2005 International Symposium on Nonlinear Theory and its Applications (NOLTA2005)
Bruges, Belgium, October 18-21, 2005

Synchronized chaotic intermittent and spiking behavior in coupled map chains.

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Abstract—
We study phase synchronization effects in a chain of non-identical chaotic oscillators with a type-I intermittent behavior. Two types of parameter distribution: linear and random, are considered. The typical occurring futures are the onset and existence of global (all-to-all) and cluster (partial) synchronization with increase of coupling. Increase of coupling strength can also lead to desynchronization phenomena, i.e. global or cluster synchronization is changed into a regime where synchronization is intermittent with incoherent states. Then the regime of fully incoherent non-synchronous state - spatio-temporal intermittency appears.

1. Introduction

The study of cooperative behavior in ensembles of chaotic oscillators is a topical problem of nonlinear dynamics. Chaotic synchronization in such spatially extended systems has been considered for populations of locally and globally coupled maps [1, 2, 3, 4, 5, 6, 7] as well as for ensembles of locally and globally coupled continuous-time chaotic oscillators [8, 9, 10, 11]. The theoretical knowledge obtained has been often applied to describe dynamical processes in various biological and physical systems.

Among basic types of synchronization (complete and generalized) *chaotic phase synchronization* (CPS) is a subject of active investigations (see [12]). CPS in ensembles of locally coupled chaotic elements was firstly studied in chains of weakly diffusively coupled chaotic Rössler oscillators [10]. Time-discrete systems were also under study. Synchronization phenomena in ensembles of locally coupled circle maps were considered in [7]. Many phenomena observed in populations of periodic oscillators were found there too, especially to mention the formation of several clusters of mutually synchronized elements and global synchronization. The study of CPS requires the existence of equations for the evolution of phase variables (as it is for coupled Rössler oscillators or circle maps) or at least the existence of appropriate definition of phases [13]. However, there are so far no unambiguous methods to obtain such equations and definitions. But in some cases specific properties of the chaotic attractors allows to define the phases of chaotic oscillations in a rather simple way. Besides oscillators, where chaos appears through a period doubling cascade, it is possible to introduce a suitable phase for typical systems with intermittent-like behavior, especially for systems with type-I intermittent chaotic oscillations, or spiking neurons [14]. In this paper we investigate the collective dynamics in chains of such maps. Our study is motivated by high importance of understanding mechanisms behind the transition from low-dimensional chaos (which may correspond to synchronized chaotic systems) to developed (spatio-temporal) turbulence that often looks like intermittent chaotic behavior. The paper is organized as follows. In Sec.II we shortly describe the behavior of the quadratic map generating chaotic type-I intermittent behavior, introduce definitions of the phase and the frequency of oscillations, and give criteria for synchronization in chains of coupled maps. Synchronization phenomena as well as synchronization-desynchronization transitions with linear and random distribution of control parameter are discussed in Secs. III and IV. The results are summarized in Sec.V.

2. Model of coupled intermittent maps. Phase and frequency. Synchronization criteria

In the focus of this study is the synchronization problem in chains of coupled non-identical maps with the intrinsic type-I intermittent chaotic behavior. In order to measure the degree of synchronized motion, we will first introduce frequency and phase of intermittent oscillations. Chaotic intermittent motion has a distinct *characteristic time scale* (CTS). For type-I intermittency a very large laminar stage (with duration τ) is followed by a very short turbulent stage (with duration T) and then the next laminar stage begins. Sometimes (for example, in the model map studied below) the turbulent stage has only one jump from a practically fixed variable value and back. This event is reminiscent of firing - a special behavior, which is typical for neuronal systems. Regarding this specific character of behavior we will distinguish between the laminar and the firing stages. The average length of the laminar stage (ALLS) for a single element is defined as [15]

$$\langle \tau_0 \rangle \propto \frac{1}{\sqrt{\varepsilon - \varepsilon^{cr}}}, \quad (1)$$

where ε is a bifurcation parameter and ε^{cr} is the critical value when chaos sets in [16]. For coupled maps studied below ALLS can be calculated numerically as:

$$< \tau >= \lim_{N\to\infty} \frac{1}{N} \sum_{k=1}^{N} (k_{l+1} - k_l), \qquad (2)$$

where k_l is the moment when the lth laminar stage sets in or in other words when the lth firing occurs. One can also introduce *a phase of the intermittent oscillations*, attributing to each interval between the starts of the laminar stage (or in other words between two firings) a 2π phase increase:

$$\varphi^k = 2\pi \frac{k - k_l}{k_{l+1} - k_l} + 2\pi l, \; k_l \leq k < k_{l+1}, \qquad (3)$$

where k is discrete time.

The presence of a CTS and a suitable phase allows to formulate the problem of chaotic phase synchronization in ensembles of coupled units with intermittent behavior. So, if CTS $< \tau_j >$ or the corresponding frequencies

$$\Omega_j = 2\pi / < \tau_j > \qquad (4)$$

of all units become equal, this manifests their global 1:1 *frequency entrainment*. If the conditions

$$|\varphi_l^k - \varphi_m^k| \; < \; Const \qquad (5)$$

for all k are fulfilled, one can speak about a 1:1 *phase locking* between the lth and the mth units.

Let us demonstrate mutual phase synchronization of chaotic intermittent oscillations for a chain of diffusively locally coupled non-identical quadratic 1-D maps:

$$\begin{aligned} x_j^{k+1} &= f_j(x_j^k) + \\ & d(x_{j-1}^k - 2x_j^k + x_{j+1}^k), \qquad (6) \\ & j = 1, ..., N, \end{aligned}$$

where, N is the number of elements in the chain, $f_j(x)$ consists of the standard quadratic part that produces a laminar motion and a somewhat arbitrary chosen return part that acts as a firing stage:

$$f_j(x) = \begin{cases} \varepsilon_j + x + x^2, & \text{if } x \leq 0.2, \\ g(x - 0.2) - \varepsilon_j - 0.24, & \text{if } x > 0.2 \end{cases} \qquad (7)$$

Here g regulates the coherence properties of the chaotic attractor. In case $g < 5$ the laminar stage duration is distributed in a rather narrow band, i.e. the chaotic behavior is highly coherent, but for $g > 5$ this distribution is rather broad. We will focus on the case of a coherent chaotic attractor and set $g = 2$. We remind that the uncoupled map ($d = 0$ in (6)) demonstrates a type-I intermittent behavior for $\varepsilon_j > 0$, i.e. $\varepsilon_j^{cr} = 0$.

The parameter ε_j defines CTS in the individual j-th oscillator. In our study we treat two cases: (i) a linear distribution of the parameter ε_j: $\varepsilon_j = \varepsilon_1 + \Delta\varepsilon(j-1)$, where $\Delta\varepsilon$ is the parameter mismatch between neighboring elements, and (ii) a random uniform distribution of natural frequencies in the range $[\varepsilon_1, \varepsilon_1 + \Delta\varepsilon(N-1)]$. We assume free-end boundary conditions:

$$x_0^k(t) = x_1^k(t) \quad ; \qquad x_{N+1}^k(t) = x_N^k(t) \qquad (8)$$

for all k.

3. Linearly distributed control parameter. Soft transition to global synchronization

First, a chain with a linear distribution of the parameters ε_j is explored. The evolution of the observed frequencies Ω_j in dependence on the coupling is presented in Fig. 1. In all diagrams with an increase of coupling from zero the tendency to a more coherent behavior is clearly seen. Then in dependence on the mismatch $\Delta\varepsilon$, global synchronization is observed (Fig. 1a) or is not (Fig. 1b,c). But in all cases the increase of coupling leads to a fully incoherent behavior. The detailed analysis of the frequency distribution Ω_j vs coupling shows that the transition to global synchronization is smooth, i.e. a gradual adjustment of frequencies is observed. The reason of such "soft" route to global synchronization is the existence of two quite different time scales: slow laminar stage and fast firing stage. It is well known (see, for instance [17]) that the appearance and interaction of many time scales (at least two) can lead in the oscillatory systems to a chaotic behavior. Another consequence of the slow-fast motion is a large value of the frequency of global synchronization. It is close to the maximal individual frequency [18]. The reason for this effect is the following. For a sufficiently large coupling the strong change (firing) of the dynamical variable in the elements close to the right end of the chain is faster than in other elements. This provokes analogous strong change of the dynamical variable in the neighboring element which also provokes his neighbor and so on. This process leads to a sequential firing in all elements in the chain.

A detailed analysis of synchronization - desynchronization transitions is presented for the case of randomly distributed parameter ε_j in the next section.

4. Randomly distributed control parameter. Transition to spatio-temporal intermittency

For randomly distributed ε_j, the evolution of the observed frequency distribution is shown in Fig. 2. Three types of transitions to global synchronization is observed here: (i) two adjacent elements (clusters) with close frequencies can be easily synchronized and a new cluster appears; (ii) nonlocal synchronization can occur, i.e. an element (a cluster of elements) becomes synchronized not to a nearest-neighbor element (cluster), but to some other element (cluster) having a close rotation number. At that the observed frequencies of the elements (clusters) in-between

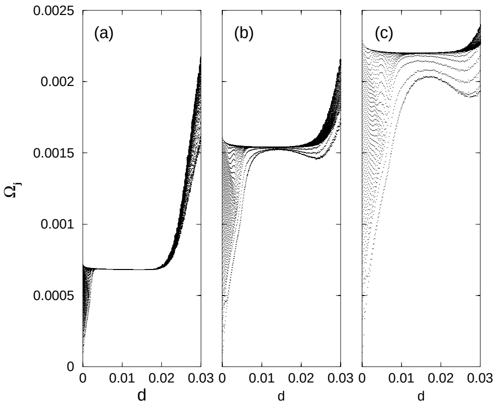

Figure 1: The evolution of Ω_j (4) in dependence on coupling for $\varepsilon = 0.000001$ and for three different values of $\Delta\varepsilon$ in the chain of 50 coupled maps. (a) $\Delta\varepsilon = 0.000001$; (b) $\Delta\varepsilon = 0.000005$; (c) $\Delta\varepsilon = 0.00001$

are considerably different; (iii) one element (group of elements) at the edge of one cluster can go to another neighboring cluster. Similar to the case of linearly distributed parameters ε_j in case of random distribution of ε_j the regime of global synchronization can disappear with the increase of coupling. At the some critical value d^* this regime becomes unstable. In the chain triangular embeddings are formed. The onset of such embeddings in some places in the chain leads to the propagation of firing processes in one or more typically in both directions. Propagating firing fronts are usually unstable and new triangular embeddings are appearing and this process repeats. Therefore the domains with a large synchronized intermittency are changed by domains of complex spatio-temporal behavior, which in the presented context we call spatially turbulent regime. This spatially turbulent regime appears suddenly and extends to the whole chain, then it suddenly disappears and in the whole chain the regime of synchronized intermittency is again realized. With an increase of coupling the duration of the spatially turbulent regime grows and correspondingly the duration of the synchronized regime becomes shorter. After some critical value d^{**}, the synchronized regime is no more observed and the regime of fully developed spatio-temporal intermittency (STI) sets on. The rich spatio-temporal dynamics in the synchronous and non-synchronous regimes is illustrated in Fig. 3. The left panel corresponds to a non-synchronous behavior (small values of coupling). There are several clusters of mutually synchronized elements. Only panel (b) corresponds to a synchronous regime. Panel (c) corresponds to the intermittency of synchronized and turbulent regimes. Panels (d) and (e) show highly developed STI. The tendency to the complication of collective oscillations with increase of coupling is clearly seen. In all plots the darker regions mark higher values of the presented variables.

It is interesting to analyze these observed processes by using our phase definition (3). Hence, we can state that in the regimes of perfect (Fig. 3(b)) and intermittent (Figs. 3(c)) chaotic phase synchronization, the phase distribution φ_j is a sequence of intervals with constant phase, separated by $\pm 2\pi$-kinks. The position of the kinks at constant time corresponds to a phase slips. In the synchronous regimes the phase slips appear with the frequency of synchronization. In the non-synchronous regimes phase slips appear suddenly and rather fast.

A fully incoherent state - STI - is one of the most fascinating phenomena appearing in a wide range of extended systems in several experimental situations, such as chemical reactions [19], Rayleigh-Benard convection [20], planar Couette flow [21], fluid flows between rotating electrical cylinders [22], Taylor-Couette flows [23] etc as well as in theoretical models, as coupled map lattices [24] or partial differential equations [25]. As in the mentioned theoretical works STI appears in the presented model due to the relatively strong interaction of many units. The specific property in our observation consists in the existence of a transient regime from fully coherent (synchronous) to fully non-coherent (turbulent) behavior.

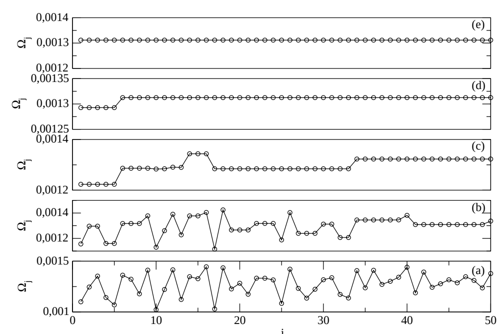

Figure 2: The evolution of observed frequencies Ω_j (4) for different couplings (a) $d = 0$, (b) $d = 0.0005$, (c) $d = 0.001$, (d) $d = 0.0015$, and (e) $d = 0.0025$. $\varepsilon = 0.000001$, $\Delta\varepsilon = 0.0000001$, and $N = 50$

5. Conclusions

In conclusion, we have found the existence of global and cluster phase synchronization effects in a chain of non-identical chaotic oscillators with a type-I intermittent behavior. A very important feature is that an increase of the coupling strength can also lead to desynchronization phenomena, i.e. global or cluster synchronization is changed by a regime where synchronization is intermittent with the incoherent state. Then a regime of fully incoherent non-synchronous state, spatio-temporal intermittency, appears. Our results elucidate complex and intriguing collective dynamics of intermittent and spiking spatially extended systems, and may be used in applied problems like developed (spatio-temporal) turbulence and complex behavior in neurobiological networks.

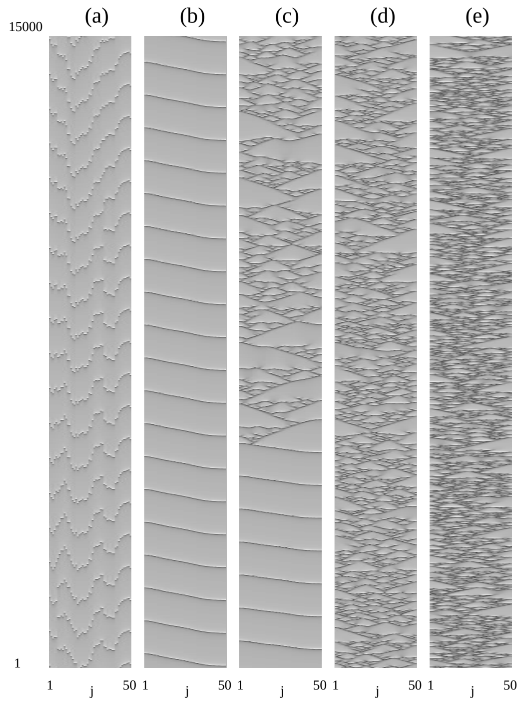

Figure 3: Space time plots of x_j for synchronous (b) and non-synchronous regimes (a,c,d,e) for ε_j randomly distributed in the interval $[0.000005; 0.000015]$. $N = 50$, $d = 0.001$ (a), $d = 0.04$ (b),$d = 0.0056$ (c),$d = 0.07$ (d),$d = 0.15$ (e).

Acknowledgments

M.I. acknowledge financial support of INTAS Ref. Nr 04-83-2816. M.I. and G.O. acknowledge financial support of RFBR (Projects No. 03-02-17543 and 05-02-90567).

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