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Multi-Attractors From Nonautonomous Chaotic Oscillators

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Abstract— **A technique for designing nonautonomous chaos generators capable of producing multiple attractors is presented. The technique is based on proper multi-level-logic pulse-excitation. A forced Van der Pol equation and a forced active tank resonator are given as design examples.**

I. Introduction

The generation of multiple chaotic attarctors distant but statuary in space from a principle chaos generating engine (chaotic oscillator) has been a topic of both theoretical and practical interest [1]-[5]. Most of the effort was directed towards the generation of multi-scroll attractors due to the familiarity with the famous double-scroll attractor whether generated from the original Chua's circuit [2] or more general models and circuits [6], [7]. In all, the idea is to introduce multiple break-points in the nonlinear folding function within the chaos generator. Each break-point sets a unique equilibrium point in space around which chaotic trajectories evolve. This technique, employed in [1]-[5] and similar works, is basically a static DC approach where the number of possible equilibrium points is limited by the DC supply.

In [8] and [9], it was shown that equilibrium points with fixed positions in space can also be generated by using a binary pulse-exciting source without adding break-points to the internal nonlinearity, which in [8] and [9] was kept as a two-level-logic digital inverter. With this approach, the resulting chaos generators are classified as nonautonomous.

In this work, we show that multiple chaotic attractors can be generated by suitably introducing a composite multi-level-logic pulse-exciting source into a chaotic oscillator structure. The number of equilibrium points depends on both the number of logic levels, which in turn depend on the number of frequencies of the generating sources and their amplitudes. Two design examples are given and experimentally verified.

II. Multi-level Logic Excitation

First, we demonstrate the generation of multi-scrolls from a forced Van der Pol equation by using a multi-logic-level exciting source. The classical Van der Pol equation is given by

$$\ddot{x} + f(x)\dot{x} + x = f(t) \tag{1}$$

where $f(t)$ is usually a sinusoidal periodic force of the form $f(t) = A\sin\phi t$ and $f(x)$ is a quadratic nonlinearity of the form $k\cdot(x^2-1)$. Evidently, the location of the equilibrium points of this system are time varying in space.

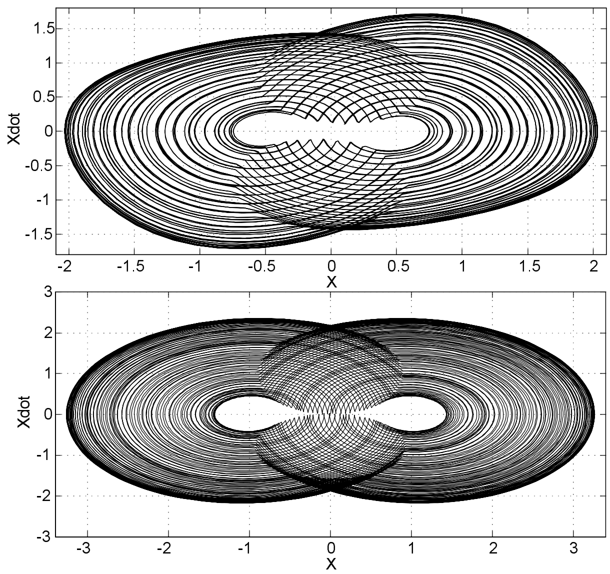


Figure 1: Double-scroll from a binary pulse-excited Van der Pol equation.

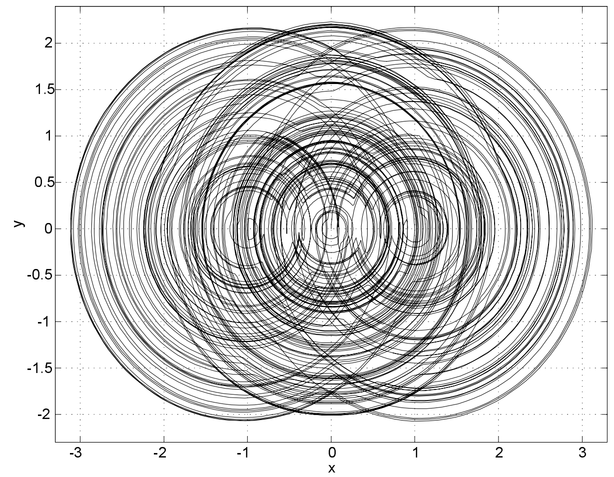
Now consider using a periodic binary pulse exciting force of the form $f(t) = A\mathrm{sgn}(\sin\phi t)$. In this case, the system has two fixed time-invariant equilibrium points located at $(x_0, y_0) = (\pm A, 0)$. These two points are clearly visible in the upper subplot of Fig. 1 which is obtained from numerical simulations of (1) with $A = 0.5, k = 0.25$ and $\phi = 0.7$. It is also possible to maintain similar behavior by using a binary switching-type nonlinearity of the form $f(x) = k\cdot\mathrm{sgn}(x)$, instead of the quadratic one, as shown in the lower subplot of Fig. 1 with $A = 1, k = 0.05$ and $\phi = 0.65$. Equation (1) in this case may be re-written in the canonical form

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & -a \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 0 \\ b \end{pmatrix} \tag{2}$$

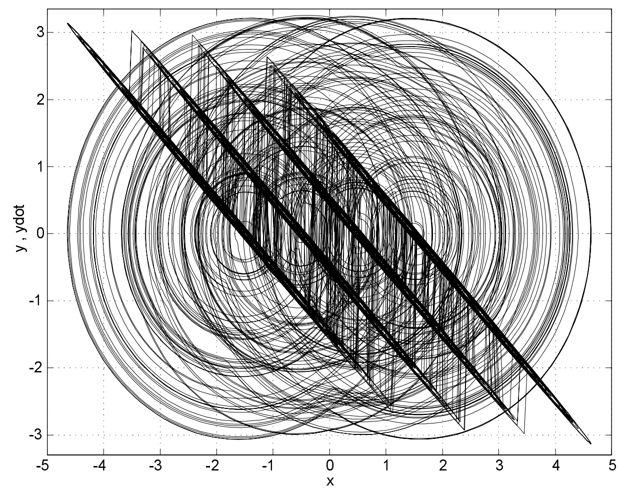
$$\text{where } a = \begin{cases} k & x \geqslant 0 \\ -k & x < 0 \end{cases} \quad \& \quad b = \begin{cases} A & \sin\phi t \geqslant 0 \\ -A & \sin\phi t < 0 \end{cases} \tag{3}$$

The two equilibrium points are still located at $(x_0, y_0) = (b, 0) = (\pm A, 0)$. Note that the core engine driving oscillations in this system is a quadrature sinusoidal oscillator with oscillation condition $a = 0$ and oscillation frequency $\omega_0 = 1$. The role of the external excitation is only to force the system to alternate between the two equilibrium points with alternation frequency ϕ without affecting the state transition matrix.

Instead of using a binary pulse, which is a two-level-logic source, we may use a three-level-logic source obtained by combining the outputs of two binary sources with different frequencies. In particular, the signal $f(t) = \frac{1}{2}[\mathrm{sgn}(\sin\phi_1 t) + \mathrm{sgn}(\sin\phi_2 t)]$ has three possible values namely $0, +1$ and -1. Using this $f(t)$ in the above system with $\phi_1 = 0.65, \phi_2 = 0.63$ and $k = 0.03$, a three-scroll chaotic attractor is observed, as shown in Fig. 2(a). A four-scroll can be generated using $f(t) = \frac{1}{2}[\mathrm{sgn}(\sin\phi_1 t) + 2{\cdot}\mathrm{sgn}(\sin\phi_2 t)]$, as shown in Fig. 2(b) for the same values of ϕ_1 and ϕ_2. This signal has four possible levels; ± 0.5 and ± 1.5. Note that in Fig. 2(b) we have plotted x versus y and $\dot{y}$ for clarity.



(a)



(b)

Figure 2: 3-scroll and 4-scroll attractors from the forced Van der Pol equation using a dual-composite excitation with frequency separation $\Delta\phi = 0.02$.

In Fig. 3: 5,6,7 and 8-scroll attractors are generated respectively using the excitation $f(t) = \frac{1}{2}[A_1\mathrm{sgn}(\sin\phi_1 t) + A_2\mathrm{sgn}(\sin\phi_2 t) + A_3\mathrm{sgn}(\sin\phi_3 t)]$ with the same ϕ_1 and ϕ_2 and with $\phi_3 = 0.61$. The signal amplitudes (A_1, A_2, A_3) are respectively $(1, 1, 2)$, $(1, 2, 2)$, $(1, 2, 3)$ and $(1, 2, 4)$ and the corresponding logic levels are $(0, \pm 1, \pm 2)$, $(\pm 2.5, \pm 1.5, \pm 0.5)$, $(0, \pm 3, \pm 2, \pm 1)$ and $(\pm 3.5, \pm 2.5, \pm 1.5, \pm 0.5)$.

It is clear that the proposed technique relies on a composite exciting source formed of two or more periodic pulse signals essentially with different oscillation frequencies. Hence, the generation of n-scrolls may be transformed from a static DC problem into an amplitude and frequency-shift problem. Note in the above example that we have an inter-source frequency separation $\Delta\phi = 0.2$. If the composite exciting source $f(t)$ is formed of m periodic pulse-trains then the maximum number of scrolls which can be generated is 2^m.

Recalling (2), it is clear that we may also generate n-scroll-grid attractors by modifying the displacement vector to be $\begin{pmatrix} b_x \\ b_y \end{pmatrix}$ where b_x and b_y are defined as in (3). Figure 4 shows a 2x2 scroll-grid obtained with $b_x = A_x\mathrm{sgn}(\sin\phi_x t)$ and $b_y = A_y\mathrm{sgn}(\sin\phi_y t)$; $(A_x, A_y, \phi_x, \phi_y) = (1, 1, 0.63, 0.65)$. Three special cases may occur: (i) $\phi_x = 0$, $\phi_y \neq 0$ (ii) $\phi_y = 0$, $\phi_x \neq 0$ (iii) $\phi_x = \phi_y \neq 0$. The first case will give rise to the horizontally-inclined double-scroll of Fig. 1 while the second case will give rise to the vertically-inclined double-scroll shown in Fig. 5. The third case gives rise to a double-scroll inclined by 45^0, as shown also in Fig. 5. The general case where $\phi_x \neq \phi_y$ results in the scroll-grid of Fig. 4. In general, for $\phi_y > \phi_x$, the grid tends to rotate as a whole in the clockwise direction; the opposite occurs for $\phi_x > \phi_y$.

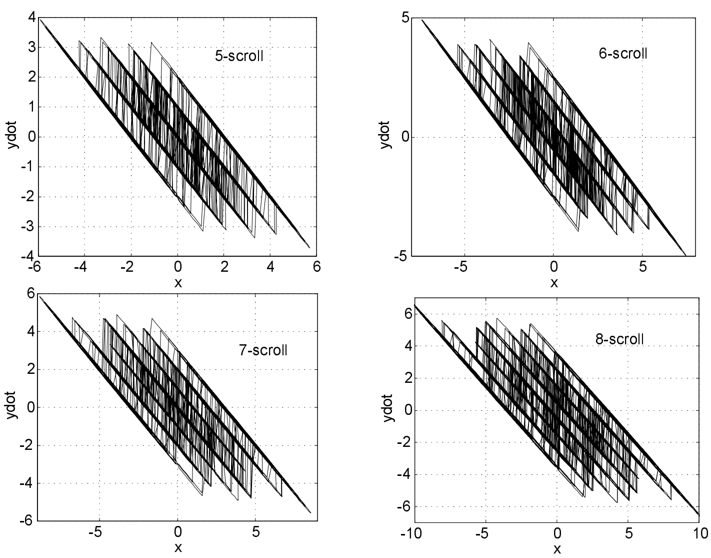


Figure 3: Generation of 5,6,7 and 8-scroll attractors using a triple composite exciting signal wih frequency separation $\Delta\phi = 0.02$.

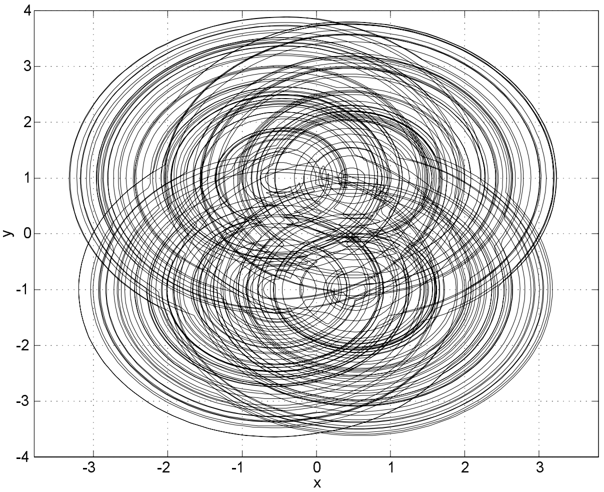


Figure 4: 2x2-scroll-grid attractor.

III. The Nonautonomous-Autonomous Transformation

It is well known that to generate sinusoidal oscillations, at least a second-order system is needed. The simplest such system is given by

$$\ddot{z} + \phi^2 z = 0 \tag{4}$$

which yields $z(t) = \sin(\phi t)$. Using this sinusoidal waveform as an input to a two-level-logic comparator, a periodic source of the form $f(t) = A\text{sgn}(\sin \phi t) = A\text{sgn}(z)$ can be obtained. Hence, it is clear that if we consider the exciting source of an order n nonautonomous system as part of the system, it can be transformed into an autonomous system of order $n+2$ using (4). The above forced Van der Pol system is thus essentially of order four where in (3) $b = A\text{sgn}(z)$.

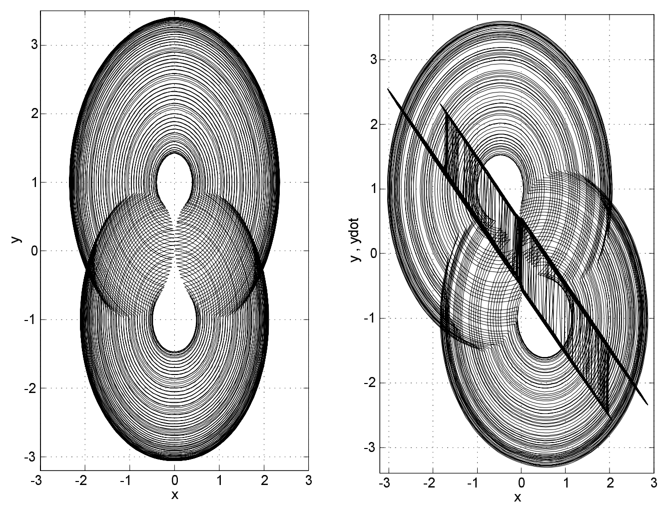


Figure 5:. Vertically-inclined and 45^0-inclined double-scrolls in the two special cases $(\phi_x, \phi_y) = (0.65, 0)$ and $(0.65, 0.65)$ respectively.

IV. Multi-level Excitation of an Active Tank Resonator

A second example is given in this section based on the nonautonomous pulse-excited active tank resonator introduced in [10] and modelled by

$$\begin{aligned} \dot{x} &= \alpha[-y - (\beta+\gamma)x + \gamma\text{sgn}(x) + \beta f(t)] \quad (5a) \\ \alpha\dot{y} &= x + y \quad (5b) \end{aligned}$$

α, β and γ are circuit-related parameters [10]. Here, we consider $f(t)$ as a composite excitation given by

$$f(t) = \frac{1}{m}\sum_{1}^{i} A_i \text{sgn}(\sin(\phi_i t)) \tag{6}$$

In [10], the special case $i = 1$ was studied and the four equilibrium points in this case were found to be $(x_0, y_0) = \frac{-\gamma\pm\beta}{\gamma+\beta-1}(1, -1)$. The special case $\beta = \gamma$ implies that two of these equilibria coincide with the origin $(0, 0)$. The eigenvalues at all points are identical since $f(t)$ only changes the position of the equilibrium point but does not affect the state transition matrix. Here, we demonstrate the two cases $i = 2$ and $i = 3$ fixing $\alpha = 50, m = 3$ and $\beta = \gamma = 1$. Figures 6(a)-6(d) show the generated 2,3,4 and 5 multi-chaotic attractors. The corresponding $(\phi_1^2, \phi_2^2, \phi_3^2)$ are respectively: (0.1,0.2,0) for Figs. 6(a) and 6(b) and (0.1,0.15,0.2) for Figs. 6(c) and 6(d). Note that we give the values of ϕ_i^2 since (4) was used to implement each element of $f(t)$. The amplitudes (A_1, A_2, A_3) were taken as (1,1,0), (1,2,0), (1,1,2) and (1,2,3) respectively for the four figures. Of course, if $\beta \neq \gamma$ the number of attractors can be increased further, as shown in Fig. 6(e) where 6 attractors are observed. Here, $(A_1, A_2, A_3) = (1, 2, 3)$, $\beta = 1$ and $\gamma = 1.5$. Seven attractors can also be observed with $A_3 = 4$ instead of $A_3 = 3$.

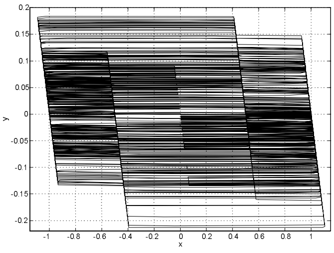
V. Experimental Verification

Figure 7(a) shows an experimental setup to verify the multi-attractors in a pulse-excited tank resonator. Two pulse exciting sources with amplitudes (V_{P1},V_{P2}) and frequencies (f_{P1}, f_{P2}) are shown. This corresponds to $i = 2$ in (6). Extra exciting sources can be added in parallel for $i > 2$. The passive components were fixed as $L = 40mH$, $C = 4.7pF$ and $r = R_F = R_{P1} = R_{P2} = 2.2k\Omega$.

Figure 7(b) shows a 2-attractor observed when $V_{P1} = V_{P2} = 1.5V$ while the source frequencies are $f_{P1} = 120kHz$ and $f_{P2} = 166kHz$. These values correspond to $A_1 = A_2 = 0.5$, $\phi_1^2 = 0.1$ and $\phi_2^2 = 0.2$. A 3-attractor is shown in Fig. 7(c) when $V_{P1} = 1V$, $V_{P2} = 1.9V$, $f_{P1} = 90kHz$ and $f_{P2} = 140kHz$. By adding one more source with frequency $f_{P3} = 166kHz$, five and six attractors can be obtained, as shown in Figs. 7(d) and 7(e) respectively.

VI. Conclusion

We have described a nonautonomous technique to generate multi-chaotic attractors. Of course, it is possible to combine this technique with the traditional static multiple break-point technique to give a larger number of equilibria.



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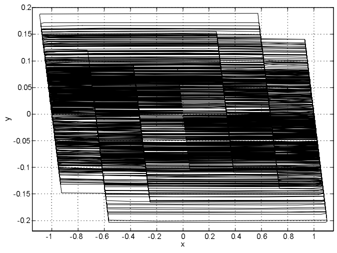

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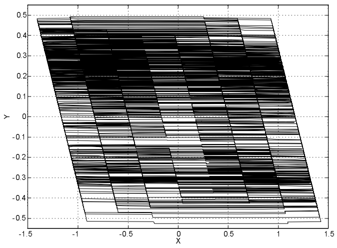

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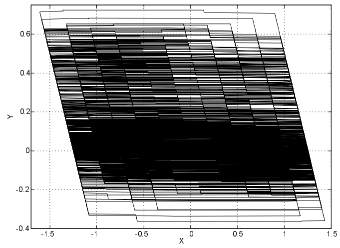

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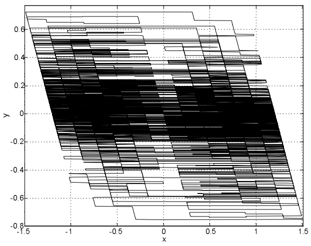
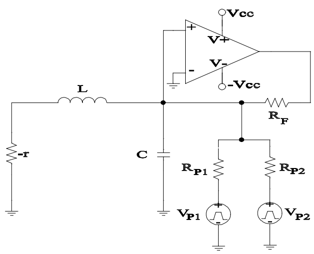


(e)

Figure 6: 2,3,4,5 and 6 attractors from a pulse excited active resonator.



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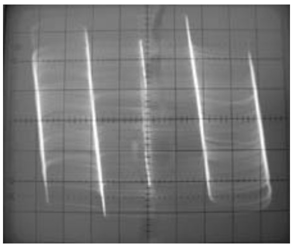

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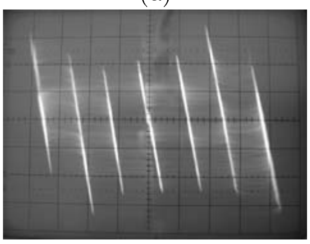

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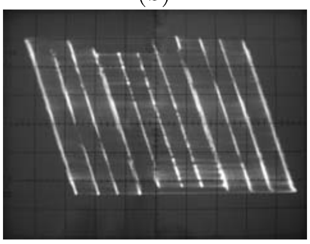

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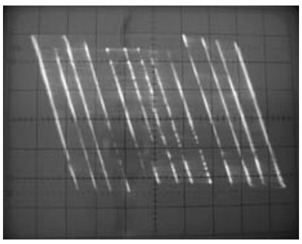

(e)

Figure 7: (a) Compositely excited resonator; (b)-(e) 2,3,5 and 6 attractors (Xaxis: 0.5V/div, Yaxis: 0.75V/div).

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