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# An Improved Pose Estimation Algorithm for Nearly Coplanar Points

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**Abstract** To uniquely determine the position and orientation of a calibrated camera from a single image with respect to known scene structure, pose estimation algorithms have been developed. However, these algorithms usually suffer from pose ambiguity problem. When all the object points are coplanar, algorithms have been presented to solve this problem. In this paper, we show that pose ambiguity also exists for non-coplanar object points, especially for nearly coplanar points. Based on an analysis of the cause of pose ambiguity for nearly coplanar points, we proposed an improved algorithm to solve this problem. Simulation results and experiments on real images demonstrate the effectiveness of our proposed pose estimation algorithm.

## 1. Introduction

The aim of pose estimation is to determine the position and orientation between a camera and an object. It has many applications in computer vision, such as hand-eye robot systems, augmented reality, and photogrammetry and so on.

In the literature, several approaches have been proposed. Most of them work for arbitrary 3D object configurations, and some have been extended to planar object [3, 4, 5, 7, 8, 9]. For points from planar object, pose ambiguity usually arise because it is a degenerate case. With attention being paid to this degenerate configuration, two coplanar algorithms have been developed to solve coplanar pose ambiguity problem. Oberkamp et al. [1] first discussed the pose ambiguity for coplanar points. They gave a straightforward interpretation and developed an algorithm based on scaled orthographic projection. Schweghofer and Pinz [2] analyzed the pose ambiguity problem for coplanar points using the general case of perspective projection. They chose the correct pose with minimum reprojection error from two possible minima. However, with noise increasing, the two possible minima may be very close to each other. In other words, the

correctness rate will decrease regardless of the selected strategy.

In this paper, we show that pose ambiguity also exists for non-coplanar object points, especially for nearly coplanar points. In this situation, traditional pose estimation algorithms may induce large error and the existing coplanar algorithms can not be adopted directly. Based on an analysis of the cause of pose ambiguity and the influence of coplanar points and non-coplanar points, we point that pose ambiguity has a compact relationship with the distribution of object points. Based on this observation, an improved algorithm is presented to solve pose ambiguity for nearly coplanar points.

The rest of this paper is organized as follows: Section 2 provides a thorough analysis of the pose ambiguity problem. We describe our improved pose estimation algorithm for nearly coplanar points in Section 3. Section 4 presents experimental results using both synthetic and real data sets, and compares our algorithm with the state-of-the-art pose estimation algorithms. Conclusions are drawn in Section 5.

## 2. Pose Ambiguity

The perspective projection camera model can be described as

$$l_i \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix} = RP_i + T = \begin{bmatrix} R & T \end{bmatrix} \begin{bmatrix} P_i \\ 1 \end{bmatrix} \quad (1)$$

Where  $P_i = [X_i \ Y_i \ Z_i]^T$  and  $v_i = [x_i \ y_i]^T$  are the coordinates of corresponding object points and normalized image points, respectively.

Pose estimation is then to seek the optimal rotation matrix  $R$  and translation vector  $T$ . In the literature; there exist linear algorithms and iterative algorithms to solve equation (1).

For linear algorithms, we choose Fiore's algorithm [6] as an instance to analyze the cause of pose

ambiguity. In this algorithm, we should solve a linear equation as follow

$$C^T C \alpha = 0 \quad (2)$$

where  $C$  is a matrix containing the coordinates of image points and  $\alpha$  is an unknown  $4 \times 1$  vector. The best  $\alpha$  is the eigenvector corresponding to the minimum eigenvalue of the  $4 \times 4$  matrix  $C^T C$ , we can write this matrix as

$$C^T C = P(WW^T \otimes D^T D)P^T$$

Where we use " $\otimes$ " to denote component wise multiplication, with

$$P = \begin{bmatrix} p_1 & p_2 & \cdots & p_n \\ 1 & 1 & \cdots & 1 \end{bmatrix}, \quad D = [v_1, v_2, \cdots, v_n]$$

and  $W$  can be calculated from the equation

$$PW = 0 \quad (3)$$

For a nearly coplanar object scene, consisting of five points  $p_1 = [1 \ 1 \ 0]^T$ ,  $p_2 = [-1 \ 1 \ 0]^T$ ,  $p_3 = [-1 \ -1 \ 0]^T$ ,  $p_4 = [1 \ -1 \ 0]^T$ ,  $p_5 = [0 \ 0 \ 1]^T$ , the eigenvalues of  $C^T C$  are depicted in Fig.1

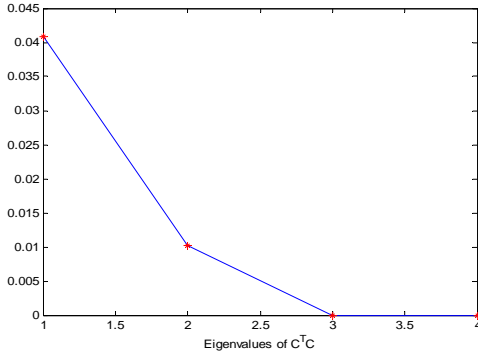


Fig.1 Eigenvalues of  $C^T C$

In Fig.1 we find that there are two minimum eigenvalues, which cause the solution of equation (2) to be unstable. As a result, pose estimation may get a wrong solution and pose ambiguity usually arise because of this.

For iterative algorithms, there are two kinds of objective function because of different projection methods chosen. One is image space error function

$$E_{is}(R,T) = \sum_{i=1}^N \left[ \left( x_i \frac{R_1 \cdot p_i + T_x}{R_3 \cdot p_i + T_z} \right)^2 + \left( y_i \frac{R_2 \cdot p_i + T_y}{R_3 \cdot p_i + T_z} \right)^2 \right] \quad (4)$$

And the other is object space error function

$$E_{os}(R,T) = \sum_{i=1}^N \left\| (I - V_i)(R p_i + T) \right\|^2, \quad V_i = \frac{v_i v_i^T}{v_i^T v_i} \quad (5)$$

We derive results for  $E_{os}$ , which has been used by Lu [3], and is easier to parameterize than  $E_{is}$ . In Lu's algorithm, the minimum of  $E_{os}$  can be determined

through several iterations. Pose ambiguity problem corresponds to situations where  $E_{is}$  or  $E_{os}$  have several local minima for a given configuration. We now illustrate it for nearly coplanar configuration. For the same five-point object scene, we change the distance  $\|T\|$  between the camera and the object continually, while keeping all the other parameters fixed, and then we observe the variation trend of  $E_{os}$  around some axis, as depicted in Fig.2.

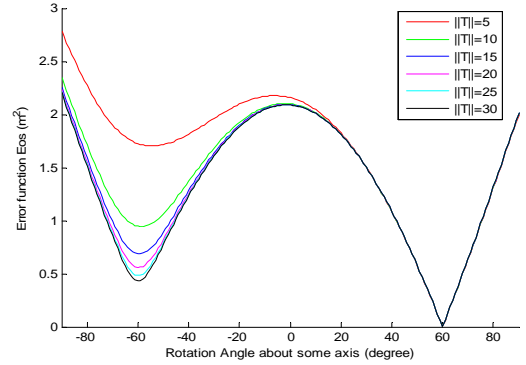


Fig.2. Error function for varying distances  $\|T\|$

In the literature, it has been proved that there may be at most two local minimum for coplanar points, which is called coplanar pose ambiguity. In Fig.2 we can see that, for nearly coplanar points, with the change of  $\|T\|$ , pose ambiguity problem also exists.

### 3. Robust Algorithm

Based on the above analysis of pose ambiguity, we observe that pose ambiguity have a close relationship with the distribution of object points, not only for coplanar points but also for 3D points, especially when the image or object noise level is high.

For the special case of nearly coplanar distribution, where most points are in a plane and only one or several points are far away from the plane, we realize that the influences of coplanar points and non-coplanar points are different and a robust pose estimation algorithm should consider both influences comprehensively. Therefore, pose estimation can be done firstly using the points in a plane, which usually results in two possible solutions because of pose ambiguity. Then we can decide on the correct pose from the two candidate solutions, using the reminder points (i.e. the points outside the plane).

Based on the simple idea, we propose a novel robust method to solve pose ambiguity for nearly coplanar points. Using the proposed algorithm, pose parameters can be uniquely determined and pose ambiguity can be avoided. The novel algorithm is summarized as:

- 1) Select the majority of object points, which reside in a main plane, using the SVD method to find the

largest singular value and the second largest singular value.

- 2) Calculate the two candidate solutions using the algorithm of Schweghofer and Pinz for the selected coplanar points.
- 3) Calculate the re-projection error according to these two local minima, in both image space and object space.
- 4) Choose the correct pose corresponding to the minimum re-projection error.

## 4. Results

We compare the effect of our approach against that of the state-of-the-art ones, both on synthetic and real world data.

### 4.1 Synthetic experiments

For all the synthetic experiments in this subsection, we use the following setup:

- 1) There are 9 object points, where 8 points are coplanar. The distance  $\|T\|=10$ .
- 2) For each test, we generate a random rotation R, the angle interval is  $[-180,180]$ .
- 3) The corresponding image points are calculated through rotation R and translation T, and the Gaussian noise is added.

For each noise level, we run 1000 independent simulations for each algorithm, such as Lu.'s algorithm, Fiore's algorithm and our proposed algorithm. Fig.3 shows the rate of finding the correct solution.

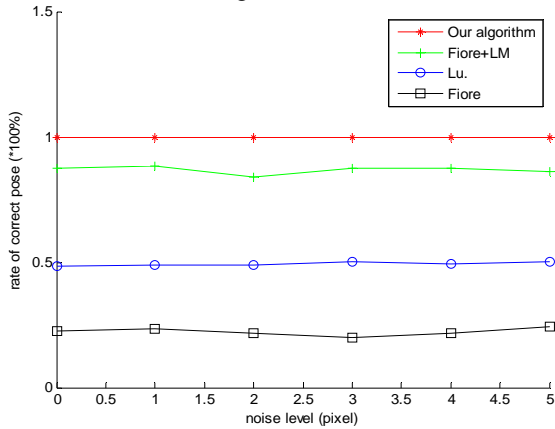


Fig.3. Rate of correct pose for different algorithms

In Fig.3, we can see that for different noise level, Lu.'s algorithm has a correct rate about 50 percent, which is because that Lu.'s algorithm gets pose through the minimum of  $E_{os}$ . However,  $E_{os}$  has two minimums for nearly coplanar points. Fiore.'s algorithm is a linear algorithm and is extremely

unstable for nearly coplanar points, thus the correct rate is very low. Fiore+LM algorithm is Fiore.'s algorithm followed with LM optimization. In Fig.2, we find that the rate has a great improvement. The slightly less robust results (about 10 percent) can be explained in this way: in a larger range, the result can be optimized to the correct pose, however, in some range; it will reach to the other wrong pose because of pose ambiguity. The correct rate of our algorithm is 100 percent, which demonstrates the effectiveness of our novel algorithm, even if the object points have nearly coplanar configuration.

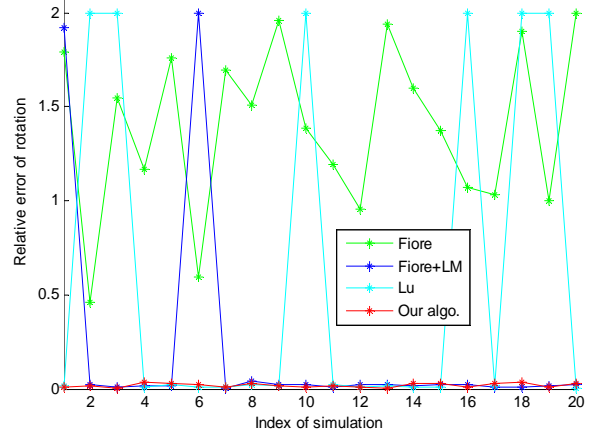


Fig.4. Relative error of rotation

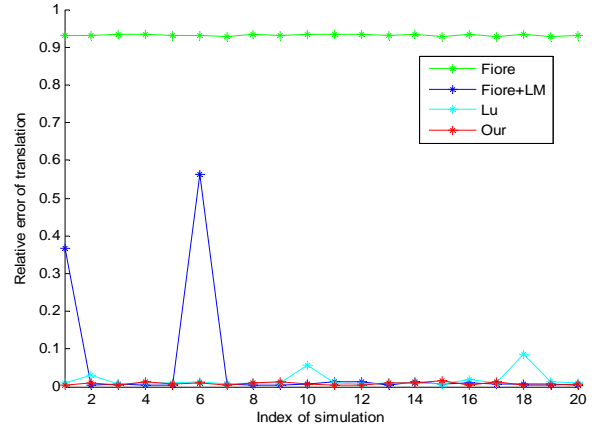


Fig.5. Relative error of translation

Fig.4 and Fig.5 present 20 results for the relative rotation error and translation error with noise=5 pixels, respectively. We can see that our algorithm is more robust than the other algorithms and the pose estimation is more accurate.

### 4.2 Real Image

As depicted in Fig.6, we select a manual model to validate the technical soundness of our algorithm.

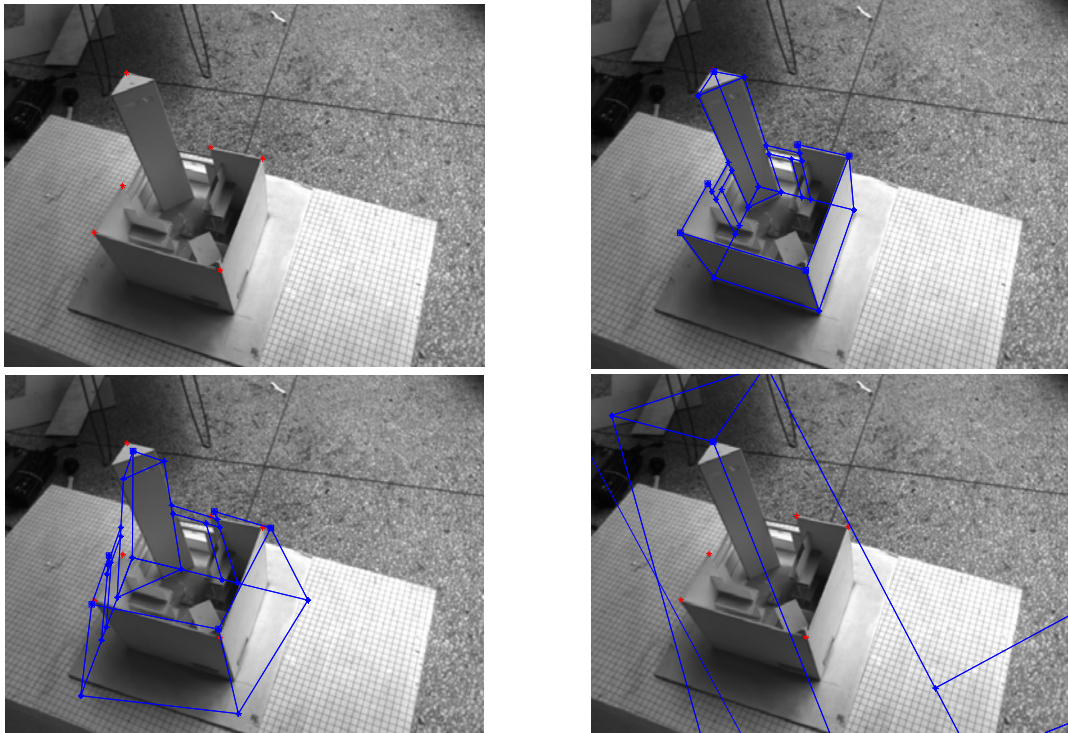


Fig.6. Experiment results for real image: top-left: model and object points; top-right: our algorithm; bottom-left: Lu algorithm; bottom-right: Fiore algorithm

There are six points, five of which are coplanar points, used to estimate pose using our proposed robust algorithm, and then project the model to the image plane, which can confirm the effectiveness and precision intuitively. From Fig.6 we can see that Fiore's algorithm has significant deviation; Lu's algorithm has a certain bias due to the wrong minimum and our algorithm matches the model very well and this means that the parameters calculated by our algorithm are the correct pose. The real image results demonstrate the effectiveness of our algorithm.

## 5. Conclusions

In this paper, based on a thorough analysis of the reason of pose ambiguities, we develop a novel robust method to solve pose ambiguity problem for nearly coplanar points. According to the analysis, when we need to estimate pose in practice, the distribution of object points should be considered to avoid pose ambiguity. When designing the configuration of object points, it would be better for the uniform distribution in all directions, which can decrease the rate of pose ambiguity and then increase the robustness of algorithm. If the configuration is confirmed, our novel algorithm can be used to ensure the robustness and precision of pose parameters. This novel algorithm should be relevant for many applications in AR and autonomous navigation.

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