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Generation of *n*-scroll attractors by Josephson junctions

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Abstract—In this paper Josephson junctions are used in order to generate *n*-scroll attractors. Instead of synthesizing the nonlinearity of *n*-scroll attractors via linear components, we propose this paper to make use of the nonlinearity of the Josephson junction itself.

1. Introduction

Since the observation of *n*-double scroll attractors [9] from a generalized Chua's circuit [2], multi-scroll attractors [13, 5] have received considerable attention. Short after the presentation of *n*-double scroll attractors, *n*-scroll attractor have been studied by Suykens *et al.* [8] which also allows for an odd instead of an even number of scrolls. Then an experimental confirmation of *n*-double scroll and *n*-scroll attractors were given followed by the circuit realization of 2-double scroll attractor [1] and 5-scroll attractor [12]. In *n*-scroll attractors the scrolls are only located along one state variable direction in state space. The family of scroll grid attractors has been introduced where this is no longer a restriction [11]. These families allow that the scrolls can be located in any state variable direction. Currently multi-scroll chaotic attractors is a common umbrella for the collection of *n*-scroll, *n*-double scroll, and families of scroll grid chaotic attractors.

A design of multi-scroll chaotic attractor is basically a problem of design of multiple equilibrium points in a scroll base chaotic attractor by modifications of nonlinear characteristic. In [9] the modification is done by introducing additional breakpoints in the piecewise nonlinear characteristic of Chua's circuit in order to obtain *n*-double scroll attractors. In [10] the same modification is done by the sine functions. In [11] and [6] authors used a collection of the step functions and smooth hyperbolic tangent functions for the modifications, respectively.

In fact the design and realization of multi-scroll attractors depends on synthesizing the nonlinearity with an electrical circuit. The question arises whether there exists an electrical device that can naturally allow to design a multi-scroll chaotic attractor. In this paper we want to argue that Josephson junctions [4] are suitable candidate devices that possess such a nonlinearity. Josephson Junctions are superconducting devices that can generate high frequency oscillations. In [3] chaotic dynamics from Josephson junction have been reported. In this paper, in the first introduced model we employ the nonlinearity of the junctions in a *n*-scroll attractor. Then we use also the dynamics of the junctions in a second model for generating *n*-scroll attractor. The phase difference of the junctions will be one of the state variable of the resulting *n*-scroll attractor.

This paper is organized as follows. In Section 1 we propose *n*-scroll attractors obtained as 1-D scroll grid attractors using a sine function. In Section 2 two new models are introduced by making use of the Josephson junction nonlinear characteristic and the phase dynamics together with its nonlinearity.

2. *n*-scroll attractors via the sine function

In [10] a sine function was replacing the nonlinear characteristic of Chua's circuit. With the sine function different numbers of scrolls can be designed. Here we apply a similar approach to a simple $M_x + N_x + 1$-scroll attractor

$$\begin{cases} \dot{x} &= y \\ \dot{y} &= z \\ \dot{z} &= -ax - ay - az + af(x) \end{cases} \tag{1}$$

where

$$f(x) = \sum_{i=1}^{M_x} g_{\frac{(-2i+1)}{2}}(x) + \sum_{i=1}^{N_x} g_{\frac{(2i-1)}{2}}(x) \tag{2}$$

and

$$g_\theta(\zeta) = \begin{cases} 1, & \zeta \geq \theta, \quad \theta > 0 \\ 0, & \zeta < \theta, \quad \theta > 0 \\ 0, & \zeta \geq \theta, \quad \theta < 0 \\ -1, & \zeta < \theta, \quad \theta < 0. \end{cases} \tag{3}$$

$x, y, z \in \mathbb{R}$, $\zeta \in \mathbb{R}$ [13]. In [6] the authors have replaced the nonlinear function (3) by a smooth hyperbolic tangent

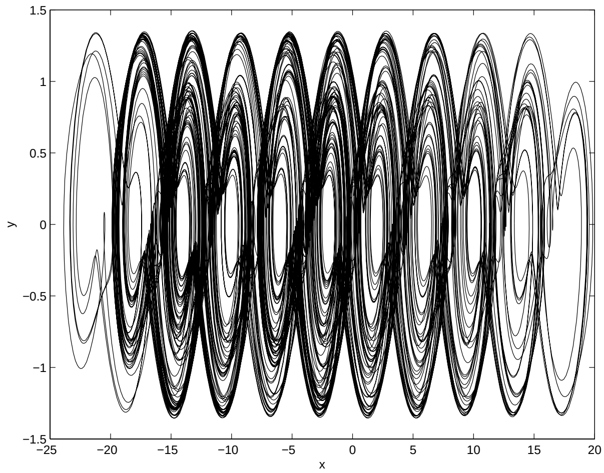

Figure 1: 11-scroll attractor from (4) ($a = 0.3$, $b = 0.25$, initial conditions $(.1, .2, .3)$)and numerical simulations are performed in Matlab (`ode23`) until $t = 6000$.

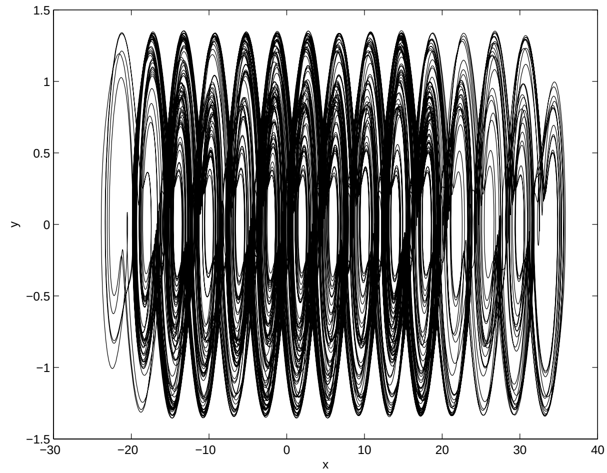

Figure 2: 15-scroll attractor from (4) ($a = 0.3$, $b = 0.25$, initial conditions $(.1, .2, .3)$) and $t = 9000$.

function. Here we replace the nonlinearity by a sine function. Hence the new model is described by

$$\begin{cases} \dot{x} &= y \\ \dot{y} &= z \\ \dot{z} &= -ay - az + ag(x) \end{cases} \tag{4}$$

where

$$g(x) = \sin(2\pi bx) \tag{5}$$

and $b \in \mathbb{R}$. Figure 1. shows 11-scroll attractors obtained from the system (4) for $a = 0.3$, $b = 0.25$, initial conditions $(.1, .2, .3)$ and numerical simulations are performed in Matlab using a Runge-Kutta integration rule (`ode23`) until $t = 6000$. Figure 2 shows 15-scroll attractors obtained from the same system with the same parameter when the numerical simulations are performed until $t = 9000$. The reason to obtain different number of scrolls for the different simulation time is indeed that the system is an n-scroll attractor for the given parameters and the number of scrolls (n) is defined by the system itself. The sine function in [10], which is defined by

$$h(x) = \begin{cases} \frac{b\pi}{2s}(x - 2ac), & \text{if} \quad x \geq 2ac \\ -b\sin\left(\frac{x\pi}{2a} + d\right), & \text{if} \quad -2ac < x < 2ac \\ \frac{b\pi}{2s}(x + 2ac), & \text{if} \quad x \leq -2ac. \end{cases}, \tag{6}$$

is used for a given interval of the variable (x) of the nonlinearity and the scrolls are obtained within this interval. There is no restriction here to locate the scrolls within an interval, therefore the number of scrolls can not be known.

3. Josephson Junctions and n-scroll attractors

Josephson junctions are highly nonlinear superconducting electronic devices. It is also well-known that Josephson Junctions are superconducting devices that can generate high frequency oscillations [7]. The aim here is basically to use the nonlinearity of a Josephson junction in the model 5. The current in a Josephson junction is described by

$$I = I_c \sin\phi \tag{7}$$

where

$$\dot{\phi} = kV. \tag{8}$$

Here ϕ is the phase difference and V is the voltage across the junction. In a superconducting Josephson junction k is defined by the fundamental constants $k = \frac{2e}{h}$ (h is Planck's constant divided by 2π and e is the charge on the electron).

Instead of designing a sine function for the n-scroll attractor which was discussed in the previous section here the nonlinearity of the Josephson junction (7) is applied to the model (4). For that reason the current in the Josephson junction is chosen as $g(x)$ and the voltage across the junction set to y state variable. Hence the new model with the Josephson junction is given by

$$\begin{cases} \dot{x} &= y \\ \dot{y} &= z \\ \dot{z} &= -ay - az + \frac{a}{2Ic}I \\ \dot{\phi} &= ky \end{cases} \tag{9}$$

where I is given in Eq. (7). Figure 3. shows 10-scroll attractors from the system (9) for $a = 0.1$ and $k = 1$. Furthermore in Figure 4 n-scroll attractors from the same system (9) are shown for $a = 0.1$ and $k = 2$. A zoom of Figure 4. is shown in Figure 5.

The model (9) can also be described by

$$\begin{cases} \dot{\phi} &= ky \\ \dot{y} &= z \\ \dot{z} &= -ay - az + \frac{a}{2Ic}I. \end{cases} \tag{10}$$

In this case the Josephson junction is integrated to the system and phase difference (ϕ) is one of the state variable of

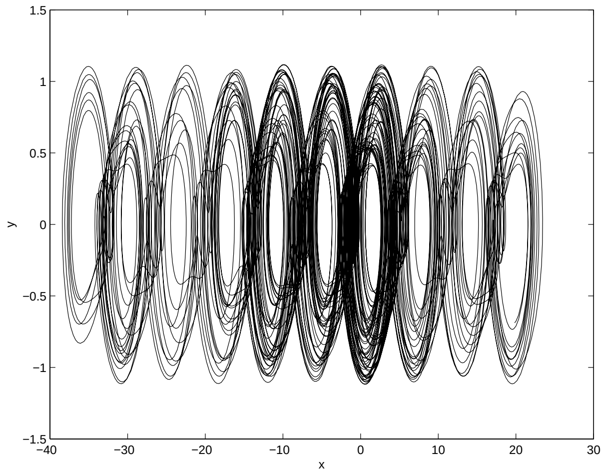

Figure 3: n-scroll attractor from the model (9) with $k = 1$, $a = 0.1$.

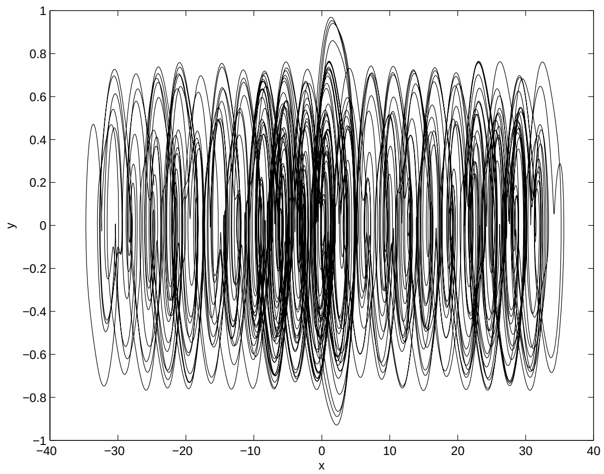

Figure 4: n-scroll attractor from the model (9) with $k = 2$, $a = 0.1$.

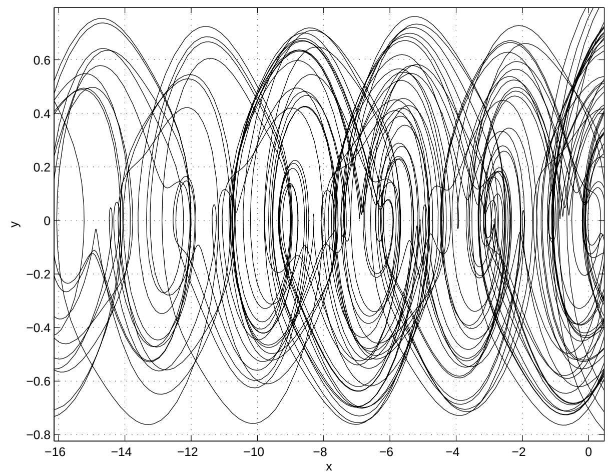

Figure 5: Zoom of Figure 4.

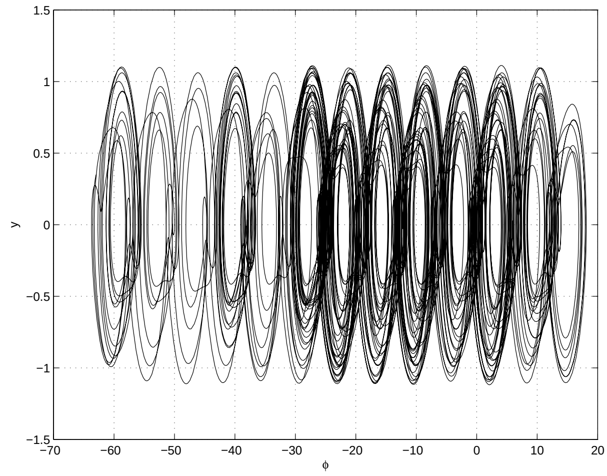

Figure 6: n-scroll attractor from the model (10) with $k = 1$, $a = 0.1$.

the n-scroll attractor (10). Figure 6 and 7 show the phase portrait of an n-scroll attractor obtained from this model with $k = 1$ and $k = 3$, respectively ($a = 0.1$ in both).

4. Conclusions

Based on a model with sine function nonlinearity, a new model for generating n-scroll attractors has been proposed that makes use of Josephson junction nonlinearity. The Josephson junction is integrated into the model itself such that the phase difference of the junctions is one of the state variables of the model. The results have been illustrated with computer simulations.

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References

[1] P. Arena, S. Baglio, L. Fortuna, and G. Manganaro. Generation of n-double scrolls via cellular neural networks. *Int. J. Circuit Theory and Applications*, 24:241–252, 1996.

[2] L. O. Chua. Chua's circuit 10 years later. *Int. J. Circuit Theory and Applications*, 22:279–305, 1994.

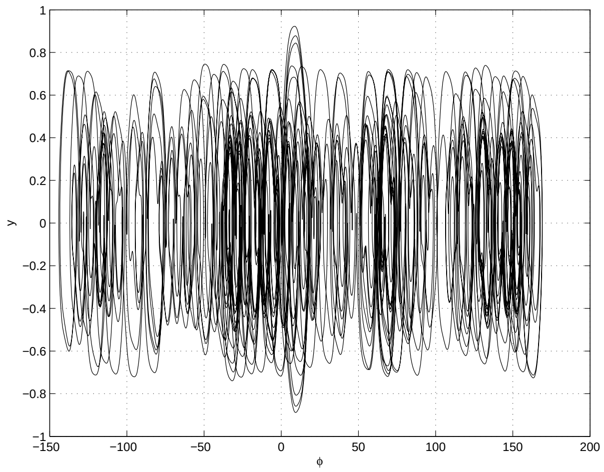

Figure 7: *n*-scroll attractor from the model (10) with $k = 3$, $a = 0.1$.

[3] S. K. Dana, D. C. Sengupta, and K. D. Edoh. Chaotic dynamics in Josephson junction. *IEEE Trans. Circuits and Systems-I*, 48(8):990–996, 2001.

[4] K. K. Likharev. *Dynamics of Josepson junction and circuit*. Gordon and Breach, New York, 1986.

[5] J. Lu and G. Chen. Multi-scroll chaos generation: Theories, methods and applications. *Int. J. Bifurcation and Chaos*, 16(4), 2006. in press.

[6] S. Ozoğuz, A. S. Elwakil, and K. N. Salama. *n*-scroll chaos generator using nonlinear transconductor. *Elecronics Letters*, 38(14):685–686, 2002.

[7] S. H. Strogatz. *Nonlinear Dynamics and Chaos*. Addison Wesley, 1996.

[8] J. A. K. Suykens, A. Huang, and L. O. Chua. A family of n-scroll attractors from a generalized Chua's circuit. *Archiv für Elektronik und Ubertragungstechnik*, 51(3):131–138, 1997.

[9] J. A. K. Suykens and J. Vandewalle. Generation of *n*-double scrolls (*n*= 1,2,3,4,...). *IEEE Trans. Circuits and Systems-I*, 40:861–867, 1993.

[10] K. S. Tang, G. Q. Zhong, G. Chen, and K. F. Man. Generation of *n*-scroll attractors via sine function. *IEEE Trans. Circuits and Systems-I*, 48(11):1369–1372, 2001.

[11] M. E. Yalçın, S. Ozoğuz, J. A. K. Suykens, and J. Vandewalle. Families of scroll grid attractors. *Int. J. Bifurcation and Chaos*, 12(1):23–41, 2002.

[12] M. E. Yalçın, J. A. K. Suykens, and J. Vandewalle. Experimental confirmation of 3- and 5-scroll attractors from a generalized Chua's circuit. *IEEE Trans. Circuits and Systems-I*, 47(3):425–429, 2000.

[13] M. E. Yalçın, J. A. K. Suykens, and J. Vandewalle. *Cellular neural networks, multi-scroll chaos and synchronization*, volume 50 of *Nonlinear Science A*. World Scientific, Singapore, 2005.