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Synchronization and desynchronization in chaotic spiking chain ensembles

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Abstract—We study processes of synchronization and desynchronization in chains of chaotically spiking neuronal maps. We introduce phase and frequency characteristics to apply phase chaotic synchronization formalism. We find that in addition to classical transition to synchronization at moderate coupling strength, desynchronization occurs as the coupling is further increased. These results throw light upon collective phenomena in extended neuronal systems.

1. Introduction

Synchronization of chaotic oscillations is a fundamental nonlinear phenomenon observed in natural and artificial physical systems. This process implies pulling in frequencies of interacting self-sustained oscillators, exhibiting frequency mismatch in absence of interaction [1]. At present, the theory of synchronization of chaotic oscillators with a dominant frequency is basically complete. However, synchronization in systems with developed chaos, when several dominant frequencies exist, is practically unstudied. Apart from theoretical interest, there are important applied problems, like spatio-temporal disorder in extended media, coupled laser systems, coherent dynamics of neural ensembles, which strongly stimulate analysis of synchronization in multiple time scale systems.

In our talk we will present recent results on synchronization in ensembles of neural spiking oscillators with 1D chain coupling topology. We will study different synchronization-desynchronization transitions that occur as the coupling strength is varied. We will demonstrate, that apart of the desynchronization-synchronization transition, that typically occurs in case of weakly coupled self-sustained oscillators with frequency mismatch, there exists a synchronization-desynchronization transition, which takes place at relatively strong coupling and can be characterized as a spatio-temporal intermittency. We will also show that the second transition leads to the appearance of multiple time-scale oscillations in the chain [4].

2. The model

In simulations we next study a chain of locally coupled non-identical model maps (replicating neural spiking activity) proposed in [2]:

$$\begin{aligned} x_j^{k+1} &= f(x_j^k, x_j^{k-1}, y_j^k) + \tfrac{1}{2}d(x_{j+1}^k - 2x_j^k + x_{j-1}^k), \\ y_j^{k+1} &= y_j^k - \mu(x_j^k + 1) + \mu\sigma_j + \mu\tfrac{1}{2}d(x_{j+1}^k - 2x_j^k + x_{j-1}^k), \\ j &= 1, ..., N, \end{aligned} \quad (1)$$

where x_j and y_j are the fast and slow variables respectively. $\mu = 10^{-3}, \sigma_j$, and $\alpha = 3.5$ are the parameters of the individual map, d is the coupling. The function $f(\cdot,\cdot,\cdot)$ has the form:

$$f(x^k, x^{k-1}, y^k) = \begin{cases} \alpha/(1 - x^k) + y^k, \text{ if } x^k \leq 0, \\ \\ \alpha + y^k, \text{ if } 0 < x^k < \\ \alpha + y^k \text{ and } x^{k-1} \leq 0, \\ \\ -1, \text{ if } x^k \geq \alpha + y^k \\ or\ x^{k-1} > 0 \end{cases} \quad (2)$$

3. The framework for analysis

In dependence on the parameters the individual dynamics of the map (in 1 $d = 0$) is ranging from a regular spiking to a chaotic spiking or bursting behavior and can, therefore, be used for the effective modelling of neuron-like elements. Several main spatio-temporal regimes (including pulse and spiral wave propagation) for networks of identical maps (1),(2) were presented in [3]. Here, we show synchronization phenomena in a chain of locally coupled *non-identical* maps. As well as for maps with a type-I intermittent behavior the phase and frequency of oscillations can be defined by

$$\begin{aligned} \varphi_j^k &= 2\pi\frac{k - k_{j,m}}{k_{j,m+1} - k_{j,m}} + 2\pi m_j, \\ k_{j,m} &\leq k < k_{j,m+1}, \end{aligned} \quad (3)$$

where k is discrete time, $k_{j,m}$ is the moment of the m-th spike in the j-th neuron, and

$$\Omega_j = \lim_{k\to\infty} \frac{\varphi_j^k - \varphi_j^0}{k} \quad (4)$$

4. Results

In numerical simulations we observe two types of transitions between asynchronous and synchronous regimes [4]:

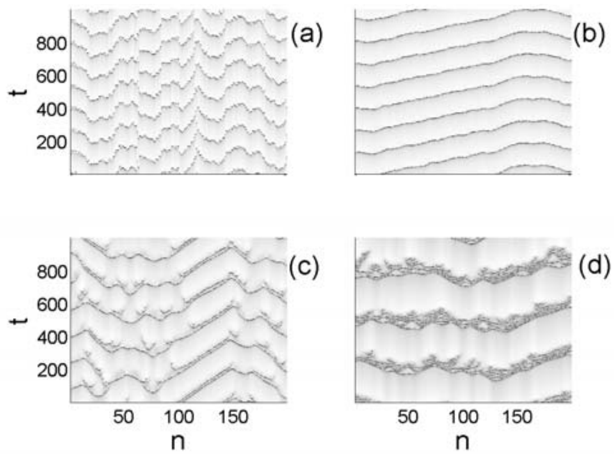

Figure 1: Space time plots of x_j for synchronous (b) and non-synchronous regimes (a,c,d) for σ_j randomly distributed in the interval [0.15; 0.16]. $N = 200$, $d = 0.005$ (a), $d = 0.05$ (b),$d = 0.09$ (c),$d = 0.2$ (d).

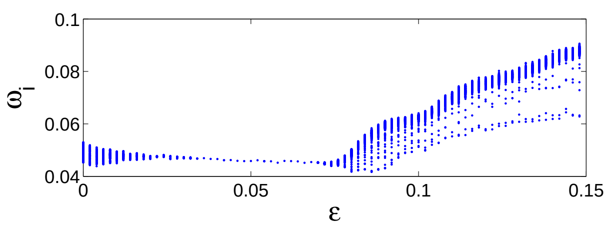

Figure 2: Dependence of the averaged frequencies Ω_i on the coupling strength ε

- At relatively low coupling ($\varepsilon \sim 0.035$) chaotic spikes get phase-synchronized.
- At twice larger coupling ($\varepsilon \sim 0.07$) they get desynchronized.
- the latter transition leads to appearance of phase-synchronized bursts (with rare imperfections). Inside bursts spikes are not synchronized, giving rise to a fractal-like structure.

The corresponding spatio-temporal patterns are shown in Fig.1. We also illustrate these regimes by plotting averaged frequencies Ω_i versus coupling strength ε (Fig.2).

Simulating dynamics of 2-dimensional networks we observed the same synchronization-desynchronizaiton transitions. In Fig.3 we show dependence of the average frequencies of spiking vs. coupling strength and in Fig.4 we present snapshots of the lattice for different coupling strengthes. Again we observe transition from non-synchronized spiking (Fig.4(a)) to synchronized spiking in the form of a spiral wave (fig.4(b)). Then we observed desynchronization (Fig.4(c)) seen as a spiral wave with non-smooths fronts, and finally the onset of chaotic bursting behavior (Fig.4(d)).

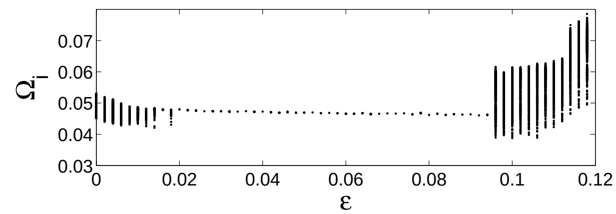

Figure 3: Dependence of the averaged frequencies Ω_i on the coupling strength ε in 2D lattice $N = 25 \times 25$.

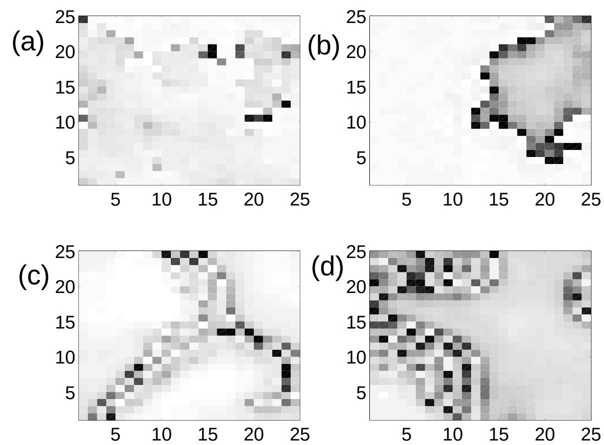

Figure 4: Snapshots of x_j for synchronous (b) and non-synchronous regimes (a,c,d) for 2D lattice. $N = 25 \times 25$, $d = 0.01$ (a), $d = 0.06$ (b),$d = 0.12$ (c),$d = 0.2$ (d).

5. Conclusion

In conclusion, we have found the existence of two types synchronization-desynchronization transitions with increase of coupling in 1D and 2D lattices of locally coupled non-identical maps demonstrating spiking activity. It is important to note that synchronous state loses its stability through an avalanche-like process, which results in appearance of the multiple time-scale oscillations. These findings elucidate complex dynamics of neural systems and improves understanding of the role of coupling in these ensembles.

Acknowledgments

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