2005 International Symposium on Nonlinear
Theory and its Applications (NOLTA2005)
Bruges, Belgium, October 18-21, 2005

Initial estimates for block structured nonlinear systems with feedback

J. Schoukens*, J. Swevers, J. Paduart*, D. Vaes**, K. Smolders**, R. Pintelon**

(*)Vrije Universiteit Brussel, Department ELEC, Pleinlaan 2, B1050 Brussels, Belgium, email: Johan.schoukens@vub.ac.be
(**) Katholieke Universiteit Leuven, PMA, Celestijnenlaan 300B, B3001 Heverlee, Belgium

Abstract - This paper studies the identification of a block structured nonlinear Wiener-Hammerstein system that is captured in the feedforward or the feedback path of a feedback loop. Nonparametric initial estimates are generated for the three dynamic blocks, modelled by their frequency response function, and the static nonlinear system. The method can be applied to input/output data resulting from random or periodic excitations.

1. Introduction

Identification of nonlinear dynamic systems (NL) is a very difficult problem because no universal valid model structure is available as it is for linear systems. For that reason block structured models that describe the NL system as a connection of linear dynamic and static nonlinear blocks [1], [6], [7] are very popular. Initially the attention was focused on Wiener and Hammerstein systems, followed more recently by Wiener-Hammerstein and Hammerstein-Wiener systems. All these systems are open loop systems, there is no nonlinear feedback present. Such systems can not describe many phenomena that are observed in practice: a shifting resonance frequency or a changing damping as a function of the input amplitude of the excitation. To include also these phenomena, a feedback around the nonlinearity should be added. This leads to the system as shown in Fig.1. In this figure G_0, R_0, S_0 are linear dynamic systems and f_0 is a static nonlinear system. The aim of this paper is to provide nonparametric initial estimates G_0, R_0, S_0 (the frequency response functions) and for f_0 (static nonlinearity) starting from a set of measured input and output data $r(k), y(k)$, $k = 1, \ldots, N$.

2. Structure selection: an indistinguishability problem

A detailed study reveals that it is impossible to distinguish [5] from input/output data only between the two structures in Fig.1. It is even impossible to identify G_0 : an arbitrary part of the dynamics of the feedback branch can be shifted to the feedforward branch by changing at the same time the static nonlinear characteristic f_0. Table 1 gives the relations between the different structures and their linear dynamic and static nonlinear blocks. From identification point of view, it is impossible to make a

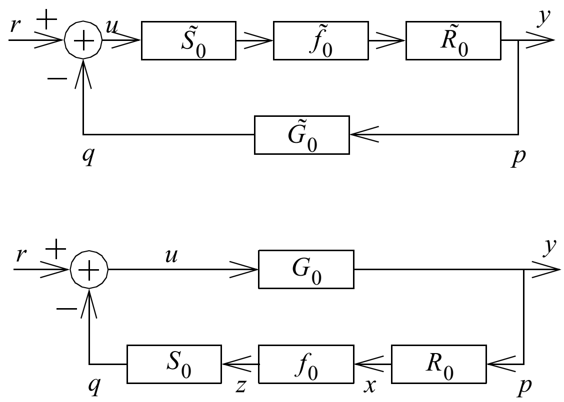

Fig.1: Examples of a block structured feedback model. Top: nonlinearity in the feedforward path; bottom: nonlinearity in the feedback path.

Table 1:Equivalencies between the structures in Fig.1

System(s)	Relations
Shift the NL from the feedback to the feedforward path (top&bottom fig.)	$\tilde{f}_0 = f_0^{-1}$, $\tilde{G}_0 = G_0^{-1}$, $\tilde{S}_0 = S_0^{-1}$, $\tilde{R}_0 = R_0^{-1}$
FF structure (top fig.): shift the dynamics from the feedforward to the feedback path	$\tilde{G}_0 \rightarrow \tilde{G}_0 + \alpha \tilde{S}_0^{-1} \tilde{R}_0^{-1}$ $\tilde{f}_0 \rightarrow \tilde{f}_0 - \alpha$, with α an arbitrary parameter
FB structure (bottom fig.): shift the dynamics from the feedback to the feedforward path	$G_0 \rightarrow G_0 + \alpha S_0^{-1} R_0^{-1}$ $f_0 \rightarrow (f_0^{-1} - \alpha)^{-1}$, with α an arbitrary parameter

physical interpretation from input/output measurements only, additional assumptions or prior information are needed.

In this paper we select without any loss of generality the bottom structure with the static nonlinearity in the feedback as default block structure.

3. Nonparametric identification of the dynamics of the feedforward and the feedback path

The first step of the initialization procedure is to generate initial estimates for the dynamics G_0 (FF) and $G_2 = R_0 S_0$ (FB). In a second step R_0, S_0 will be identified together with an initial estimate for f_0.

It is shown that a wide class of nonlinear systems can be approximated by its best linear approximation plus a nonlinear noise source [2], [9], [10] if the system is driven by Gaussian noise. For a Wiener-Hammerstein system, symbolically denoted as

$$q = S_0 f_0(R_0 p), \tag{1}$$

the best linear approximation is the product

$$\alpha S_0 R_0 = \alpha G_2, \text{ with } \alpha \text{ a constant.} \tag{2}$$

The constant α depends on the power spectrum of the input. This result will be used here as an approximation to the FB-branch of the closed loop system. The reader should notice that even if the reference signal $r(t)$ is Gaussian distributed, it is not guaranteed to be the case for the driving signal $p(t)$ of the FB. However, to generate initial estimates, all approximations are allowed as long as it results in reasonable estimates. Applying (2) to the feedback branch leads in this case to:

$$G_{CL} \approx \frac{G_0}{1 + \alpha G_0 G_2}. \tag{3}$$

Measure the best linear approximation $G_{CL}(i, j)$ for different amplitudes of the input, where i is the frequency index ω_i, and j is the amplitude index. The inverted measurements are used as entries for the matrix g_{CL}^{-1}:

$$g_{CL}^{-1} = \begin{bmatrix} . & . & . \\ . & \frac{1}{G_{CL}(i, j)} & . \\ . & . & . \end{bmatrix}, \tag{4}$$

where it should be noted that

$$\frac{1}{G_{CL}(i, j)} = \frac{1}{G_0(i)} + \alpha_j G_2(i). \tag{5}$$

The kernel idea is to note that g_{CL}^{-1} is a rank 2 matrix, and an SVD can be used to retrieve the basis vectors. After proper scaling of the right singular vectors using a linear regression, the following estimates are found for the dynamics $\hat{G}_{FF}$ of the feedforward path and $\hat{G}_2$ of the feedback path:

$$\begin{aligned} \hat{G}_{FF}^{-1} &= \frac{1}{G_0} + \beta_1 G_2 \\ \hat{G}_2 &= \beta_2 G_2 \end{aligned}. \tag{6}$$

with β_1 and β_2 arbitrary parameters. This was shown for β_1 in the indistinguishibiltiy study (Section 2., Table 1). A change of β_2 leads to an appropriate variation of the scaling of $\hat{f}$ such that the total gain of the feedback path remains constant.

4. Separating the dynamics and the static nonlinearity in the nonlinear branch

Once initial estimates are available for the FF and FB dynamics, it is possible to calculate the initial estimates $\hat{p}_{\text{lin}}$, $\hat{q}_{\text{lin}}$ from r, y:

$$\hat{p}_{\text{lin}} = y, \text{ and } \hat{q}_{\text{lin}} = r - \hat{G}_{FF}^{-1}(y). \tag{7}$$

Next a nonparametric initial estimate for the Wiener-Hammerstein FB-branch can be generated [11]. A detailed discussion of this method is out of the scope of this paper.

5. Experimental verification

In this paper the procedure that is explained in Section 3 will be first illustrated on a known electrical circuit. Next a mechanical structure with a strong nonlinear behaviour will be analysed.

5.1 The silverbox example

The Brussels silverbox is an electrical circuit that basically consists of linear second order system in the FF, and a static nonlinear feedback:

$$\frac{d^2 y}{dt^2} + a_1 \frac{dy}{dt} + a_0 y = u, \ u = r - q, \ q = p^3. \tag{8}$$

Of course the actual realized circuit is not in perfect agreement with (8), for example, we noticed in the measurements also the presence of a small quadratic term $y(t)^2$. A detailed study of this nonlinear system can be found in [8]. The system is excited with a Gaussian noise sequence with a slowly increasing amplitude and measured in 40 000 points. This record is split in 4 successive subrecords and each of these is used to identify the best linear approximation $G_{\text{CL}}(., j)$, $j = 1, \ldots, 4$ (Fig.2). In order to smooth the resulting FRF, a parametric 2nd order model is identified $G_{\text{CL}}(\omega, \theta_j)$ for each of the subrecords and this model is used to setup the matrix g_{CL}^{-1} in (4). The FRF of

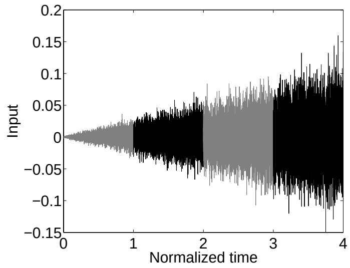

Fig.2: Input signal split in four successive sub-records with growing amplitude.

$G_{CL}(\omega, \theta_i)$ is shown in Fig.3. For increasing ampli-

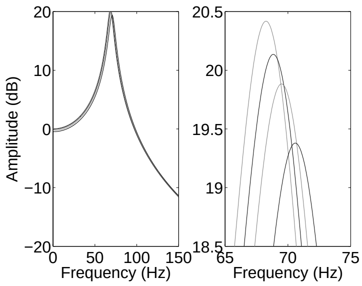

Fig.3: G_{CL} for the four experiments

tudes of the excitation, the resonance frequency shifts to the right, while at the same time the amplitude drops.

Next the estimates $\hat{G}_{FF}$, $\hat{G}_2$ are estimated (putting the nonlinearity in the feedback) and shown in Fig.4. As

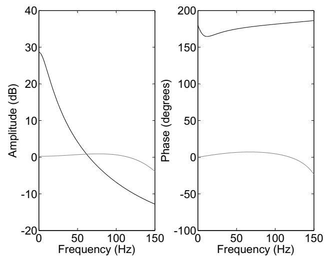

Fig.4: FRF of $\hat{G}_{FF}$ (black) and $\hat{G}_2$ (gray) branch.

could be expected from the physical insight, $\hat{G}_2$ is quite flat. On the other hand it is impossible to recognize a 2nd order system in $\hat{G}_{FF}$. This is due to the indistinguishability problems that are explained before. Any physical interpretation of the FF dynamics is lost, even if we would know that the nonlinearity is in the FB.

Next the optimal scaling factor β_j was estimated in least squares sense from $G_{CL}(i, \theta_j)$ and the original models $G_{CL}(\omega, \theta_j)$ were reconstructed using (6). A very good match was found with a relative error below - 50 dB. Notice that the complex FRF in Fig.3 changes with about 20% (relative variation -15 dB). This proofs that the model (6) can be used as a valid approximation to describe the amplitude dependent linear approximation in this case.

Eventually, an intial estimate of the static nonlinearity $z = f_0(x)$ is estimated using the nonparametric Wiener-Hammerstein initialization method [11]. In Fig.5 the nonparametric scatter plot x, z is shown together

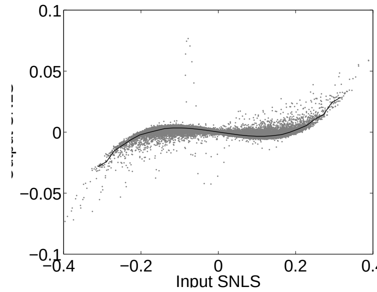

Fig.5: Nonparametric estimate of the static nonlinearity. Black dots: median.

with the median values (determined in 30 intervals). The linear dependency was taken out in order to show clearly the nonlinear behaviour. Although the cloud looks quite scattered, a more detailed analysis shows that most points are close to the median values which proves that a good initial model is obtained.

5.2 The halfcar test setup

The second experiment is made on the Leuven halfcar test setup (see http://www.mech.kuleuven.be/lnvr/halfcar/). This is a mechanical setup that is a scale model of the rear axis of a car suspension. The system is driven by two shakers (one for each wheel). In this test, one of the two shakers is kept at a fixed position, the other one is used to excite the system at 4 different amplitudes ($r_{RMS} = 2.3, 3.1, 3.8, 4.5$) with a multisine. This is a periodic signal with a user imposed spectrum. A similar analysis as in the previous section is made. In this case the resonance frequency is not shifting but the damping decreases significantly as can be seen in Fig.6. Again G_{CL} is decomposed in a FF and FB rep-

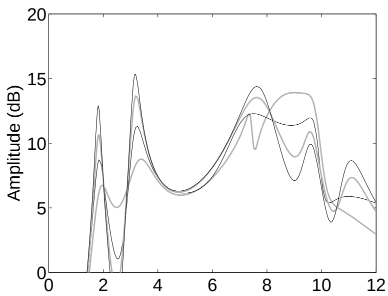

Fig.6: G_{CL} for the four experiments.

resentation and next the optimal scaling factors β_j are determined. The resulting models $G_{CL}(\omega, \beta_j)$ are compared to the original models $G_{CL}(\omega, \theta_j)$. As can be seen, there is again a (very) good match. Only for the smallest amplitude the error is larger. This is due to the fact that the damper in the system has a strong nonlinear behaviour around the origin. It becomes

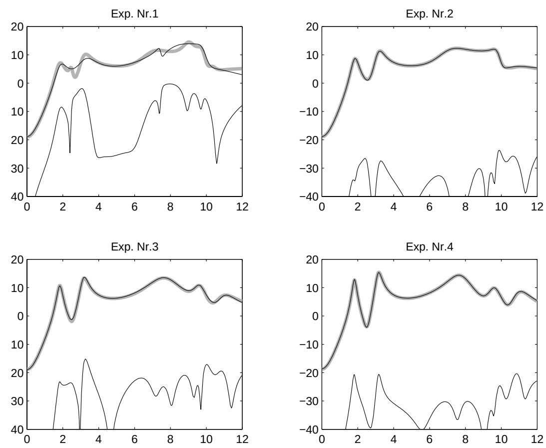

Fig.7: Error $\left|G_{\mathrm{CL}}(\omega, \theta_j) - G_{\mathrm{CL}}(\omega, \beta_j)\right|$ for the four experiments (amplitude in db versus freq. in Hz). Top black and gray line: measurement and model. Bottom line: the error.

more linear with a lower equivalent damping for larger amplitudes.

Next we tried to model the FB-branch as a Wiener-Hammerstein system, but this failed. Seemingly the nonlinear feedback branch has a much more complex structure due to the presence of the damper that can not be captured by a Wiener-Hammerstein model. This suggest that the proposed procedure is also valid for a more general class of nonlinearities than the Wiener-Hammerstein systems. Possible generalizations would be the NFIR models as described by Enqvist and Ljung [4], or systems where the siso static nonlinearity is replaced by a static nonlinear relation that depends on more than one input, for example position and acceleration as was suggested in [3].

6. Conclusion

In this paper the identification of block structured nonlinear feedback systems is studied. First it is shown that it is impossible to retrieve from input-output data only the internal structure. An infinite number of equivalent systems can be proposed that all explain equally well the input-output relations. Next a method is proposed to identify nonparametric models for the linear dynamic and static nonlinear blocks of the structure. These can be used in a second step to initialise a nonparametric optimization procedure. Finally the method is illustrated on two experiments. It is shown that the method allows to describe the variations of the best linear approximation using a single parameter that is set by the input amplitude. It also turned out that even in the case that a Wiener-Hammerstein nonlinear branch is not rich enough to give a full description of the system, it is still possible to use the method to estimate the linear dynamics of the FF and FB branch.

7. Acknowledgement

This work was supported by the Flemish government (GOA-ILiNos), the FWO and the Belgian government as a part of the Belgian program on Interuniversity Poles of Attraction (IUAP V/22).

REFERENCES

[1] Billings S.A. and Fakhouri, S.Y. (1982). Identification of systems containing linear dynamic and static nonlinear elements, *Automatica*, 18, 15-26, 1982.

[2] Bussgang, J.J. (1952). *Crosscorrelation functions of amplitude-distorted Gaussian signals*. MIT Technical report. P.No 216.

[3] Duym, S. (1997). An alternative force state map for shock absorbers. Proc. Inst. Mech. Eng. Part D - Journal of Automobile engineering, 211, 175-179.

[4] Enqvist, M. and Ljung, L. (2005). Linear approximations of nonlinear FIR systems for separable input processes. *Automatica*, 41, 459-473.

[5] Godfrey K.R. and Chapman M.J. (1990). Identifiability and indistinguishability of linear compartmental models. Mathematics and Computers in Simulation 32, 273-295.

[6] Korenberg, M.J. and Hunter, I.W. (1986). The identification of nonlinear biological systems: LNL cascade models. *Biol. Cybern.*, 55, 125-134.

[7] Korenberg, M.J. (1991). Parallel cascade identification and kernel estimation for nonlinear systems. *Annals of Biomedical Engineering*, 19, 429-455.

[8] Pintelon, R. and Schoukens J. (2001). *System Identification. A Frequency Domain Approach.* IEEE-press, Piscataway.

[9] Schoukens, J., Dobrowiecki, T. and Pintelon R. (1998). Identification of linear systems in the presence of nonlinear distortions. A frequency domain approach. *IEEE Trans. Autom. Contr.*, Vol. 43, No.2, 176-190.

[10] Schoukens, J., Pintelon, R., Dobrowiecki, T. and Rolain, Y. (2005). Identification of linear systems with nonlinear distortions. *Automatica*, 41, 451-504.

[11] Schoukens J., R. Pintelon, J. Paduart, and G. Vandersteen. Nonparametric initial estimates for Wiener-Hammerstein systems. Technical note VUB-2005.