

# Interpolation of Communication Distance in Urban and Suburban Areas

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*Abstract- This paper is concerned with a numerical method to interpolate the distribution of communication distance function in urban and suburban areas. According to the 1-ray model, it is shown that the propagation order of distance  $\beta$  depends only on the height of transmitting antenna and the amplitude modification factor  $\alpha$  can be derived from the path loss obtained theoretically or experimentally. We propose a numerical method for estimating communication distance functions in urban and suburban areas by employing the path loss computed by the Okumura-Hata model. Numerical examples are shown to demonstrate the effectiveness of the proposed method.*

## I. INTRODUCTION

Recently, we have proposed the 1-ray model characterized by the amplitude modification factor  $\alpha$  and the propagation order of distance  $\beta$  as well as the field matching factor  $\gamma$  in order to numerically simulate electromagnetic (EM) wave propagation in complicated EM environments such as urban and suburban areas or along random rough surfaces (RRSs) [1]. Originally the parameter  $\beta$  was proposed by the Okumura-Hata model when introducing the empirical equations for the path loss in urban, suburban and open areas in Japan [2].

According to the Okumura-Hata model [2],  $\beta$  depends only on the antenna height of base station (BS), and the path loss of EM wave propagation is given only in the urban, suburban and open areas. As a result, the 1-ray model combined with the Okumura-Hata model enables us to evaluate propagation characteristics only in these three areas. In order to deal with the EM propagation in other regions, however, it is necessary to propose a new method by use of a numerical interpolation.

It is evident that the path loss is closely associated with the complexity of propagation environments especially in the urban areas with high-rise buildings. In this paper, we propose a new algorithm to construct the communication distance function in urban and suburban areas which plays an important role for allocating base stations optimally in a complicated EM environment.

Section 1 is the introduction of the present paper. Section 2 reviews the propagation characteristics of the 1-ray model. Section 3 deals with the path loss of the 1-ray model. Section 4 shows numerical examples for the gross floor area ratio in

Fukuoka city and the amplitude modification factor as well as the communication distance function. Section 5 concludes the present investigation.

## II. 1-RAY MODEL

The 1-ray model in far zone ( $r \gg \lambda$ ) yields the electric field expression as follows [3],[4]:

$$\mathbf{E}(r) = 10^{\alpha/20} 10^{(\beta-1)\gamma/20} r^{(1-\beta)} \mathbf{E}_i(r) \quad (1)$$

where  $r$  is the distance from source to receiver and the field matching factor  $\gamma$  [dB] as well as the field matching distance  $\Gamma$  [m] are defined by

$$\gamma = 20 \log_{10} \Gamma, \quad \Gamma = r |\mathbf{E}_t(r)| / |\mathbf{E}_i(r)| \quad (2)$$

The incident electric field in the free space is given by [3],[4]

$$\mathbf{E}_i(r) = \sqrt{30 G_s P_s} \sin \theta_s \boldsymbol{\Theta}^v(\mathbf{r}, \mathbf{p}_s) e^{-j\kappa_0 r / r} \quad (3)$$

and the total electric field above a uniform ground plane is given by [3],[4]

$$\mathbf{E}_t(r) = \mathbf{E}_i(r) + \sqrt{30 G_s P_s} \sin \theta_0 \mathbf{e}_r(r) e^{-j\kappa_0 r / r_0} \quad (4)$$

where the time dependence  $e^{j\omega t}$  is assumed. Moreover,  $\kappa_0$  is the wave number in the free space, and  $r_0$  is the total distance of the reflection ray above the uniform ground plane.

The transmitting and receiving antennas are small dipoles with gain  $G_s = G_r = 1.5$  and direction vectors  $\mathbf{p}_s$  and  $\mathbf{p}_r$ . Other detailed discussions for notations can be seen elsewhere [5]. Then the received power is given by

$$P_r = \lambda^2 G_r |\mathbf{E}(r) \cdot \mathbf{p}_r| / 4\pi Z_0, \quad Z_0 = \sqrt{\mu_0 / \epsilon_0} \quad (5)$$

where  $\lambda$  is the wavelength and  $Z_0$  is the intrinsic impedance of the free space.

Fig.1 shows the received powers for  $\alpha = 0$  with parameter  $\beta = 1.0, 1.5$  and  $2.0$  where we have assumed that the base

station (BS) antenna height is  $h_b = 30$  [m] and the mobile station (MS) antenna height is  $h_m = 1.5$  [m].

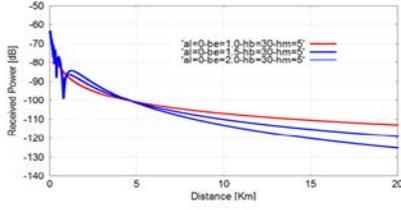


Figure 1 Received power computed using  $\gamma$  versus distance with a parameter  $\beta$ .

It is interesting that the three curves coincide at  $r = \Gamma$  defined in Eq.(2) which can be approximated in far zone as follows;

$$\gamma = \gamma_0 = 20 \log_{10} \Gamma_0, \quad \Gamma = \Gamma_0 = 2\kappa_0 h_b h_m. \quad (6)$$

The accuracy of Eq.(6) can be numerically confirmed as shown in Fig.2 depicting the normalized field matching length  $\Gamma/\Gamma_0$  versus the normalized distance  $r/\Gamma_0$ . It is demonstrated that the three curves for different BS antenna heights,  $h_b = 30, 100$  and  $200$  [m], are almost coincident with each other, and 94.4% accuracy at  $r/\Gamma_0 = 1$  together with 99.6% accuracy at  $r/\Gamma_0 = 5$  has been achieved.

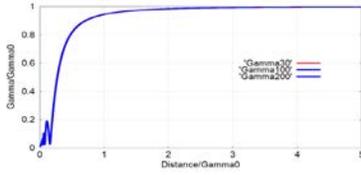


Figure 2  $\Gamma/\Gamma_0$  vs.  $r/\Gamma_0$  for three different BS antenna heights.

Thus, we can conclude that  $\gamma$  in Eq.(1) can be replaced by  $\gamma_0$  defined by Eq.(6) for  $r > \Gamma_0$ , and the electric field can be rewritten as follows:

$$\mathbf{E}(r) = 10^{\alpha/20} 10^{(\beta-1)\gamma_0/20} r^{(1-\beta)} \mathbf{E}_i(r). \quad (7)$$

Fig.3 shows received powers computed by Eq.(7) with the same parameters as Fig.1. It is evident that Fig.3 is in good agreement with Fig.1 for  $r > \Gamma_0$ , and a monotonic property is well depicted for  $r < \Gamma_0$ .

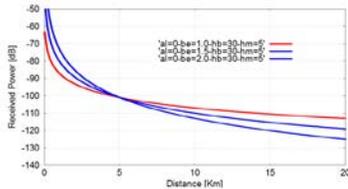


Figure 3 Received powers computed by using  $\gamma_0$ .

### III. PATH LOSS

The path loss expressed in [dB] is given by the difference between the transmitted power  $P_s$  and the received power  $P_r$ . This definition of the path loss in [dB] is summarized as follows [2];

$$L_p \text{ [dB]} = P_s \text{ [dBW]} - P_r \text{ [dBW]} = P_s \text{ [dBW]} - |\mathbf{E}| \text{ [dBV/m]} + 10 \log_{10} Z_0 - 10 \log_{10} (\lambda^2 / 4\pi). \quad (8)$$

Eq.(8) indicates that received electric field intensity can be obtained from the path loss as long as the input power  $P_s$  is specified. Substituting Eq.(7) into Eq.(8) leads to

$$L_p \text{ [dB]} = -\alpha - (\beta-1)\gamma_0 + 20\beta \log_{10} R + 20 \log_{10} f_c + 60\beta + 20 \log_{10} (4\pi/300) \quad (9)$$

where conversions of units have been made in accordance with the Okumura-Hata model; that is  $f$  [Hz]  $\rightarrow$   $f_c$  [MHz] and  $r$  [m]  $\rightarrow$   $R$  [Km] [2]. It should be noted that Eq.(9) indicates that  $\alpha$  can be obtained analytically as far as the path loss  $L_p$  and  $\beta$  are known theoretically or experimentally.

#### A. Evaluation of $\beta$

One of the interesting features of the Okumura-Hata model is that  $\beta$  depends only on BS antenna height  $h_b$  as follows:

$$\beta = 2.25 - 0.33 \log_{10} h_b \quad (10)$$

Now we consider a theoretical derivation of Eq.(10) from the 1-ray model discussed so far. According to Eq.(7), an enhancement of field intensity of the 1-ray model occurs when the BS and/or MS antenna heights are increased. This is why its amplitude is proportional to  $\Gamma_0$  given in Eq.(6).

Now we assume that the field enhancement caused by the increment of BS antenna height from  $h_b^{\min}$  to  $h_b$  may be compensated by the propagation order of distance  $\beta$  at the smallest distance denoted by

$$\Gamma_0^{\min} = 2\kappa_0 h_b^{\min} h_m^{\min} \quad (11)$$

where  $h_m^{\min}$  is the minimum MS antenna height. Then we have the following relation

$$h_b h_m / (h_b^{\min} h_m^{\min}) = (\Gamma_0^{\min})^{2-\beta}. \quad (12)$$

Taking the logarithm of Eq.(12) leads to

$$\beta = 2 + [\log_{10}(h_b h_m / (h_b^{\min} h_m^{\min})) - \log_{10}(K)] / \log_{10}(\Gamma_0^{\min} / h_b) - \log_{10}(h_b) / \log_{10}(\Gamma_0^{\min}) \quad (13)$$

where  $K = h_m / h_m^{\min}$ .

Fig.4 depicts an enhancement of the field intensity due to the increase of BS antenna height from 30 to 100 [m]. According to the Okumura-Hata model [2],  $h_b$  ranges from 30

to 200 [m] and so we could choose  $h_b^{min} = 30$  [m]. On the other hand,  $h_m$  varies from 1 to 10 [m] and so we could choose  $K = 5$  since the median of MS antenna height is 5 [m]. Moreover, since Eq.(6) provides  $\Gamma_0^{min} \approx 1.005$  [Km] for  $f_c = 800$  [MHz], Eq.(13) can be re-written as follows:

$$\beta \approx 2 + 0.26 - 0.33 \log_{10}(h_b). \quad (14)$$

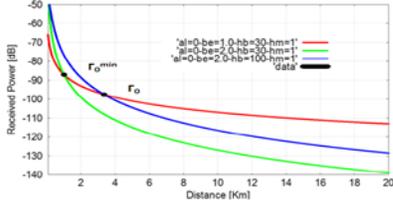


Figure 4. Received powers with different BS antenna heights. .

Fig.5 shows an example of  $\beta$  calculated by Eq. (14) in comparison with that computed by Eq.(10). It is found that the two results are in good agreement and the 1-ray model could provide us an physical insight of the Okumura-Hata model..

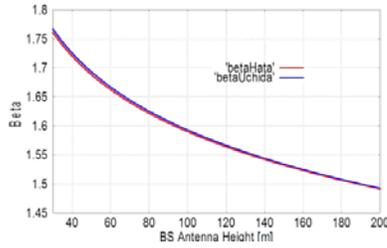


Figure 5 Proposed  $\beta$  compared with the Okumura-Hata model.

### B. Evaluation of $\alpha$

Since  $\beta$  is given by Eq.(10), Eq.(9) provides us an explicit expression for  $\alpha$  as follows:

$$\alpha = -L_p \text{ [dB]} - (\beta - 1)\gamma_0 + 20\beta \log_{10} R + 20 \log_{10} f_c + 60\beta + 20 \log_{10} (4\pi / 300). \quad (15)$$

The most important feature of Eq.(15) is described as  $\alpha$  can be evaluated if and only if the value of path loss  $L_p$  is given theoretically or experimentally [8], [9].

Based on the Okumura-Hata model, for examples, when  $f_c = 800$  [MHz],  $h_b = 30$  [m] and  $h_m = 1.5$  [m], Eq.(15) provides us  $\alpha_u = -37.3$  [dB],  $\alpha_s = -27.7$  [dB] and  $\alpha_o = -9.3$  [dB] in urban, suburban and open areas, respectively. However, it should be noted that Eq.(15) is restricted only to the urban areas of a large city or a medium-small city together with suburban and open areas, since the Okumura-Hata model provides the path loss only in these three areas. In other regions, we have to interpolate the value of  $\alpha$  numerically [10].

### C. Communication Distance

Now we assume that the antenna orientation is arranged so that the maximum received power can be obtained. Let  $E_{min}$  be the minimum detectable electric field intensity and  $D_c$  the maximum communication distance. Then Combining Eq.(3) and Eq.(7) leads to the following expression

$$D_c = 10^{\alpha/20} \beta \times 10^{(\beta-1)\gamma_0/20} \beta \times (30G_s P_s)^{1/2} \beta \times (E_{min})^{-1/\beta} \quad (16)$$

Consequently, we can evaluate the communication distance by Eq.(16), when the amplitude modification factor  $\alpha$  is known theoretically or experimentally. One method to estimate communication distance is to employ the known path loss as described in Eq.(15).

## IV. NUMERICAL EXAMPLES

The statistics of buildings and houses in a city of Japan are described in terms of two parameters; one is the building coverage ratio denoted by  $DK$  indicating the total covered area on all floors of all buildings on a certain site area, and the other is the floor area ratio denoted by  $DY$  indicating the ratio of the total floor area of buildings to the site area. Moreover, these statistical quantities are classified into two categories, net and gross values. The term "net" means that only building site is considered, and the term "gross" means that not only building site areas but also road and park areas are included for the two statistical parameters. We have obtained the gross floor area ratio  $GDY$  from the Fukuoka local government as shown in Fig.6.

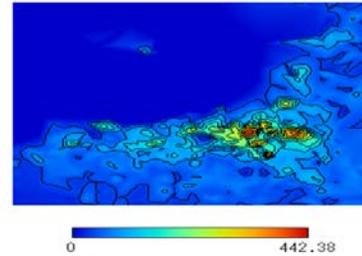


Figure 6. Gross floor ration in Fukuoka city.

Now we assume that the amplitude modification factor  $\alpha$  could be strongly correlated with the gross floor area ratio  $GDY$ . Based on this assumption, we can estimate  $\alpha$  in terms of the interpolation with respect to  $GDY$  as follows [11]:

$$\alpha = \frac{(y-y_s)(y-y_o)}{(y_u-y_s)(y_u-y_o)} \alpha_u + \frac{(y-y_u)(y-y_o)}{(y_s-y_u)(y_s-y_o)} \alpha_s + \frac{(y-y_u)(y-y_s)}{(y_o-y_u)(y_o-y_s)} \alpha_o \quad (17)$$

where  $y$  is given by a function of  $GDY$  data  $x$  as follows:

$$y_p = x_p^{0.4}, \quad (p=u,s,o). \quad (18)$$

Fig.3 shows an example of interpolated  $\alpha$  in Fukuoka city where we have selected "Tenjin" as an urban area with the amplitude modification factor  $\alpha_u = -37.3$  [dB] and the gross

floor area ratio  $x_u=435.8$ , "Hakozaki" as a suburban area with  $\alpha_s=-27.7$  [dB] and  $x_s=87.2$  and "Heiwa" as an open area with  $\alpha_o=-9.3$  [dB] and  $x_o=5.2$ . It is worth noting that "Tenjin" is a commercial district and "Heiwa" is a graveyard area.

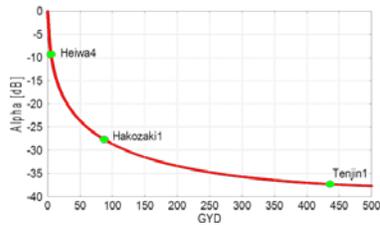


Figure 7 Interpolated  $\alpha$  in Fukuoka city.

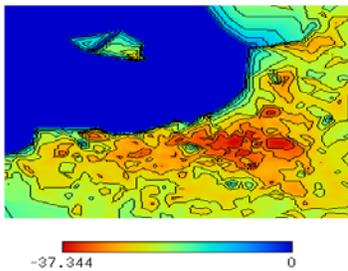


Figure 8. Distribution of  $\alpha$  in Fukuoka city.

Fig.8 shows an example of distribution of the amplitude modification factor  $\alpha$  interpolated from the gross floor area ratio in Fukuoka. It should be noted that  $\alpha = 0$  [dB] is assumed in the sea region and the maximum attenuation  $\alpha=-37.3$  [dB] is observed in the urban area.

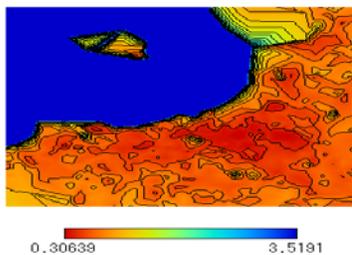


Figure 9. Distribution of communication distance in Fukuoka.

Fig.9 shows an example of distribution of interpolated communication distance in Fukuoka in a 2D-way. It should be noted that the unit of communication distance is expressed as  $D_c$  [Km] which is computed by use of  $\alpha$  distribution shown in Fig.8. The distribution of communication distance as shown in this figure can be considered to be a candidate for the communication distance function which is useful for an optimal allocation of base stations in urban areas [12].

## V. CONCLUSION

First, we have reviewed the 1-ray model in order to estimate propagation characteristics in complicated EM environments

such as urban and suburban areas or random rough surfaces. Second, we have discussed the path loss in conjunction with the amplitude modification factor  $\alpha$  and the propagation order of distance  $\beta$ , and also we have proposed a numerical method to evaluate these two parameters. Third, we have shown the gross floor area ratios in Fukuoka city by using the vector interpolation, and also we have proposed a method to relate the amplitude modification factor  $\alpha$  and the gross floor area ratio  $GDY$ . Numerical results have been shown in a 2D way to demonstrate the example of a distribution of communication distance or a communication distance function in Fukuoka city.

More detailed discussions are required on the statistical relationship between  $(\alpha, \beta)$  and  $(GDK, GDY)$ . It deserves as a future investigation.

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