than a harf of the firing period of the hub neuron. The phase response curves (PRCs) of the Morris-Lecar neuron represent that the "phase delay"(resp. phase advance) is caused effectively by injecting the inhibitory(resp. excitatory) synaptic at timing about a harf of the firing period. Our result shows similar phenomenon as the PRCs.

On the other hand, the firing period of the outer neurons are hardly affected by excitatory synapse following the synaptic current injection. However, the firing period of the hub neuron becomes long by inhibitory synaptic injection, then the injection of excitatory is slow. Then, we can observe the shorter firing period of the outer neurons than these criteria firing periods. Moreover, the firing period of coupled neurons becomes shorter than that of uncoupled neurons. Hence, the neurons active with the higher synchronous firing frequency than f_f by injecting the synaptic current.

Figures 5(a) and 5(c) show that neurons cannot synchronize in $g_{\text{syn}} = 0$. For $g_{\text{syn}} = 10$, neurons acquire synchronization at the equal or higher firing frequency than 20[Hz], since the firing period of coupled neurons becomes shorter than 50[Hz].

Figure 5(d) shows the case of the asynchronous firing between the neurons. The firing periods of the neurons do not converge, so the neurons do not synchronize when $I_{\text{ext0}} = 100$ (corresponding to withe region in Fig. 2).

From Figs. 5(a)-5(c), we can find that when the excitatory synaptic injection causes the phase advance of the outer neurons effectively, the neurons easily achieve synchronization.

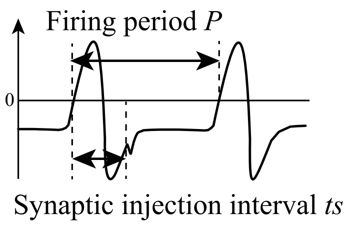


Figure 4: Definition of the firing period P and the synaptic injection interval ts.

2.2. Conclusion

We have considered synchronization in excitatory and inhibitory coupled neurons in this study. This model is structured with the ladder; the hub and outer neurons are excitatory and inhibitory neurons, respectively. We have obtained that the neurons have wide paramater region in which stable 1:2 synchronous firing is observed as the external current of the hub neuron is lager than that of outers. Hence, neurons have higher synchronous firing frequency than criteria frequencies of neurons.

Table 1 shows the synchronous firing frequencies of coupled excitatory-excitatory(EE), inhibitory-inhibitory(II) and inhibitory - excitatory(IE) neurons In this case, criteria frequencies of neurons are 20[Hz]. From this table, we can find that coupled IE neurons have the highest firing frequency in these couplings. It is higher than the firing frequency of coupled EE type. This result is supported by the physiological experiment in [3]. Hence, we guess that in the neuronal netwrok, the IE coupling motifs play a key role to generate the high firing frequency.

Moreover, by analyzing the firing period and synaptic injection interval, we can represent the process of synchronization by synaptic injection. From this analysis, we show that when the excitatory synaptic injection from the hub neuron causes the phase advance of the outer neurons effectively, the neurons easily achieve synchronization at the high firing frequency.

Our open problems are studying the system applied the more physiological neuron model and synchronization in the large scale network consisting of IE coupling motifs.

Table 1: Synchronous firing frequency

hub neuron-outer neuron	period[msec]	frequency[Hz]
excitatory-excitatory(EE)	50.8	19.7
inhibitory-inhibitory(II)	53.2	18.8
excitatory-inhibitory(EI)	46.6	21.5

Appendix

The Morris Lecar equation is described as follow:

$$\begin{aligned} C_{\text{M}}\frac{dV_i}{dt} &= -g_{\text{L}}(V_i - V_{\text{L}}) - g_{\text{Ca}}M_{\infty,i}(V_i - V_{\text{Ca}}) \\ &- g_{\text{K}}N_i(V_i - V_{\text{K}}) \quad (5) \\ \frac{dN_i}{dt} &= \frac{N_{\infty i} - Ni}{\tau_{N_i}} \quad (6) \\ (i &= 0, 1, 2), \end{aligned}$$

where, $g_{\text{Ca}} = 4.0[\text{mS/cm}^2]$, $g_{\text{K}} = 8[\text{mS/cm}^2]$, $g_{\text{L}} = 2[\text{mS/cm}^2]$, $V_{\text{Ca}} = 120[\text{mV}]$, $V_{\text{K}} = -80[\text{mV}]$, $V_{\text{L}} = -60[\text{mV}]$, $C_{\text{M}} = 20[\mu\text{F/cm}^2]$ and $\phi = 1/15[\text{s}^{-1}]$. $M_{\infty i}$, $N_{\infty i}$ and τ_{N_i} are shown as follow:

$$\begin{aligned} M_{\infty i} &= 0.5[1 + \tanh\{(V_i - V_{\text{a}})/V_{\text{b}})\}] \quad (7) \\ N_{\infty i} &= 0.5[1 + \tanh\{(V_i - V_{\text{c}})/V_{\text{d}})\}] \quad (8) \\ \tau_{N_i} &= 1/[\phi \cosh\{(V_i - V_{\text{c}})/2V_{\text{d}})\}]. \quad (9) \end{aligned}$$

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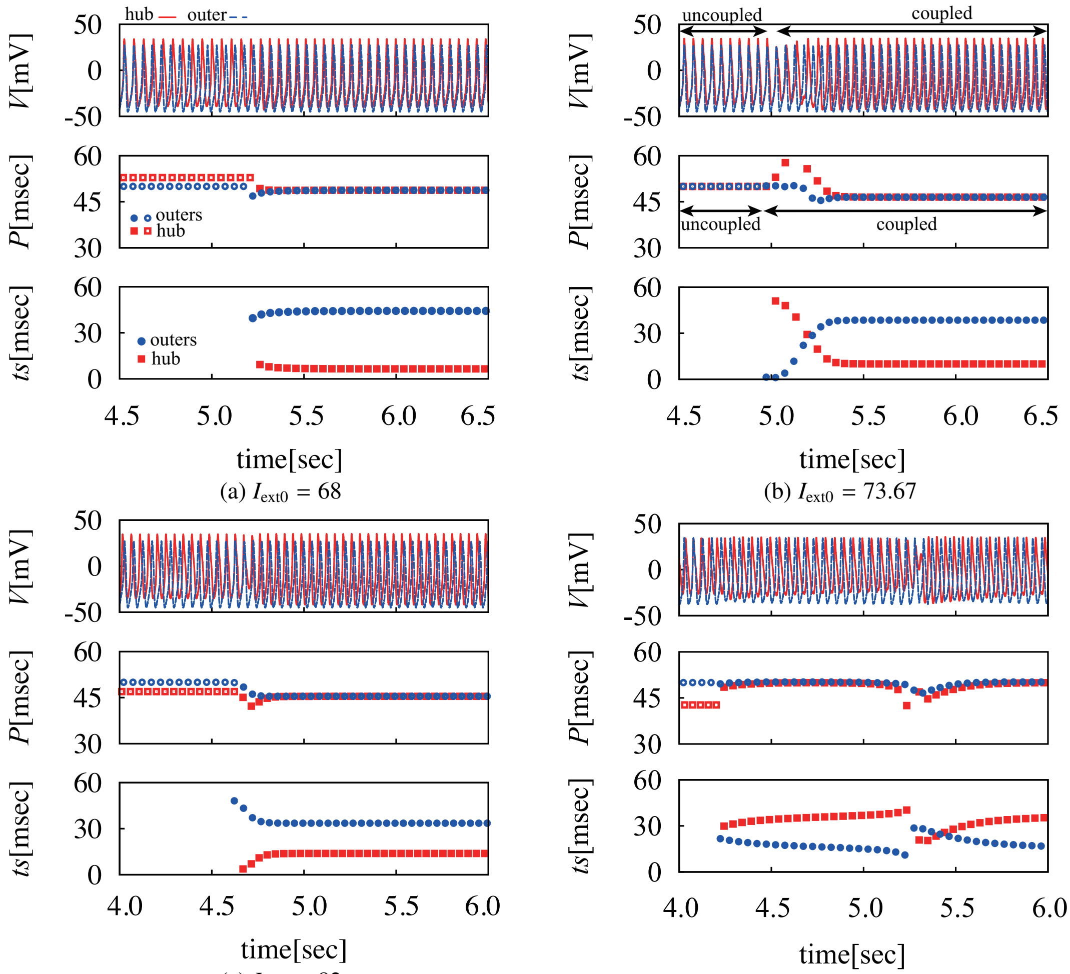

(a) $I_{\text{ext0}} = 68$ (b) $I_{\text{ext0}} = 73.67$ (c) $I_{\text{ext0}} = 82$ (d) $I_{\text{ext0}} = 100$

Figure 5: Firing period P and synaptic injection interval ts of neurons. Upper figures are membrane potentials of the neurons. The red and bule curves indicate the membrane potentials of hub and outer neurons, respectively. The middle figures denote the firing periods P of the hub (square) and outer (circle) neuros. The open and filled symbols illustrate the firing periods P of uncoupled and coupled neurons, respectively. The lower figures show the synaptic injection interval. The symbols filled red and blue show that intervals of synaptic current injection ts of the hub and outer neurons.

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