IEICE Proceeding Series

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Vol. 2 pp. 73-76
Publication Date: 2014/03/18
Online ISSN: 2188-5079

2013 International Symposium on Nonlinear Theory and its Applications
NOLTA2013, Santa Fe, USA, September 8-11, 2013

Optimal Pricing Algorithm Based on Steepest Descent Method for Electricity Market with Battery and Accumulator

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Abstract—This paper considers a day-ahead market with batteries and accumulators to level power generation. First, we model a consumer with battery, a generator with battery, and an accumulator that each act to maximize their own profit. Then, we not only propose the optimal pricing algorithm based on dual decomposition and the steepest descent method but also prove the stability of this algorithm. Finally, we demonstrate the effectiveness of the algorithm through some numerical simulation.

1. Introduction

While energy demand keeps increasing, global warming and energy problems are becoming global issues. Because of this paradigm shift, a smart-grid system with demand-side management (DSM) is required in order to use electricity efficiently. One of the DSM techniques uses the electricity price, which is called dynamic pricing (DP).

DP sets the power price for each individual hour, or for shorter time intervals, to control power consumption. Several dynamic pricing schemes have already been proposed, such as time-of-use pricing (TOU) [1] and critical peak pricing (CPP) [2], in which the electric power company sets the price at given times or peak times. Consumers use power in accordance with a predetermined price plan, and generators supply power in balance with the demand.

In this paper, we treat an electricity market that is independent of the electric power company. Because this market can set an optimal price based on the power gap and treat each player independently, it can apply to a smart-grid that involves distributed generation with sources like wind power generators [3, 4].

There are two kinds of electricity markets: day-ahead (DA) markets, which set a rough price the day before; and real-time (RT) markets, which suppress the gap between the DA market and the real generation or consumption of power. RT markets focus on stabilization of the power systems [5, 6]; therefore, we treat a DA market to level the power generation.

The other technique for leveling power generation is the storage system, which charges with power at off-peak times and discharges power at peak times. A storage system may include not only an accumulator like in pumped-storage power generation but also an aggregate of batteries like those in notebook PCs and electric vehicles.

We consider a model that includes an independent accumulator, a consumer with battery, a generator with battery, and a DA market. Consumer, generator, and accumulator each act to maximize their own profit. We define the market model by applying dual decomposition to a centralized market model. Then, we solve the market problem by the steepest descent method. We prove the stability of the model and demonstrate the effectiveness by numerical simulation.

2. Problem Formulation

The market solves for an optimal price as follows. Here Fig. 1 shows the concept of the model.

1. The market sets the price $\boldsymbol{\lambda}$ and conveys this to each player.
2. The consumer, generator, and accumulator make their plans and convey these plans to the market.
3. If the supply and the demand are not matched, the market resets the price and conveys the price again.
4. Steps 2 and 3 are repeated until the gap is sufficiently small.

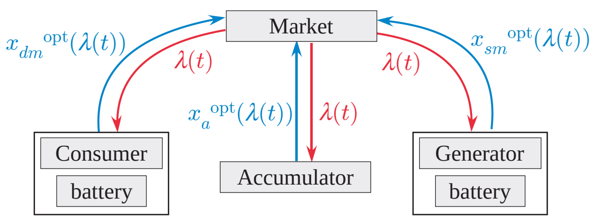

Figure 1: Concept of model

We define parameters as follows: a bullet (•) represents any player; d indicates the consumer, s the generator, and a the accumulator. For example, $x_{dm,k}$ is the power that the consumer buys from the market at time k.

$x_{\bullet m,k}$: power bought or sold at time k
($x^{\text{opt}}_{\bullet m,k}(\lambda_k)$ is the optimal value at price λ_k)
λ_k : power price
$v_k(x)$: utility function of consumer
$c_k(x)$: cost function of generator
$[d_k^{\min}, d_k^{\max}]$: bounds of power consumption
$[s_k^{\min}, s_k^{\max}]$: bounds of power generation
$x_{\bullet p,k}$: power discharged from or charged to ($x_{\bullet p,k} < 0$ for charged and $x_{\bullet p,k} > 0$ for discharged)
($x^{\text{opt}}_{\bullet p,k}(\lambda_k)$ is the optimal value at price λ_k)
$[p_{\bullet,k}^{\min}, p_{\bullet,k}^{\max}]$: bounds of charging and discharging
$b_{\bullet,k}$: battery level at time k
$b_{\bullet,0}$: initial battery level
$[b_{\bullet k}^{\min}, b_{\bullet k}^{\max}]$: bounds of battery level
$\pi_{\bullet k}(x_{\bullet p,k})$: cost function of overcharge/overdischarge

3. Player Models

In this section we make assumptions for the consumer, generator, and storage system.

Assumption 1. The function $v(x_d)$ is $C^2[0,\infty)$, strictly increasing, and strictly convex. The function $c(x_s)$ is $C^2[0,\infty)$, strictly increasing ,and strictly concave. The bounds of the consumer and generator satisfy $d_k^{\min} < s_k^{\max}$ or $s_k^{\min} < d_k^{\max}$. The function $\pi_{\bullet k}(x_{\bullet,k})$ is $C^2[p_{\bullet k}^{\min}, p_{\bullet k}^{\max}]$, strictly convex, and satisfies $\pi_{\bullet k}(0) = 0$.

3.1. Player models without batteries

First, we define models without batteries. The consumer buys power according to (1) and the generator sells power according to (2).

$$x^{\text{opt}}_{d,k}(\lambda_k) = \arg \max_{d_k^{\min} \le x_{d,k} \le d_k^{\max}} v_k(x_{d,k}) - \lambda_k x_{d,k} \tag{1}$$

$$x^{\text{opt}}_{s,k}(\lambda_k) = \arg \max_{s_k^{\min} \le x_{s,k} \le s_k^{\max}} \lambda_k x_{s,k} - c_k(x_{s,k}) \tag{2}$$

$\lambda_k x_{d,k}$ is the cost to buy energy, and (1) means that consumers buy $x^{\text{opt}}_{d,k}(\lambda_k)$ to maximize their own welfare. Similarly, $\lambda_k x_{s,k}$ is the value of energy sales, and (2) means that generators sell $s(\lambda)$ to maximize their own welfare.

The consumer model with price inelasticity of demand is represented as follows:

$$x^{\text{opt}}_{d,k}(\lambda_k) = \mu_1 d_{1,k} + \mu_2 \dot{\tilde{v}}_k^{-1}(\lambda_k(t)) \tag{3}$$

where $\mu_1 d_{1,k}$ is the price inelasticity of demand and $\mu_2 \dot{\tilde{v}}_k^{-1}(\lambda_k(t))$ is the price elasticity. Note that μ_1 and μ_2 represent the proportions of elastic and inelastic demand, and $\dot{\tilde{v}}$ is a virtual utility function. To model with (3), we define the utility function $v_k(x_{d,k})$ as follows:

$$v_k(x_{d,k}) = \mu_2 \tilde{v}_k\left(\frac{x_{d,k} - \mu_1 d_{1,k}}{\mu_2}\right) \tag{4}$$

3.2. Consumer model

The power consumed is the sum of what the consumer has bought from the market and discharged from a battery. Therefore, the consumer model with a battery is represented as (6).

$$\boldsymbol{x}_d^{\text{opt}}(\boldsymbol{\lambda}) = \begin{bmatrix} \boldsymbol{x}_{dm}^{\text{opt}T} & \boldsymbol{x}_{dp}^{\text{opt}T} \end{bmatrix}^T \tag{5}$$

$$= \arg \max_{x_{dm,k}, x_{dp,k}} \left[\sum_{k=0}^{N-1} v_k(x_{dm,k} + x_{dp,k}) - \lambda_k x_{dm,k} - \pi_{d,k}(x_{dp,k}) \right] \tag{6}$$

$$\text{s.t. } d_k^{\min} \le x_{dm,k} + x_{dp,k} \le d_k^{\max}, \quad \forall k \tag{7}$$

$$p_{d,k}^{\min} \le x_{dp,k} \le p_{d,k}^{\max}, \quad \forall k \tag{8}$$

$$A\boldsymbol{x}_{dp} - \boldsymbol{b}_d \le 0 \tag{9}$$

A and $\boldsymbol{b}_\bullet$ can be represented as follows:

$$A = \left[\begin{array}{cccc} 1 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 1 & \cdots & \cdots & 1 \\ \hline -1 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ -1 & \cdots & \cdots & -1 \end{array}\right], \quad \boldsymbol{b}_\bullet = \left[\begin{array}{c} b_{\bullet 0} - b_{\bullet 0}^{\min} \\ \vdots \\ b_{\bullet 0} - b_{\bullet N-2}^{\min} \\ b_{\bullet 0} - b_{\bullet N-1}^{\min} \\ \hline b_{\bullet 0}^{\max} - b_{\bullet 0} \\ \vdots \\ b_{\bullet N-2}^{\max} - b_{\bullet 0} \\ b_{\bullet N-1}^{\max} - b_{\bullet 0} \end{array}\right] \tag{10}$$

where lines $1, \ldots, N$ of $\boldsymbol{b}_\bullet$ are lower bounds of battery level and lines $N+1, \ldots, 2N$ are upper bounds.

3.3. Generator model

The power generated is the sum of what the generator has sold to the market and charged to a battery. Therefore, the generator model with a battery is represents as (12).

$$\boldsymbol{x}_s^{\text{opt}}(\boldsymbol{\lambda}) = \begin{bmatrix} \boldsymbol{x}_{sm}^{\text{opt}T} & \boldsymbol{x}_{sp}^{\text{opt}T} \end{bmatrix}^T \tag{11}$$

$$= \arg \max_{x_{sm,k}, x_{sp,k}} \left[\sum_{k=0}^{N-1} \lambda_k x_{sm,k} - c_k(x_{sm,k} - x_{sp,k}) - \pi_{s,k}(x_{sp,k}) \right] \tag{12}$$

$$\text{s.t. } s_k^{\min} \le x_{sm,k} + x_{sp,k} \le s_k^{\max}, \quad \forall k \tag{13}$$

$$p_{s,k}^{\min} \le x_{sp,k} \le p_{s,k}^{\max}, \quad \forall k \tag{14}$$

$$A\boldsymbol{x}_{sp} - \boldsymbol{b}_s \le 0 \tag{15}$$

3.4. Accumulator model

Here we make another assumption for the accumulator.

Assumption 2. The accumulator manages charging, discharging, and losses from overcharging or overdischarging.

The accumulator produces a profit when power is neither consumed nor generated. The accumulator model is represented as (16).

$$\arg \max_{\boldsymbol{x}_{ap}} \left[\sum_{k=0}^{N-1} \lambda_k x_{ap,k} - \pi_a(x_{ap,k}) \right] \tag{16}$$

$$\text{s.t. } p_{a,k}^{\min} \le x_{ap,k} \le p_{a,k}^{\max}, \quad \forall k \tag{17}$$

$$A\boldsymbol{x}_{ap} - \boldsymbol{b}_a \le 0 \tag{18}$$

4. Market Model

We introduce the following assumptions for the power network and the market, respectively.

Assumption 3. 1. Resistive losses in the transmission and distribution lines are negligible. 2. The line capacities are high enough that congestion will not occur.

Assumption 4. The market does not know the function $v(x_d)$.

According to these assumptions, the market allows each player to decide on a power plan.

To define the market model, we regard the power network as one centralized player, and then we decompose this model into the proposed model with three players and a market.

The overall player wants to maximize profit with no gap between supply and demand. Therefore, the problem is represented as (19).

$$\max_{\boldsymbol{x}} \sum_{k=0}^{N-1} \Big[\Big\{ v_k(x_{dm,k} + x_{dp,k}) - \pi_{d,k}(x_{dp,k}) \Big\} + \Big\{ -c_k(x_{sm,k} - x_{sp,k}) - \pi_{s,k}(x_{sp,k}) \Big\} + \Big\{ -\pi_a(x_{ap,k}) \Big\} \Big] \quad (19)$$

$$\boldsymbol{x} = \left[\boldsymbol{x}_{dm}^T, \boldsymbol{x}_{dp}^T, \boldsymbol{x}_{sm}^T, \boldsymbol{x}_{sp}^T, \boldsymbol{x}_{ap}^T \right]^T \quad (20)$$

s.t. (7)–(9), (13)–(15), (17)–(18),

$$\boldsymbol{x}_{dm,k} - \boldsymbol{x}_{sm,k} - \boldsymbol{x}_{ap,k} = \boldsymbol{0} \quad (21)$$

Using (21) and the Lagrange multiplier $\lambda_{0,k}$, we rewrite this primal problem as the min-max dual problem.

$$\min_{\boldsymbol{\lambda}_0} \max_{\boldsymbol{x}} \sum_{k=0}^{N-1} \Big[\Big\{ v_k(x_{dm,k} + x_{dp,k}) - \pi_{d,k}(x_{dp,k}) \Big\} + \Big\{ -c_k(x_{sm,k} - x_{sp,k}) - \pi_{s,k}(x_{sp,k}) \Big\} + \Big\{ -\pi_a(x_{ap,k}) \Big\} \Big] - \boldsymbol{\lambda}_0^T \left(\boldsymbol{x}_{dm} - \boldsymbol{x}_{sm} - \boldsymbol{x}_{ap} \right) \quad (22)$$

s.t. (7)–(9), (13)–(15), (17)–(18)

The variables in (7)–(9) consist of $x_{dm,k}$ and $x_{dp,k}$. Similarly, the variables in (13)–(15) consist of $x_{sm,k}$ and $x_{sp,k}$, while the variable in (17)–(18) consists of $x_{ap,k}$. Therefore, we can divide (22) into four player problems.

$$\begin{cases} \underset{x_{dm,k}, x_{dp,k}}{\arg\max} \left[\sum_{k=0}^{N-1} v_k(x_{dm,k} + x_{dp,k}) - \lambda_{0,k} x_{dm,k} - \pi_{d,k}(x_{dp,k}) \right] \\ \text{s.t. } (7)\text{–}(9) \end{cases} \quad (23)$$

$$\begin{cases} \underset{x_{sm,k}, x_{sp,k}}{\arg\max} \left[\sum_{k=0}^{N-1} \lambda_{0,k} x_{sm,k} - c_k(x_{sm,k} - x_{sp,k}) - \pi_{s,k}(x_{sp,k}) \right] \\ \text{s.t. } (13)\text{–}(15) \end{cases} \quad (24)$$

$$\begin{cases} \arg\max_{\boldsymbol{x}_{ap}} \left[\sum_{k=0}^{N-1} \lambda_{0,k} x_{ap,k} - \pi_a(x_{ap,k}) \right] \\ \text{s.t. } (17)\text{–}(18) \end{cases} \quad (25)$$

$$\min_{\boldsymbol{\lambda}_0} \boldsymbol{\lambda}_0^T \left(\boldsymbol{x}_{dm} - \boldsymbol{x}_{sm} - \boldsymbol{x}_{ap} \right) \quad (26)$$

When we treat the Lagrange multiplier λ_0 as the price λ, (23)–(25) are the same as the model of the consumer, generator, and accumulator. Hence, we can divide the model using dual decomposition.

4.1. Properties of the model

Here we confirm the model in detail. In order to solve the dual problem with the optimization tool, we confirm that the solution of the decomposed model is the same as that of the primal problem and that the dual problem function is convex and differentiable. For simplicity, we rewrite the dual problem as follows:

$$\text{Primal:} \quad \max_{\boldsymbol{x}} f(\boldsymbol{x}) \quad (27)$$

$$\text{s.t. } h(\boldsymbol{x}) = \boldsymbol{0}, \quad \boldsymbol{x} \in X \quad (28)$$

$$\text{Dual:} \quad \min_{\boldsymbol{\lambda}} \varphi(\boldsymbol{\lambda}) \quad (29)$$

$$\varphi(\boldsymbol{\lambda}) = \max_{\boldsymbol{x}} \{ L(\boldsymbol{x}, \boldsymbol{\lambda}) | \boldsymbol{x} \in X \} \quad (30)$$

$$L(\boldsymbol{x}, \boldsymbol{\lambda}) = f(\boldsymbol{x}) - \boldsymbol{\lambda}^T h(\boldsymbol{x}) \quad (31)$$

$$\boldsymbol{x} = \left[\boldsymbol{x}_{dm}^T, \boldsymbol{x}_{dp}^T, \boldsymbol{x}_{sm}^T, \boldsymbol{x}_{sp}^T, \boldsymbol{x}_{ap}^T \right]^T,$$

$$f(\boldsymbol{x}) \Leftrightarrow \sum_{k=0}^{N-1} \Big[\Big\{ v_k(x_{dm,k} + x_{dp,k}) - \pi_{d,k}(x_{dp,k}) \Big\} + \Big\{ -c_k(x_{sm,k} - x_{sp,k}) - \pi_{s,k}(x_{sp,k}) \Big\} + \Big\{ -\pi_a(x_{ap,k}) \Big\} \Big],$$

$$h(\boldsymbol{x}) \Leftrightarrow \boldsymbol{x}_{dm,k} - \boldsymbol{x}_{sm,k} - \boldsymbol{x}_{ap,k} = \boldsymbol{0},$$

$$X \Leftrightarrow (7)\text{–}(9),\ (13)\text{–}(15),\ (17)\text{–}(18)$$

5. Proposed Algorithm

The market updates the price by using the steepest descent method based on the plans of the consumer, generator, and accumulator. We propose an iterative algorithm as follows:

$$\boldsymbol{\lambda}(t+1) = \boldsymbol{\lambda}(t) + \gamma(\boldsymbol{x}_{dm}^{\text{opt}}(\boldsymbol{\lambda}) - \boldsymbol{x}_{sm}^{\text{opt}}(\boldsymbol{\lambda}) - \boldsymbol{x}_{ap}^{\text{opt}}(\boldsymbol{\lambda})) \quad (32)$$

Here we summarize this algorithm.

Algorithm 1.
1. The market sets the initial price $\lambda(0)$.
2. The consumer, generator, and accumulator calculate the plan of each player from the electricity price based on (6), (12), and (16).
3. The market updates the price based on (32).
4. Steps 2 and 3 are repeated until the gap is sufficiently small according to (33).

$$(\boldsymbol{x}_{dm}^{\text{opt}}(\boldsymbol{\lambda}(t)) - \boldsymbol{x}_{sm}^{\text{opt}}(\boldsymbol{\lambda}(t)) - \boldsymbol{x}_{ap}^{\text{opt}}(\boldsymbol{\lambda}(t))) \approx \boldsymbol{0} \quad (33)$$

We now prove the stability of this algorithm.

Theorem 1. *When we denote the optimal price by $\boldsymbol{\lambda}^*$, the algorithm is stable when the step-size γ satisfies the following inequality:*

$$0 < \gamma \leq \frac{2(\boldsymbol{\lambda}^* - \boldsymbol{\lambda}(t))^T h(\boldsymbol{x}^{\text{opt}}(\boldsymbol{\lambda}(t)))}{\| h(\boldsymbol{x}^{\text{opt}}(\boldsymbol{\lambda}(t))) \|^2} \quad (34)$$

Proof. Proof can be done via similar calculation in [7] □

6. Numerical Simulation

In this simulation, although the consumer does not have a battery, the generator has a battery. We compare the market models with and without a storage system [7] to demonstrate the effectiveness of a storage system.

We set $\tilde{v}(x_d) = a\log(x_d+1)$. Then, based on Section 3.1 and Assumption 1, we define the utility function and cost function as follows:

$$v_k(x_d) = \mu_2 a \log\left(\frac{x_d - \mu_1 d_{1,k}}{\mu_2} + 1\right) \tag{35}$$

$$c(x_s) = b x_s^2 \tag{36}$$

For $d_{1,k}$ we use power consumption data provided by the Tokyo Electric Power Company [8], and we regard $\mu_1 d_1$ as the price inelasticity of demand. TOU pricing plan is shown in Fig. 2.

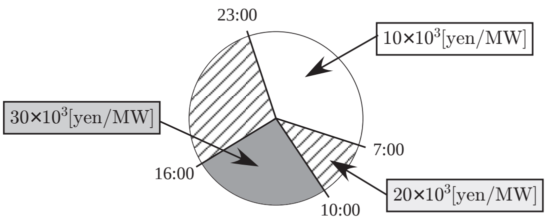

Figure 2: TOU pricing plan

We set $[b_{N-1}^{\min}, b_{N-1}^{\max}] = [b_0, b_0]$ [MWh] and designed the simulation not to increase the profit based on the initial battery level. We chose the others parameters as follows:

$N = 24$,
$a = 6.7682 \times 10^8$, $b = 0.2341$, $\xi = 0.1$,
$\mu_1 = 0.67$, $\mu_2 = 0.2$, $\gamma = 0.1$
$[d_k^{\min}, d_k^{\max}] = [0, \infty]$[MWh] $\forall k$,
$[s_k^{\min}, s_k^{\max}] = [0, \infty]$[MWh] $\forall k$,
$[p_{\bullet,k}^{\min}, p_{\bullet,k}^{\max}] = [-1000, 1000]$ [MWh] $\forall k$,
$[b_{\bullet k}^{\min}, b_{\bullet k}^{\max}] = [1000, 10000]$ [MWh]
$(k = 0, \ldots, N-2)$,
$[b_{\bullet,N-1}^{\min}, b_{\bullet,N-1}^{\max}] = [b_{\bullet,0}, b_{\bullet,0}]$ [MWh],
$b_{\bullet,0}$ [MWh]

The result of the simulation are shown in Fig. 3. A horizontal axis represents time k and a vertical axis represents the power generated or bid on. For example, the value at time 0:00 represents the power generated or bid on between 0:00 and 1:00. The blue line indicates the power sold without the accumulator, and the dotted purple line indicates the power sold with the accumulator. The red line indicates the power generated, which equals the power sold minus the power in the battery plan of the generator. At peak times, from 10:00 to 15:00, the proposed method reduces the peak more than the previous method. Furthermore, the reduction is actually more in generating power than in selling power.

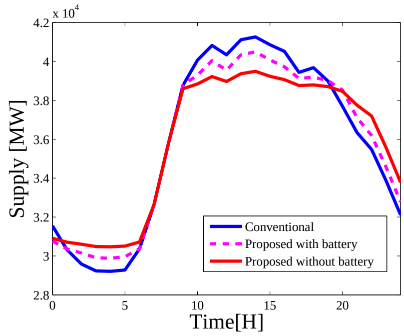

Figure 3: Power supply

7. Summary and Future Work

In this paper, we modeled an electricity market with a battery, and we proposed an algorithm to compute the optimum price. First, we modeled the consumer, generator, and accumulator to maximize their own profit, and we modeled the market by using dual decomposition. Then, we proposed an algorithm based on the steepest descent method, and we confirmed its stability using Lyapunov stability theory. Through a numerical simulation, we confirmed that our algorithm levels the power generation, matches the power balance, and increase the profit of the overall player.

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