

Simulation of distributed-lumped networks by the CIP method

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Abstract—The CIP (Cubic-Interpolated Propagation) method is a method to solve a kind of partial differential equations (PDEs) with high accuracy, in particular, for the case of steep setup input like a pulse wave, but is not well-known in the field of circuit analysis. The LSI (Large-Scale-Integrated) circuits consist of many distributed elements which can be represented by partial differential equations and lumped (and often nonlinear) elements like transistors. This paper demonstrates the application of the CIP method to these distributed-lumped circuits.

1. Introduction

The increasing developments in high-speed networks including LSIs (Large Scale Integrated circuits) highlight the previously negligible effects of interconnects to signal waveforms, such as ringing, signal delay, distortion, reflections, and crosstalk[2]. Here the interconnects consist of transmission lines governed by PDEs and lumped-constant (often nonlinear) elements (e.g., transistors) represented by algebraic equations or ODEs and are called mixed distributed-lumped networks. So the analysis of these interconnect networks corresponds to solving PDEs with boundary conditions represented by ODEs or algebraic equations and is important for the design of LSI circuits.

Signals impressed to these interconnects are nearly trapezoidal pulses with very short rise/fall time (e.g., 0.1ns, which corresponds to 10 GHz) and transmit them through the networks by repeating reflections at mismatched points. Efficient and accurate analysis of mixed distributed-lumped networks is in general difficult, in particular for the case of steep rise/fall waveform.

There are many methods to simulate these mixed distributed-lumped networks[3]. The techniques are classified into two categories; one is based on modeling all transmission lines (distributed networks) by lumped-constant networks, and the other is based on direct numerical analysis of the above PDEs. Asymptotic waveform evaluation (AWE) and asymptotic equivalent circuit (AEC) belong to the first category, while the finite-difference time-domain method

(FDTD) and finite element method (FEM) are classified into the second one.

The Cubic-Interpolated Propagation (CIP) method possesses the advantage in solving PDEs with high accuracy for the case of steep setup trapezoidal pulse wave[1]. However as far as the authors know, the CIP has not been used for the analysis of the interconnected networks.

The principle of the CIP method is illustrated in Fig.1 : (a) The solid line represents an initial profile and the dashed line is an exact solution after advection¹, (b) Only the values of the advected solutions at sampling points are illustrated ; (c) When solutions are linearly interpolated, numerical diffusion appears; (d) In the CIP, spatial derivative also propagates and the profile inside a grid cell is retrieved. In this paper we apply the CIP method to simulate mixed distributed-lumped networks and compare its accuracy etc. with other conventional methods.

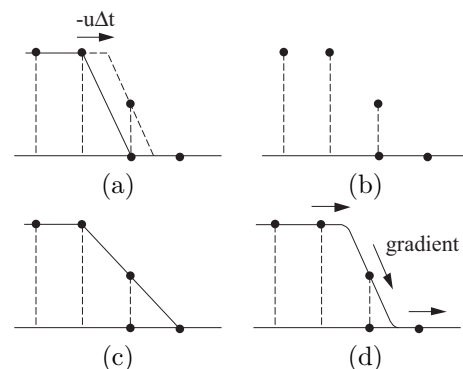


Figure 1: principle of the CIP method

2. Preliminaries

Figure 2 shows a single transmission line terminated by the lumped-constant networks consisting of linear/nonlinear elements and voltage source. The voltage and current on the single transmission line are de-

¹This word is used in meteorology and means the horizontal transfer of heat or other atmospheric properties. We use this word in the sense of *translation*.

scribed by the telegraph equation

$$-\frac{\partial v(x,t)}{\partial x} = L \frac{\partial i(x,t)}{\partial t} + Ri(x,t) \quad (1a)$$

$$-\frac{\partial i(x,t)}{\partial x} = C \frac{\partial v(x,t)}{\partial t} + Gv(x,t) \quad (1b)$$

where L , R , C , and G are inductance, resistance, capacitance, and conductance per unit length, respectively. The lumped-constant networks give boundary conditions at the $x = 0$ and $x = d$ which are the constraint between $v(0,t)$ and $i(0,t)$ ($v(d,t)$ and $i(d,t)$). For example, $v(0,t) = f(i(0,t))$, or $i(l,t) = C_0 \frac{v(l,t)}{dt}$, etc.

The equations (1a) and (1b) can be rewritten in matrix form:

$$-\frac{\partial}{\partial x} \begin{bmatrix} v \\ i \end{bmatrix} = \begin{bmatrix} 0 & L \\ C & 0 \end{bmatrix} \frac{\partial}{\partial t} \begin{bmatrix} v \\ i \end{bmatrix} + \begin{bmatrix} 0 & R \\ G & 0 \end{bmatrix} \begin{bmatrix} v \\ i \end{bmatrix} \quad (2)$$

Diagonalizing the matrix $\begin{bmatrix} 0 & L \\ C & 0 \end{bmatrix}$ in Eq.(2), we have

$$-\frac{\partial}{\partial x} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \sqrt{LC} & 0 \\ 0 & -\sqrt{LC} \end{bmatrix} \frac{\partial}{\partial t} \begin{bmatrix} a \\ b \end{bmatrix} + \frac{1}{2\sqrt{LC}} \begin{bmatrix} GL + RC & GL - RC \\ GL - RC & GL + RC \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} \quad (3)$$

where a and b are incidence and reflected voltage wave, respectively which are defined as

$$a(x,t) \equiv \frac{1}{2} \left[v(x,t) + \sqrt{\frac{L}{C}} i(x,t) \right] \quad (4a)$$

$$b(x,t) \equiv \frac{1}{2} \left[v(x,t) - \sqrt{\frac{L}{C}} i(x,t) \right] \quad (4b)$$

Thus the voltage and current on the single distributed line are expressed as

$$v(x,t) = a(x,t) + b(x,t) \quad (5a)$$

$$\sqrt{\frac{L}{C}} i(x,t) = a(x,t) - b(x,t) \quad (5b)$$

In the similar way, the boundary conditions at $x = 0$ and $x = d$ can be represented in terms of the voltage wave quantities[4],[5],[6].

Taking a derivative with respect to x of Eq.(3) yields

$$-\frac{\partial}{\partial x} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \sqrt{LC} & 0 \\ 0 & -\sqrt{LC} \end{bmatrix} \frac{\partial}{\partial t} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} + \frac{1}{2\sqrt{LC}} \begin{bmatrix} GL + RC & GL - RC \\ GL - RC & GL + RC \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \quad (6)$$

where

$$\alpha(x,t) \equiv \frac{\partial a(x,t)}{\partial x} \quad \text{and} \quad \beta(x,t) \equiv \frac{\partial b(x,t)}{\partial x}. \quad (7)$$

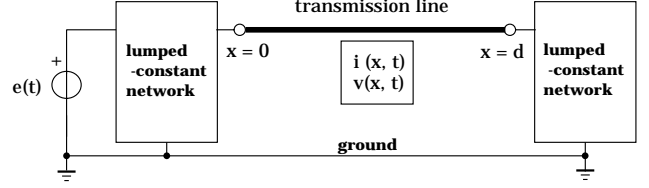


Figure 2: Mixed distributed-lumped network

3. Calculation method

The numerical solution of Eq.(3) is calculated by splitting the entire evolution into two phases; advection phase and non-advection phase. Eq.(3) can be divide into two groups of equations as follows:

$$\frac{\partial a}{\partial t} + \frac{1}{\sqrt{LC}} \frac{\partial a}{\partial x} = 0 \quad (8a)$$

$$\frac{\partial b}{\partial t} - \frac{1}{\sqrt{LC}} \frac{\partial b}{\partial x} = 0 \quad (8b)$$

and

$$\frac{\partial a}{\partial t} = -\frac{1}{2LC} \{ (GL + RC)a + (GL - RC)b \} \quad (9a)$$

$$\frac{\partial b}{\partial t} = -\frac{1}{2LC} \{ (GL - RC)a + (GL + RC)b \} \quad (9b)$$

Eq.(6) is also divided into two groups in the similar way.

In the following subsections we mention the method for computing the numerical solutions of Eq.(3) at the next time step.

3.1. Advection phase

We divide the length d into m cells and assign $m + 1$ grids on the x -axis. Thus the step length Δx is

$$\Delta x = \frac{d}{m}. \quad (10)$$

Given four values of wave quantity a and its first derivative α on the $(i - 1)$ -th and i -th grids at time $n\Delta t$, the third-order interpolated polynomial $p(x) = c_3(x - x_i)^3 + c_2(x - x_i)^2 + c_1(x - x_i) + c_0$ is determined uniquely as follows:

$$c_3 = \frac{2 \{ a_i^n - a_{i-1}^n \}}{(-\Delta x)^3} + \frac{\alpha_i^n + \alpha_{i-1}^n}{(-\Delta x)^2}$$

$$c_2 = \frac{3 \{ a_{i-1}^n - a_i^n \}}{(-\Delta x)^2} + \frac{2\alpha_i^n + \alpha_{i-1}^n}{-\Delta x}$$

$$c_1 = \alpha_i^n$$

$$c_0 = a_i^n$$

Here we denote a numerical estimate of the exact solution $a(x_i, n\Delta t)$ by a_i^n .

The partial differential equation in (8a) corresponds to the wave $a(x,t)$ and its first derivative $\alpha(x,t)$ traveling forward. Thus moving the advected profile $p\left(x_i - \frac{1}{\sqrt{LC}}\Delta t\right)$ ahead, we have

$$\begin{aligned}\tilde{a}_i^{n+1} &= p(x_i - \xi\Delta t) \\ &= c_3(-\xi\Delta t)^3 + c_2(-\xi\Delta t)^2 + c_1(-\xi\Delta t) + c_0\end{aligned}\quad (11)$$

$$\begin{aligned}\tilde{\alpha}_i^{n+1} &= \frac{dp}{dx}(x_i - \xi\Delta t) \\ &= 3c_3(-\xi\Delta t)^2 + 2c_2(-\xi\Delta t) + c_1\end{aligned}\quad (12)$$

where $\xi = 1/\sqrt{LC}$ is a propagation velocity. Values of \tilde{a}_i^{n+1} and $\tilde{\alpha}_i^{n+1}$ for $i = 1, 2, \dots, m$ are calculated on using Eqs.(11) and (12).

From Eq.(8b) the reflected wave $b(x,t)$ and its derivative $\beta(x,t)$ propagate backward. Hence values of \tilde{b}_i^{n+1} and $\tilde{\beta}_i^{n+1}$ for $i = m-1, \dots, 1, 0$ are evaluated by means of the advected polynomial $q(x)$ interpolated between the i -th and $(i+1)$ -th grids. The incidence wave \tilde{a}_0^{n+1} and reflected wave \tilde{b}_m^{n+1} are determined from the near-end and far-end boundary condition, respectively.

3.2. Non-advection phase

In this phase we solve the ordinary differential equations (9) with the initial values $\tilde{a}(t + \Delta t, x_i)$ and $\tilde{b}(t + \Delta t, x_i)$ and obtain the numerical solutions of Eq.(3) at time $(n+1)\Delta t$ as follows:

$$\begin{aligned}a_i^{n+1} &= \tilde{a}_i^{n+1} \\ &- \frac{\Delta t}{2LC} \left\{ (GL + RC)\tilde{a}_i^{n+1} + (GL - RC)\tilde{b}_i^{n+1} \right\}\end{aligned}\quad (13)$$

$$\begin{aligned}b_i^{n+1} &= \tilde{b}_i^{n+1} \\ &- \frac{\Delta t}{2LC} \left\{ (GL - RC)\tilde{a}_i^{n+1} + (GL + RC)\tilde{b}_i^{n+1} \right\}\end{aligned}\quad (14)$$

4. Numerical examples

We simulate the trapezoidal pulse propagation on the nondistortion line terminated with the linear resistor $R_L = 10(\Omega)$ (Fig.3). Per-unit-length parameters are:

$$\begin{aligned}L &= 1.0 \times 10^{-7} \quad (\text{H/m}) & C &= 1.0 \times 10^{-9} \quad (\text{F/m}) \\ R &= 1.0 \times 10^2 \quad (\Omega/\text{m}) & G &= 1.0 \quad (\text{U/m})\end{aligned}$$

The length d of the distributed line is 1 (cm). Thus the delay time equals

$$t_d = \frac{d}{1/\sqrt{LC}} = \frac{1.0 \times 10^{-2}}{1.0 \times 10^8} = 0.1(\text{ns}).$$

and the magnitude of voltage attenuates at the end of the line is

$$\exp(-d\sqrt{RG}) \approx 0.905.$$

Three methods are applied to simulate the signal propagation. The numerical solutions are illustrated in Figs.4 and 5. The rise/fall time of trapezoidal pulse denoted by t_r is 5 (ps) and 2.5 (ps), respectively.

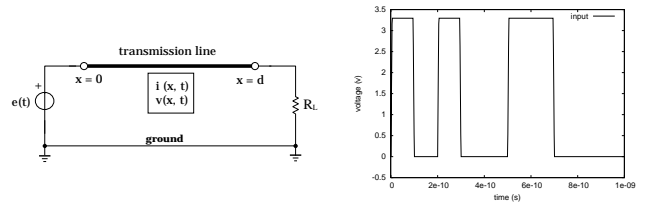


Figure 3: Mixed distributed-lumped network and input voltage waveform

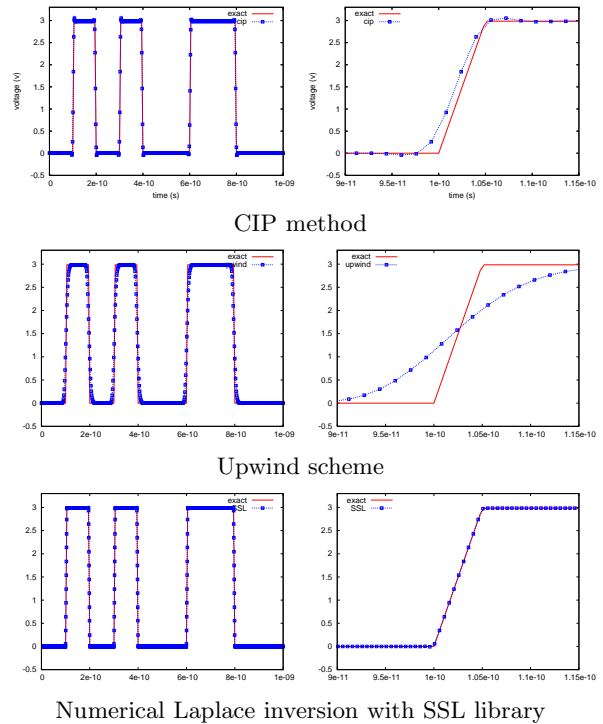


Figure 4: Numerical solutions of output voltage waveform $v(d,t)$ where $t_r = 5$ (ps), $\Delta x = 0.2$ (mm), and $\Delta t = 1.6$ (ps). The figures in the left column illustrate the whole waveform. The right-hand figures show their details around the first step slope.

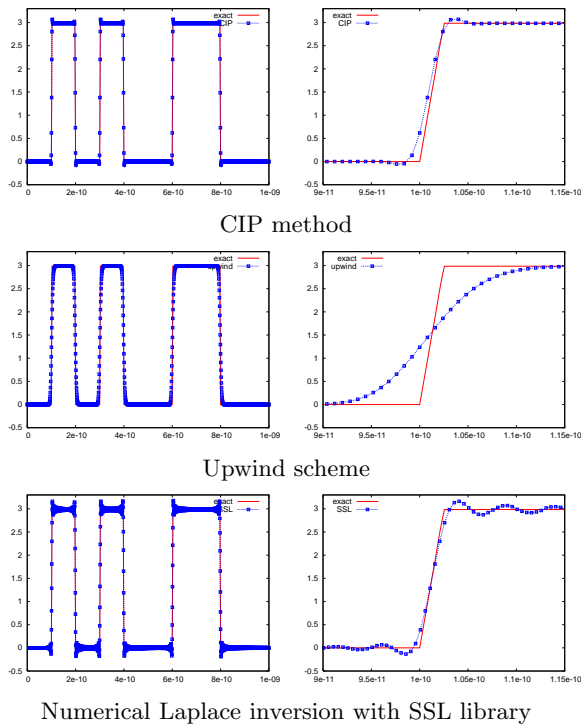


Figure 5: Numerical solutions of output voltage waveform $v(d, t)$ where $t_r = 2.5$ (ps), $\Delta x = 0.1$ (mm), and $\Delta t = 0.8$ (ps).

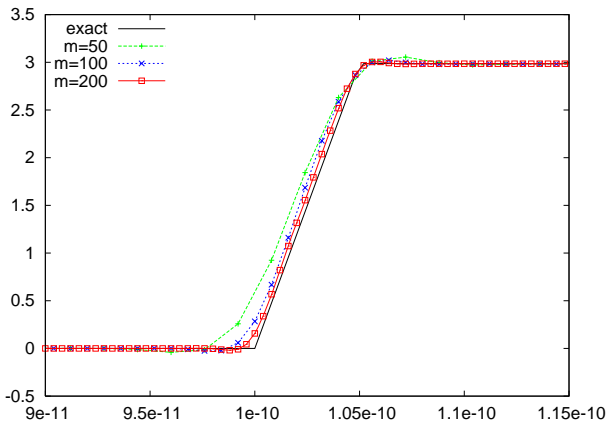


Figure 6: Numerical solutions by the CIP method with different step length where $t_r = 5$ (ps).

We can observe that the CIP method keeps the shape of the trapezoidal pulse almost accurately, while the upwind scheme (first-order difference scheme) spreads the trapezoidal pulse as shown in Figs.4 and 5. The reason why these numerical diffusions occur is explained in Fig.1. The numerical Laplace transform inversion gives the most accurate solution when $t_r = 5$ (ps). The numerical solution tends to oscillate as the rise/fall time becomes shorter

shown in Fig.5. The high-frequency components influence the convergence speed and accuracy, since the formula of numerical Laplace inversion is expressed by the alternating series on the complex plane. Comparing with the numerical Laplace inversion, the CIP method yields the solutions with high accuracy independent of the rise/fall time.

Fig.6 shows the accuracy of numerical solutions with different step length by the CIP method. The more accurate solutions are obtained as the step length becomes more smaller. We can see from Fig.6 that the CIP method has more than second order accuracy.

5. Conclusion

The CIP method has been applied to simulate the mixed distributed-lumped networks. Comparing with other methods, the CIP method can give accurate waveform irrelevant to the spectrum of the impressed trapezoidal pulse signal. Our calculation method can be easily generalized to the mixed distributed-lumped networks with multi-conductor lines which will be shown in subsequent paper.

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