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# Iterative Model of Mesoscopic Neural Populations Displaying Aperiodic Dynamics

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**Abstract**—Mesoscopic level neurodynamics study the collective dynamical behavior of neural populations. Such models are becoming increasingly important in understanding large-scale brain processes. Mesoscopic dynamics exhibit aperiodic oscillations with a much more rich dynamical behavior than fixed-point and limit-cycle approximation allow. Far from being an undesired behavior to be suppressed, researchers are beginning to explore the idea that aperiodic dynamics may be essential to the fast recognition and large capacities of biological memories. In this paper we discuss one such mesoscopic population model, based on Freeman's original K-set formulation. This model replicates the aperiodic behavior observed in biological brains. We are using this model to construct robot controllers that utilize such rich dynamics as their mechanisms for forming meanings and acting on past experiences. In this paper we introduce a discrete approximation of the original K-set continuous ODE model. We develop the discrete time model and compare its dynamical behavior in the so called K-III realm with the continuous ODE model. We then demonstrate its usefulness as a biologically inspired robot controller.

## 1. Introduction

K-sets, developed by Walter J. Freeman [7, 1], model the dynamics of the mean field (e.g. average) amplitude of a neural population. A single neural population is described by a second order, ordinary differential equation. An asymmetric, sigmoidal transfer function provides the nonlinear interaction between collections of such population units. Hierarchies of K-sets have been used to model the aperiodic, chaotic like dynamics observed in perceptual and cortical areas of the brain. The description of the K-set hierarchies are biologically motivated, and building up increasingly complex relations of recurrent, feedback relations at the K-II and K-III level produces models of the aperiodic dynamics of perceptual and cortical areas of the brain.

In this paper we develop an iterative version of the K-set neurodynamical population model. Our motivation is to develop a discrete model that replicates the dynamics of the K-sets, but in a simpler, more tractable form. The discrete simplification is useful in many areas, including the development of neural controllers for robotic agents, to explore the importance of mesoscopic neural dynamics in producing behavior.

A discrete time K-set model has many advantages over using coupled sets of continuous ODE's. Networks of such discrete dynamical components are self-contained and require no rewriting of global state equations, like those needed in solving networks of coupled ODE's. Since units are so self-contained in the discrete version, they are easily incorporated into standard neural network packages and robot simulators as basic units. Further, discrete units usually are much more efficient and will therefore be capable of performing much faster, or developing much bigger controller models, than using the corresponding continuous versions. We present our development of the discrete approximation and then show that the model is capable of replicating the types of chaotic dynamics observed in biological brains.

## 2. Discrete K-Sets Model

### 2.1. K-Sets: Continuous Differential Equation Model

The K-set dynamics were developed to model the dynamics of the mean field (e.g. average) amplitude of a neural population. A nonlinear, second order, ordinary differential equation was developed to model the dynamics of such a population. The parameters for this equation were derived by experimentation and observation of isolated neural populations of animals prepared through brain slicing techniques and chemical inhibition. The isolated populations were subjected to various levels of stimulation, and the resulting impulse response curves were replicated by the K-set equations.

The basic ODE equation of a neural population of the K-model is:

$$\alpha\beta\frac{d^2a_i(t)}{dt^2} + (\alpha+\beta)\frac{da_i(t)}{dt} + a_i(t) = net_i(t) \quad (1)$$

In this equation $a_i(t)$ is the activity level (mean field amplitude) of the $i^{th}$ neural population. $\alpha$ and $\beta$ are time constants (derived from observing biological population dynamics to various amounts of stimulation). The left side of the equation expresses the intrinsic dynamics of the K unit (which captures a neural populations characteristic responses).

On the right side of the equation are factors that allow for external network input to the population $net_i(t)$ Stimulation between populations is governed by a nonlinear transfer function. The nonlinear transfer function used in the K-models is an asymmetric sigmoid that was again derived through measurements of the stimulation between biological neural populations:

$$net_i(t) = \sum_j w_{ij} o_j(t) \quad (2)$$

$$o_j(t) = \epsilon\{1 - exp[\frac{-(e^{a_j(t)} - 1)}{\epsilon}]\} \quad (3)$$

See [1, 2] for a more complete description of the basic K-Set model.

The previous ODE and transfer function define the basic population unit and the dynamics governing the spreading of activation between populations. The real modeling power of the K-Sets is achieved by a biologically motivated definition of a hierarchy of relations, known as the K-set hierarchy. Connecting 4 interacting K units together with positive and negative feedback forms a K-II unit, which is capable of oscillatory behavior, both continuous and damped. K-II units of different frequencies connected together will keep one another from ever agreeing on a particular frequency, thus generating frustrated chaos. The K-III level of the hierarchy forms the basic unit for producing aperiodic dynamics, and for use as a model of chaotic memories.

### 2.2. KA-Sets: Discretization of Continuous K-Sets

The purpose of the discretization is to develop a model more useful in simulations with autonomous agents, thus we name it KA. We use a method for determining a discrete approximation of a sampled continuous time signal [4]. Principe et. al. [6] presents an alternative discretization based on decomposition of alpha-kernels method. The discretization presented here is more compact (thus slightly more efficient), and straightforward.

We will use the dynamics produced by the K-0 set (equation 1) as the signal that is to be approximated. The signal will be approximated using a second-order difference equation, where we look back 2 discrete time steps to develop the approximation. Equation 4 is the difference equation to be used.

$$y(t) = a_1 y(t-1) + a_2 y(t-2) + b_1 u(t-1) + b_2 u(t-2) \quad (4)$$

Here $y$ is the signal at some discrete time step and $u$ is an external input being fed into the system. The signal $y$ at time $t$ will be computed based on the signal at $y(t-1)$ and $y(t-2)$. We also use the input into the system in two previous time steps $u(t-1)$ and $u(t-2)$. Our task is to find values for the parameters $a_1$, $a_2$, $b_1$ and $b_2$ so that the difference equation approximates the dynamics of a sampled signal.

At any given time step $t$, the approximation in Equation 4 plus some $\varepsilon$ error value will equal the actual signal. We can write this as a summation:

$$y = \sum_{j=1}^{P} \beta_j X_j + \varepsilon \quad (5)$$

$$\beta_1 = a_1 \, \beta_2 = a_2 \, \beta_3 = b_1 \, \beta_4 = b_2$$
$$X_1 = y(t-1) \, X_2 = y(t-2)$$
$$X_3 = u(t-1) \, X_4 = u(t-2)$$

where P is 4 approximating using a second-order difference equation (since we have 4 parameters). $X_j$ are the signal and input to the system in the previous two time steps, and $\beta_J$ are the parameters we are to determine.

From Equation 5 we can see that the error for a single particular time step is $\varepsilon = y - \sum \beta X$. Our task, however, is to develop an approximation for all of the sampled time steps, not simply a single time step. We can write an equation for the sum squared error as:

$$S = \sum_{i=1}^{N} (y_i - \sum_{j=1}^{P} \beta_j X_{ij})^2. \quad (6)$$

Here we have $N$ discrete sampled time steps, and $S$, the sum squared error, is the sum of the errors squared. Equation 6 can be rewritten in matrix form as:

$$S = (Y - XB)^T (Y - XB) \quad (7)$$

$$Y = \begin{vmatrix} y_1 \\ \vdots \\ y_N \end{vmatrix} \quad B = \begin{vmatrix} \beta_1 \\ \vdots \\ \beta_P \end{vmatrix}$$

Equation 7 gives the error of a discrete approximation, given the $\beta$ parameters. We are attempting to find the best discrete approximation, therefore we are trying to minimize the error $S$. Taking $S$ to be 0 in Equation 7, we can solve the matrix equation for the B parameters:

$$\hat{B} = (X^T X)^{-1} X^T Y \quad (8)$$

Equation 8 states that, in order to get the error $S$ as close to 0 as possible, B needs to equal $(X^T X)^{-1} X^T Y$. Y is the signal we are trying to approximate, and X can be found if we know the previous values of Y at sampled time steps, and also previous inputs to the system in sampled time steps.

With Equation 8 we only need a sampled signal, and recordings of external stimulation, to create an approximation. Using the K-0 Equation 1 we generate a sample time series. We create a representative signal of the K-0 dynamics by varying duration of stimulation from 1 to

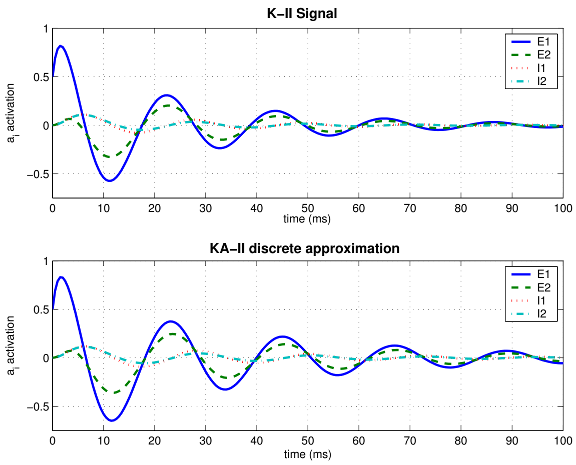


Figure 1: Comparison of continuous K-II signal (top) with the KA-II discrete approximation (bottom).

50ms in 1ms increments, and intensity of stimulation from -0.5 (inhibition) to 0.5 (excitation) in 0.01 increments. We simulate all combinations of intensity and duration of the external stimulation to the K-0. The external stimulation represents the external input to the system (u in Equation 4), and the activation of the K-0 unit represents the signal we are approximating (y in Equation 4). The sample K-0 signal and inputs were recorded and used to solve for $\hat{B}$ in Equation 8. The values of the parameters that were determined to give the least square error for the KA discrete approximation are shown in Table 1.

Table 1: KA Discrete Approximation Parameter Results

| | |
|---|---|
| $a_1$ | 1.6198 |
| $a_2$ | -0.6497 |
| $b_1$ | 0.0234 |
| $b_2$ | 0.0059 |

### 2.3. KA Discrete Dynamics Comparison

As a demonstration of the KA discrete approximation, we simulate a K-II and compare the signal with a KA-II system configured with the same values. A K-II is a combination of 4 K-0 units with mutually excitatory and inhibitory feedback. Figure 1, left, shows the results of this comparison. In both systems $w_{ee} = 0.3$, $w_{ei} = 5.0$, $w_{ie} = 0.2$ and $w_{ii} = 0.25$. Also both models were started at the same initial conditions. The K-II, simulated in Matlab on a Pentium III 2.0Ghz using Matlab's ode45 function, takes 191.57 seconds of CPU time to simulate 12 seconds. The KA-II approximation on the same system takes 52 seconds to simulate the same 12 seconds of activity.

As shown in Figure 1, the KA discretization approximates the K-set model fairly well. This good approximation occurs even though we are simulating a more complex network of 4 neural populations connected with various levels of positive and negative feedback among the units. The dynamics produced by the discretization remain relatively close to the desired system even in a system of units, even though it was only created using a sampled signal from the dynamics of a single unit.

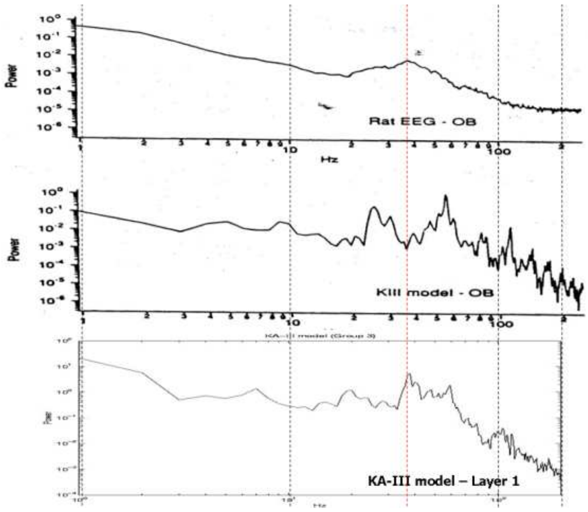


Figure 2: Comparison of power spectrum from rat (top) with continuous K-III model power spectrum (middle) and that of a KA-III discrete approximation (bottom). (Top and Middle figures taken from [5]).

The KA discrete approximation also works well at the level III of the K-set hierarchy, where aperiodic dynamics are produced that model those seen in biological brains. Figure 2, right, compares a power spectrum distribution from a rat to that of the original K-III aperiodic signal, and that produced by a KA-III. The power spectrum of the KA discretization matches that generated by the K-III in terms of a peak of power around the 40Hz range, and a slope of the power spectrum close to -2.

## 3. KA Robotic Controller

In this experiment we use a Khepera robotic agent in a virtual environment to demonstrate the use of the KA discretization as a controller. The task we choose is similar to that explored in the original Distributed Adaptive Control models of Verschure, Kröse and Pfeifer [8].

In this experiment, the goal of the agent is to learn to associate long-range distance sensory information with behaviors to learn to trigger avoidance behaviors at a distance, before the agent actually bumps into the obstacle. Therefore in the robots control architecture we also have a set of units that are connected to the long range infra-red distance sensors . The distance sensors can sense obstacles at a distance from the robot. Six KA-0 units are connected to the normal output of the distance sensors ($DS_{1-6}$ connected to

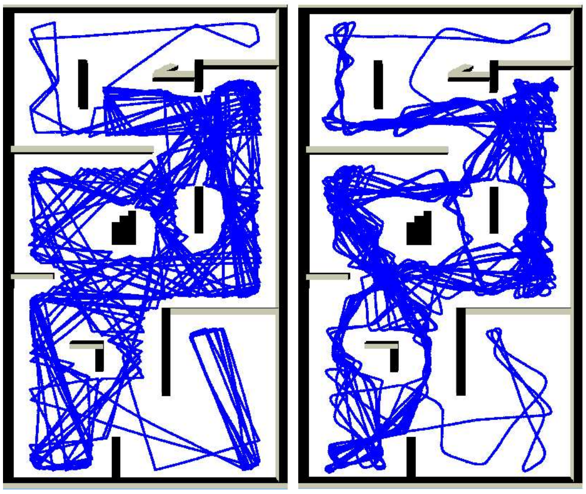

Figure 3: A comparison of typical paths created by the Hülse-Pasemann neural Schmitt trigger (Left) and the KA units (Right).

$S_{1-6}$) while six other KA-0 are connected to the inverse of the indicated distance sensor ($DI_{1-6}$ connected to $S_{7-12}$). The inverse of a distance sensor is maximally active when no obstacle is detected, and is minimally active when the sensor is right next to an obstacle. Initially the 12 sensory KA-0 are fully connected to each other with small random weights . Also the 12 KA-0 are fully connected to each of the 3 basic motor behaviors (*Turn Left, Turn Right and Move Fwd*) again with small random weights.

We use Hebbian learning and habituation on the connections between the 'Sensory' units and from the 'Sensory' to the 'Motor' units. Since these connections are initially random, typically they do not affect the behavior of the robot in the beginning. The reflexes cause the robot to move around in the environment. Later on the robot may bump into something on its left. This will cause some of the Motor behaviors to be performed, such as turning right. Since the Sensory units that are connected to sensors on the left side of the body have become stimulated while approaching the obstacle, they remain highly active when the right turn behavior is activated. This allows the strength of the connection between the Sensory unit for detection of obstacles on the left and the right turn behavior to become strengthened due to Hebbian modification because of their co-occurring excitation.

Figure 3 displays a comparison of typical paths generated in an environment using the KA architecture described previously and compared to the architecture using the dynamical Hülse-Pasemann Schmitt Triggers (HPST) [3]. We use the KA units after they have adequately learned obstacle avoidance, at which point we freeze the weights, similar to the evolved weights learned for the HPST. In general, the KA exhibits comparable performance as the HPST in this environment.

## 4. Conclusion

The discrete KA model presented here is capable of displaying all of the interesting dynamics produced by the continuous-time K models. Given similar parameter settings, the KA discretization replicates the dynamics of a similarly configured K model at the K-II level, with four units of recurrently connected excitatory and inhibitory units. Further, the KA discretization is capable of generating frustrated chaos observed at the K-III level of the hierarchy.

The KA discretization has many benefits as use for a modeling tool when compared to using the continuous model. The units are self-contained and may thus be used easily in standard neural network and robotic simulation packages. The units are also more efficient than solving continuous ODEs using approximation methods such as Runge-Kutta. We have shown in this paper that the discrete units may be used to build controllers for autonomous robots. We are currently exploring the natural production of aperiodic dynamics in such controllers, and study how they might benefit an agent in performing its behavioral tasks in a complex environment.

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## References

[1] Walter J. Freeman. *Mass Action in the Nervous System*. Academic Press, New York, NY, 1975.

[2] Walter J. Freeman. Simulation of chaotic EEG patterns with a dynamic model of the olfactory system. *Biological Cybernetics*, 56:139–150, 1987.

[3] Martin Hülse and Frank Pasemann. Dynamical neural schmitt trigger for robot control. *Lecture Notes in Computer Science, ICANN 2002*, 2415:783–788, 2002.

[4] Abdul J. Jerri. *Linear Difference Equations with Discrete Transform Methods*. Kluwer Academic, 1996.

[5] L. Kay, K. Shimoide, and Walter J. Freeman. Comparison of EEG time series from rat olfactory system with model composed of nonlinear coupled oscillators. *International Journal of Bifurcation and Chaos*, 5(3):849–858, 1995.

[6] Jose C. Principe, Vitor G. Tavares, and John G. Harris. Design and implementation of a biologically realistic olfactory cortex in analog VLSI. *Proceedings of the IEEE*, 89:1030–1051, 2001.

[7] Christine A. Skarda and Walter J. Freeman. How brains make chaos in order to make sense of the world. *Behavioral and Brain Sciences*, 10:161–195, 1987.

[8] Paul F. M. J. Verschure, B. Kröse, and Rolf Pfeifer. Distributed adaptive control: The self-organization of behavior. *Robotics and Autonomous Systems*, 9:181–196, 1992.