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Stochastic stability of a neural model for binocular rivalry

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Abstract—We provide a theoretical framework for investigating the binocular rivalry model based on the stochastic dynamical systems theory. We adopt the stochastic Lyapunov exponent as the criterion for the stochastic stability of a system. As an application of our theoretical framework, we show how the stochastic Lyapunov exponent can explain the perceptual stabilization in the model of the binocular rivalry.

1. Introduction

When the input image of the left eye is different from that of the right eye, the recognized image is not a mixture of both, but rather it is choosen from one of the input images. Furthermore, the recognized image alternates between two candidate images stochastically and spontaneously, even when the input images remain constant. This phenomenon is called binocular rivalry. Binocular rivalry is one of the most useful phenomena for studying Neural Correlates of Consciousness (NCC), because it lies at the basis of fixed visual input stimuli. After the term NCC was proposed by Crick and Koch in 1998 (see [4]), the published papers on binocular rivalry gradually increased, including review papers published in 2002 [2], 2006 [12], 2008 [11], and 2011 [3]. At this moment, binocular rivalry has become one of the attractive research fields for the study of NCC.

Since binocular rivalry looks like simply alternation between two stable recognition states, most of the theoretical studies are based on a model that describes reciprocal inhibition and adaptation. However, there is a phenomenon which cannot be explained by such traditional model. When the input image is intermittently presented, then one image stabilizes for a long time and the time-scale is much larger than the adaptation time-scale. This phenomenon is called perceptual stabilization [9], [7] and it cannot be explained by the traditional reciprocal inhibition and adaptation scheme [11]. In 2007, Noest *et al.* provided a theoretical model of perceptual stabilization [8]. However, we still do not have enough knowledge about the perceptual stabilization especially from the viewpoint of dynamical systems. Especially in the field of binocular rivalry, we have no theoretical criterion on how to measure the stochastic stability. In this report, we show the theoretical framework for investigating the binocular rivalry model based on the stochastic dynamical systems theory [1], [5].

2. Perceptual Stabilization and the Noest Model

In 2007, Noest *et al.* provided a theoretical model for perceptual stabilization [8]. The Noest model is described with the following equations,

$$\tau\frac{dH_1}{dt} = X_1(t) - (1 + A_1(t))H_1(t) + \beta A_1(t) - \gamma S(H_2), \quad (1)$$

$$\tau\frac{dH_2}{dt} = X_2(t) - (1 + A_2(t))H_2(t) + \beta A_2(t) - \gamma S(H_1), \quad (2)$$

$$\frac{dA_1}{dt} = -A_1(t) + \alpha S(H_1), \quad (3)$$

$$\frac{dA_2}{dt} = -A_2(t) + \alpha S(H_2), \quad (4)$$

where the function S is a sigmoidal function,

$$S(z) = \begin{cases} z^2/(1+z^2), & z > 0 \\ 0, & \text{otherwise.} \end{cases} \quad (5)$$

$X_1(X_2)$ is the imput image of the left (right) eye. H_i, called "local field", corresponds to the membrane potentials of neurons which relates to the perception of X_i. $A_1(A_2)$ denotes adaptation of the left (right) perception and it is modeld as "leaky integrator". Since the time scale τ is much smaller than one, A_i and H_i are slow and fast variables, respectively. α, β, γ are constant parameters. When β is equal to zero, H_i behaves as a traditional model with reciprocal inhibition $-\gamma S(H_j)$ and adaptation A_i. The input image $X_i(t)$ is not constant, but a function of time, which is described as

$$X_i(t) = \begin{cases} X_0, & 0 \le t[\text{mod } T_{on} + T_{off}] < T_{on}, \\ 0, & \text{otherwise,} \end{cases} \quad (6)$$

where X_0 is the constant amplitude of the visual input.

The Noest model (Eq.(1)–(6)) is a modification of the traditional one. They add the neural baseline term βA_i. For the traditional case, the adaptation for the dominant neuron group is always larger than the adaptation for the suppressed group. Thus, the dominant group always switches for each intermittent changing of the input with period $T_{on} + T_{off}$. However, the Noest model balances the strong

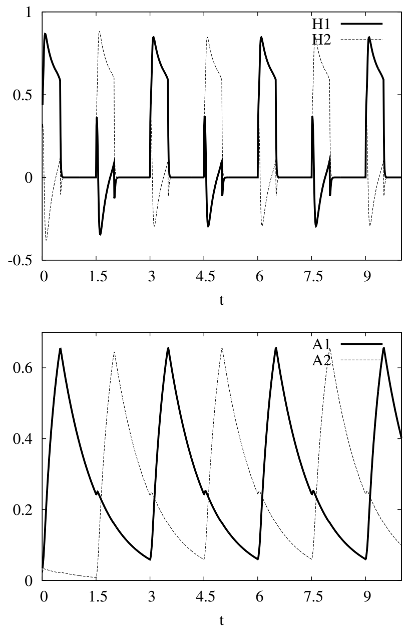

Figure 1: Time-series of H_i (upper panel) and A_i (lower panel) in the reciprocal inhibition and adaptation model. Parameters: $X_0 = 1$, $\tau = 1/50$, $\alpha = 5$, $\gamma = 10/3$, $A_1(0) = 0.03$, $A_2(0) = 0.02$, $H_1(0) = 0.1$, $H_2(0) = 0.2$, $T_{on} = 0.5$, $T_{off} = 1, \beta = 0$.

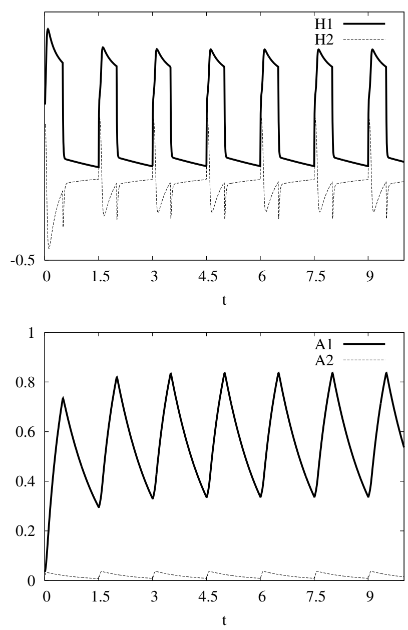

Figure 2: Time-series of H_i (upper panel) and A_i (lower panel) in the Noest model. Parameters: $X_0 = 1$, $\tau = 1/50$, $\alpha = 5$, $\gamma = 10/3$, $A_1(0) = 0.03$, $A_2(0) = 0.02$, $H_1(0) = 0.1$, $H_2(0) = 0.2$, $T_{on} = 0.5$, $T_{off} = 1, \beta = 4/(3\alpha)$.

adaptation for the domain group with its own baseline. As a result, the dominant perception persists for a long time during intermittent input change under appropriate conditions (see Fig. 1). Figure 2 shows the time-series data for the Noest model, when the input image is intermittently presented.

The Noest model can represent the phenomenon of the perceptual stabilization. However, we still do not have enough knowledge about the perceptual stabilization especially from the viewpoint of the dynamical system. Especially in the field of binocular rivalry, we have no theoretical criteria how to measure the stochastic stability. In the next section, we introduce the stochastic dynamical systems theory, which can be a powerful methodology to understand the stochastic stability in the binocular rivalry.

3. Analysis based on Stochastic Dynamical Systems Theory

3.1. Derivation of the Stochastic Lyapunov Exponent

In the paper by Noest *et al.*[8], they discuss the case when a noise term is added to β, which is related to the neural baseline. The equations of the stochastic Noest model are described as the following Ito SDE:

$$\begin{aligned} dH_1(t) &= \frac{1}{\tau}\left(X(t) - (1 + A_1(t))H_1(t) + \beta A_1(t)\right. \\ &\quad \left. -\gamma S(H_2)\right) dt + \frac{\sigma}{\tau} A_1(t) dW_t, \quad (7) \\ dH_2(t) &= \frac{1}{\tau}\left(X(t) - (1 + A_2(t))H_2(t) + \beta A_2(t)\right. \\ &\quad \left. -\gamma S(H_1)\right) dt + \frac{\sigma}{\tau} A_2(t) dW_t, \quad (8) \\ dA_1(t) &= \left(-A_1(t) + \alpha S(H_1)\right) dt, \quad (9) \\ dA_2(t) &= \left(-A_2(t) + \alpha S(H_2)\right) dt., \quad (10) \end{aligned}$$

where σ is the noise intensity. In this case, the noise term of the right hand side of Eq.(7) and Eq.(8) is $\frac{\sigma}{\tau} A_i(t) dWt$, which is multiplicative noise. The corresponding Stratonovich SDE are

$$\begin{pmatrix} dH_1(t) \\ dH_2(t) \\ dA_1(t) \\ dA_2(t) \end{pmatrix} = \begin{pmatrix} f_1(H_1, H_2, A_1, A_2) \\ f_2(H_1, H_2, A_1, A_2) \\ f_3(H_1, H_2, A_1, A_2) \\ f_4(H_1, H_2, A_1, A_2) \end{pmatrix} + \begin{pmatrix} 0 & 0 & \frac{\sigma}{\tau} & 0 \\ 0 & 0 & 0 & \frac{\sigma}{\tau} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} H_1(t) \\ H_2(t) \\ A_1(t) \\ A_2(t) \end{pmatrix} \circ dW_t. \quad (11)$$

The drift term of the above Ito SDE is the same as that of the Stratonovich SDE in this case.

The linearized SDE about a sample path $X^*(t) = (H_1^*(t), H_2^*(t), A_1^*(t), A_2^*(t))^\top$ are

$$\begin{pmatrix} dZ_1(t) \\ dZ_2(t) \\ dZ_3(t) \\ dZ_4(t) \end{pmatrix} = \begin{pmatrix} \frac{\partial f_1}{\partial H_1} & \frac{\partial f_1}{\partial H_2} & \frac{\partial f_1}{\partial A_1} & \frac{\partial f_1}{\partial A_2} \\ \frac{\partial f_2}{\partial H_1} & \frac{\partial f_2}{\partial H_2} & \frac{\partial f_2}{\partial A_1} & \frac{\partial f_2}{\partial A_2} \\ \frac{\partial f_3}{\partial H_1} & \frac{\partial f_3}{\partial H_2} & \frac{\partial f_3}{\partial A_1} & \frac{\partial f_3}{\partial A_2} \\ \frac{\partial f_4}{\partial H_1} & \frac{\partial f_4}{\partial H_2} & \frac{\partial f_4}{\partial A_1} & \frac{\partial f_4}{\partial A_2} \end{pmatrix}\Bigg|_{X^*(t)} \begin{pmatrix} Z_1(t) \\ Z_2(t) \\ Z_3(t) \\ Z_4(t) \end{pmatrix} dt + \begin{pmatrix} 0 & 0 & \frac{\sigma}{\tau} & 0 \\ 0 & 0 & 0 & \frac{\sigma}{\tau} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} Z_1(t) \\ Z_2(t) \\ Z_3(t) \\ Z_4(t) \end{pmatrix} \circ dW_t \quad (12)$$

where $Z_1(t) = H_1(t) - H_1^*(t)$, $Z_2(t) = H_2(t) - H_2^*(t)$, $Z_3(t) = A_1(t) - A_1^*(t)$, and $Z_4(t) = A_2(t) - A_2^*(t)$.

The maximum Lyapunov exponent λ_1 is numerically estimated as

$$\lambda_1 \simeq \lim_{N\to\infty} \sum_{n=1}^{N} q(S(n\Delta t)), \quad (13)$$

$$q(s) = s^\top A s + \frac{1}{2} s^\top \left(B + B^\top\right) s - (s^\top B s)^2, \quad (14)$$

$$\begin{aligned} s^\top A s = & -\frac{1}{\tau}(1 + A_1^*)(s_1)^2 - \frac{1}{\tau}\frac{dS}{dH_2}(H_2^*)s_1 s_2 - H_1^* s_1 s_3 \\ & + \frac{\beta}{\tau} s_1 s_3 - \frac{1}{\tau}\frac{dS}{dH_1}(H_1^*)s_1 s_2 - \frac{1}{\tau}(1 + A_2^*)(s_2)^2 \\ & - H_2^* s_2 s_4 + \frac{\beta}{\tau} s_2 s_4 + \alpha \frac{dS}{dH_1}(H_1^*) s_1 s_3 - (s_3)^2 \\ & + \alpha \frac{dS}{dH_2}(H_2^*) s_2 s_4 - (s_4)^2, \end{aligned} \quad (15)$$

$$\frac{1}{2} s^\top \left(B + B^\top\right) s = \frac{\sigma}{\tau}(s_1 s_3 + s_2 s_4), \quad (16)$$

$$s^\top B s = \frac{\sigma}{\tau}(s_1 s_3 + s_2 s_4). \quad (17)$$

3.2. Perceptual Stabilization and Lyapunov Exponent

Figure 3 depicts the region of the perceptual stabilization on (T_{off}, T_{on}) plane for deterministic Noest model. Figure 4 shows the stochastic Lyapunov exponent as function of T_{off} and T_{on}. All values of the stochastic Lyapunov exponent λ_1 are negative, but their absolute values differs. We can see that $|\lambda_1|$ in the region of the perceptual stabilization is slightly smaller than that in the region of the perceptual switch. It means that the perceptual switch is slightly more stable, compared with the perceptual stabilization. Furthermore, $|\lambda_1|$ takes much smaller values when (T_{off}, T_{on}) is around the boundary between perceptual stabilization. Around the boundary, the time series transiently shows perceptual stability but it converges to the perceptual switch (see Fig.5). Since the dynamics is the mixture between two different behaviors, the stochastic stability decreases. This time series also shows that the perceptual switch seems to be more stable than the perceptual stabilization.

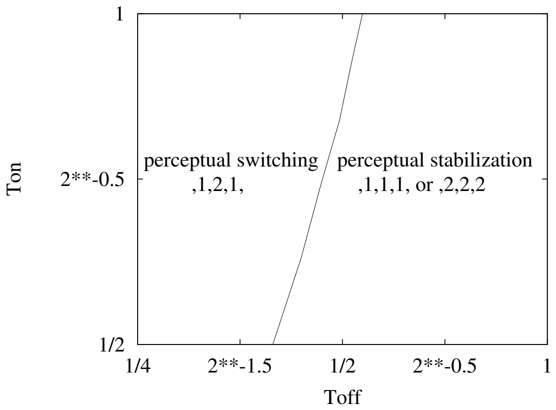

Figure 3: The region of the perceptual stabilization on (T_{off}, T_{on}) plane for deterministic Noest model. Right side from the boundary is the region of the perceptual stabilization. Parameters: $X_0 = 1$, $\tau = 1/50$, $\alpha = 5$, $\gamma = 10/3$, $A_1(0) = 0.03$, $A_2(0) = 0.02$, $H_1(0) = 0.1$, $H_2(0) = 0.2$.

4. Discussion and Conclusion

In this report, we showed that the perceptual switching is more stable than the perceptual stabilization. This contradicts our intuition because switching occurs when the system becomes unstable. Though our result means that the perceptual switching is unstable, perceptual stabilization is even more unstable. Perceptual stabilization occurs because the perception of the system is unstably forced towards one of two perceptions. It seems that the term "perceptual stabilization" is not appropriate from the viewpoint of stochastic dynamics.

It may be claimed that the stochastic dynamical systems theory is limited, because it can be used only when we know the analytical expression of the stochastic differential equation of the target dynamical system. If we can find the limit cycle and its phase resetting curve in the target system, then it is possible to apply our method, using the results in [10].

In the previous section, we interpreted the absolute value of the stochastic Lyapunov exponent as the strength of the stochastic stability. However, usually we only discuss the sign of the Lyapunov exponent, especially for the study of chaos in a deterministic dynamical system. We have already checked that the absolute value can explain the strength of the stability. For example, in the case of Brownian motion on a quartic bistable potential function, the stochastic Lyapunov exponent is always negative but the absolute value goes up and down, together with an increase in the noise intensity (not shown in this report). It means that, for low noise levels, the state detects only one attractor, but for higher noise levels, the state can detect two attractors.

At this moment, binocular rivalry has become one of the attractive research fields for the study of NCC. With the

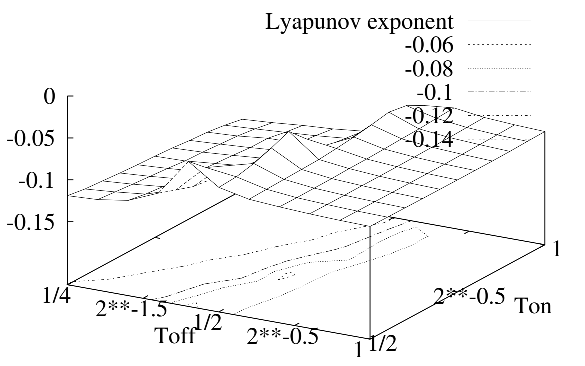

Figure 4: Stochastic Lyapunov exponent as function of T_{off} and T_{off} for stochastic Noest model with $\sigma = 0.001$. Parameters: $X_0 = 1$, $\tau = 1/50$, $\alpha = 5$, $\gamma = 10/3$, $A_1(0) = 0.03$, $A_2(0) = 0.02$, $H_1(0) = 0.1$, $H_2(0) = 0.2$.

growing interest of applying mechanisms from the brain dynamics to ICT, studying the stochastic stability of brain can also be helpful for designing new robust control mechanisms in information networks. Apart from its relevance for neural systems, the alternation between stable states may serve as a model for controlling the duty cycle in networks of sensors, subsets of which need to selectively switch on and off, depending on unpredictable environmental conditions and strict limitations of energy requirements.

In conclusion, we have provided the theoretical framework about the analysis of stochastic stability for binocular rivalry, based on stochastic dynamical systems theory. We have shown the mathematical procedure to calculate the stochastic Lyapunov exponent, step by step, as well as the application of the methodology to the Noest neuron model. Our numerical investigation shows that the stochastic Lyapunov exponent is a useful criterion for estimating the stochastic stability of perceptual stability in the case of intermittent input images. We have shown that perceptual switching is more stable than the perceptual stabilization. Before our study, there has been no theoretical criteria how to measure the stochastic stability. Therefore our methodology can shed new light on the field of the binocular rivalry from a theoretical point of view.

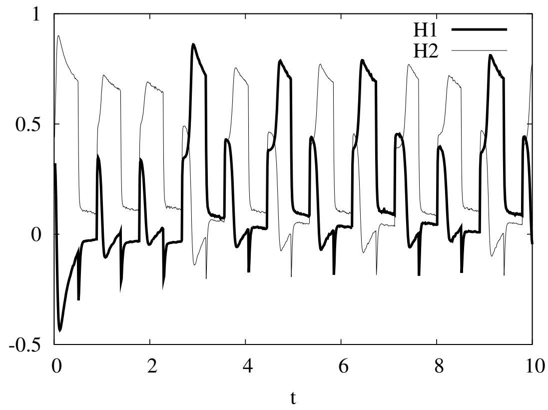

Figure 5: Time-series data of H_1 and H_2. Parameters: $X_0 = 1$, $\tau = 1/50$, $\alpha = 5$, $\gamma = 10/3$, $A_1(0) = 0.03$, $A_2(0) = 0.02$, $H_1(0) = 0.1$, $H_2(0) = 0.2$, $T_{on} = 0.5$, $T_{off} = 0.39$, $\sigma = 0.001$.

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