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Modeling of Random Delay in LAN-Based Feedback Control Systems

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Abstract– If the feedback signals of control systems are transmitted over data networks, time-varying delay times occur due to the data transport mechanism. This delay affects the behavior of closed loop control systems and has to be taken into account when designing or adjusting the controller. In the paper a simulation method for control systems including random delay is proposed. The delay is modeled using a queuing mechanism with random arrival and service time generators. The complex behavior resulting from this kind of delay is analyzed and stability conditions are demonstrated. Finally consequences for the adjustment of the controllers are derived.

1. Introduction

Control systems in buildings or factory floors use a communication network or a field bus to transmit the measurement and control data [5]. Figure 1 demonstrates this with an example. The measured data from the sensor nodes S are transmitted via the bus to the controller node C. After processing these data the controller transmits the control variable to the actor node A which accesses the plant. The repeated data transfer over the field bus causes delays which may degrade the performance of the control system or even lead to instability of the system.

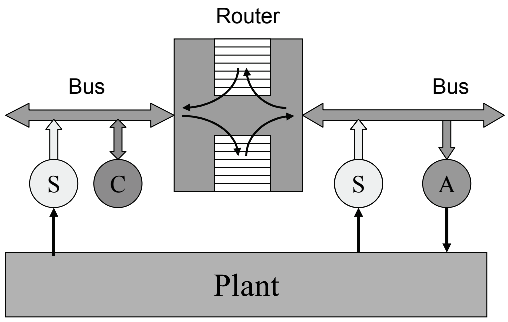


Figure 1: Field bus feedback control

As a rule the signal delay is random [3], [4]. This necessitates a statistical approach for the behavioral and performance analysis of the control system.

In this paper we propose a system model for the analysis of networked control systems. We use standard linear models for the plant and the controller and introduce a queuing mechanism to model the delay. This model at the same time preserves the time sequence of the transmitted data.

In section 2 we introduce the device models used; in section 3 typical performance measures are presented. Section 4 demonstrates a simulation system for the performance analysis of networked control systems, and section 5 shows typical analysis results.

2. System Model

2.1. Feedback Control Loop

The model of the basic structure of a single-loop feedback control system is depicted in Figure 2. Plant and controller are assumed to be continuous-time. A sampler samples the values of the process or plant output y. Sampling can be periodical or depend on the process value itself (Send On Delta). In the latter case it is assumed to be random.

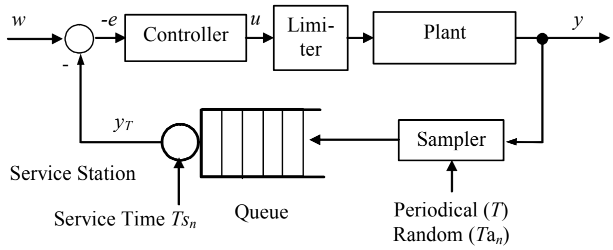


Figure 2: Model of the basic structure

The data transmission process over the network is modeled by a queuing system (queue and service station). The sampled values enter the queue (FIFO memory). After a service time interval Ts_n the each first element leaves the queue. So the delay times are generated (see paragraph 2.3). The limiter between the controller and the plant considers clipping effects of the real devices. The models of the function blocks are described in the sequel.

2.2. Models of Plant and Controller

For most applications control plants can be modeled by simple linear systems. Table 1 shows the transfer functions of plant models of I-, PT1- and PT2-type.

I	PT1	PT2
$\frac{1}{0.52T_{90}s}$	$\frac{1}{1+0.43T_{90}s}$	$\frac{1}{1+0.53T_{90}s+0.11T_{90}^2s^2}$

Table 1: Plant models

The parameter T_{90} denotes the 90% rise time of the corresponding step response. The PT2 plant model is adjusted for a slight overshoot (1%) of the step response.

For the control the standard P- and PI-types are used. The corresponding transfer function is

$$H_C(s) = \frac{Y(s)}{U(s)} = k_P + \frac{k_I}{s}, \quad (1)$$

where k_P and k_I are the proportional and the integral coefficient respectively.

2.3. Model of the Delay

Signal delays in data networks are mainly caused by the bus access control. The data transmitting units (sensors, controllers, routers) are equipped with FIFO memories where the data are stored until they are transmitted over the bus. This corresponds to a queuing model. The data queue up in the FIFO memory until they leave it at random time instants. As a rule a signal value passes several queues until it reaches its destination. Here we assume that the delay process can sufficiently be modeled by applying one single queue model. It generates the subsequent delay times and assures that the order of the transmitted data is preserved. Figure 3 depicts the delay mechanism. At the arrival time instants t_{an} the data values D_n enter the queue. At any one time the first datum in the queue is served which takes a service time T_{sn}.

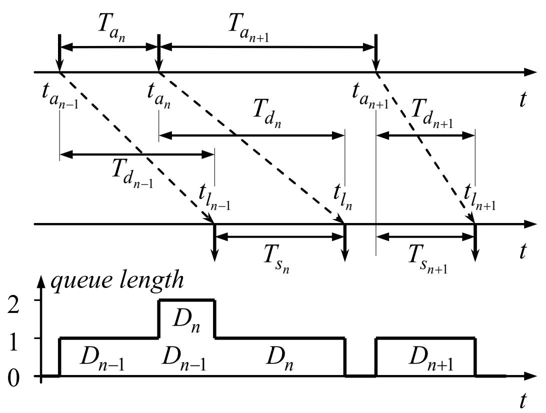


Figure 3: Delay mechanism

The arrival time instants ta_n and the arrival time intervals Ta_n are related by

$$t_{a_n} = t_{a_{n-1}} + T_{a_n} \text{ resp. } t_{a_n} = t_{a_0} + \sum_{i=1}^{n} T_{a_i}, \quad (2)$$

and, similarly for the departure time instants tl_n and the service time intervals Ts_n we have

$$t_{l_n} = t_{l_{n-1}} + T_{s_n} \text{ resp. } t_{l_n} = t_{l_0} + \sum_{i=1}^{n} T_{s_i}. \quad (3)$$

The delay time Td_n for the datum D_n is the time between the arrival time and the departure time. From Figure 3 we get for the departure time the recursive relation

$$T_{d_n} = \begin{cases} T_{d_{n-1}} + T_{s_n} - T_{a_n} & \text{for } T_{d_{n-1}} > T_{a_n} \\ T_{s_n} & \text{else} \end{cases} \quad (4)$$

or

$$T_{d_n} = T_{s_n} + \max\left(T_{d_{n-1}} - T_{a_n}, 0\right). \quad (5)$$

This relation is used to generate the subsequent delay times from the random sequences of the arrival and the service times.

2.4. Summary of Model Equations

Here we summarize the model equations assumed for the performance analysis. The control plant is described by a linear ODE, i.e. in the simplest case (PT1)

$$\tau_p \dot{y}(t) + y(t) = u(t), \quad (6)$$

where $\tau_p = 0.43T_{90}$ (see table 1) is the time constant of the plant. The PI-controller is described by

$$\begin{aligned} \dot{x}_R(t) &= -k_I e(t) \\ u(t) &= x_R(t) - k_P e(t) \end{aligned}, \quad (7)$$

where x_R is the state of the controller. The error signal is

$$e(t) = y_T(t) - w(t), \quad (8)$$

where y_T is the sampled and delayed output of the plant, given by

$$y_T(t) = \sum_{n=0}^{\infty}\left(y\left(t_{a_n}\right) - y\left(t_{a_{n-1}}\right)\right)s\left(t - t_{l_n}\right), \quad (9)$$

where a zero-order hold is assumed for the sampled signal data.

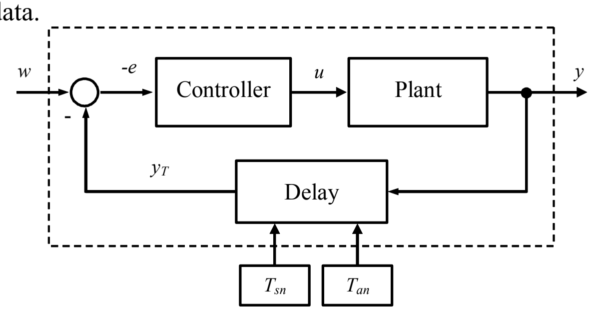


Figure 4: Block scheme of the Control System

The whole system model has a deterministic continuous-time input w and two random input sequences Ts_n and Ta_n. The latter are assumed to be independent and identically distributed random variables. Forming their pdf is used to match the statistical properties of the sampling and the delay mechanism. Due to the complex nature of the delay mechanism an analytical approach is complicated. As a rule analysis tools are used instead [1].

3. Performance Criteria

Performance criteria are used to evaluate the controller's behavior. They are preferably defined with the step response of the closed loop system, i.e. set point value w is assumed to jump at $t = 0$ and to remain constant afterwards. An ideal control system would force the process output y to track w instantaneously and perfectly suppress all disturbances.

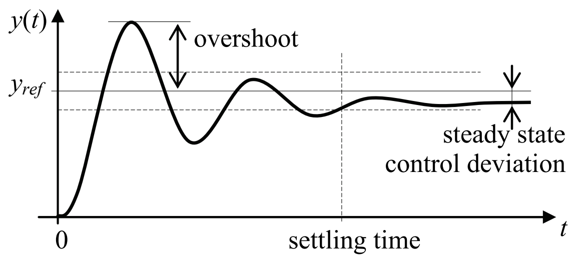


Figure 5: Step response and performance criteria

Figure 5 shows the step response of a real control system. The most important criteria describing the differences to the ideal case are the following:

Steady-State Error: This criterion describes the remaining the steady state deviation of the process output from the setpoint:

$$e_\infty = y_\infty - w. \tag{10}$$

The **Overshoot** os measures the size of the local maximum of the step response in relation to the steady state value:

$$os = \frac{y_{\max} - y_\infty}{y_\infty} \tag{11}$$

It is often given in percents. A good controller has an overshoot better than 5 %.

Settling time t_S is the time required for the step response to permanently remain within a defined tolerance range around a reference value y_{ref}:

$$t_S = \min(t) : \forall t > t_S : \left| \frac{y(t) - y_{ref}}{y_{ref}} \right| \le accuracy \tag{12}$$

The reference y_{ref} can be the setpoint value w or the steady state value y_∞ of the process output.

Integral Square Error: The deviation of the step response from a reference y_{ref} is given by $e(t) = y(t) - y_{ref}$. The integral over the squared error gives the integral square error ise

$$ise = \int_0^\infty \left(y(t) - y_{ref}\right)^2 dt, \tag{13}$$

which for a stable system and for $y_{ref} = y_\infty$ converges.

All of the criteria mentioned above deliver scalar values which depend on the controller parameters. The criterion used for a controller optimization has to be chosen according to the requirements of the application. So in the case of a room heating system a weak overshoot remains unnoticed but in the case of a light controller it may disturb.

If as in our case some parameters in the control system are random variables then the performance criteria are random variables too, and the performance has to be evaluated by statistical characteristics (expectation value, variance, probability density function) of the used performance criterion.

4. Simulation System

Based on the equations in paragraph 2.4 a simulation tool for control systems with random delay time has been developed. The tool simulates the system's step response and calculates performance criteria from chapter 3 and their statistic. This allows the study of the behavior of delayed feedback controllers and gives assistance when designing the controller and optimizing its parameters.

Figure 6 shows the user interface of the simulation tool. The control loop is depicted at the top. Here the type and the parameters of the controller and the plant, the limiter value can be adjusted. Different probability density functions can be chosen for the service time intervals. The arrival time intervals are assumed to be constant which means periodic sampling at the process output. The sampling time interval can also be adjusted.

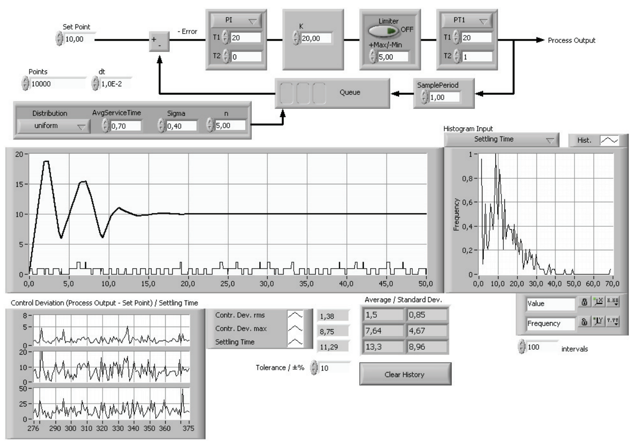


Figure 6: Simulation tool - User interface

The large graph below the block scheme depicts the system's step response and the queue length in dependence of time. The graph and the table at the bottom show the performance criteria root of integral square error, overshoot and settling time and their statistics for the number of repeatedly calculated step responses. The histogram window allows displaying the pdf of the service time intervals, of the corresponding delay times or of the performance criteria.

5. Analysis Results

In this section we show some examples of typical application cases. All times are normalized to the sampling, i.e. arrival time intervals. For the settling time a tolerance range of ± 10% around the steady state value is assumed.

Example 1:

Plant: PT1 with $\tau_p = 20$

Controller: PI with $k_P = 20$ and $k_I = 0.67$

Service time intervals: uniformly distributed in (0, 1.4), i.e. mean value 0.7, standard deviation 0.4.

Figure 7 shows a typical realisation of the step response and the queue length of the system.

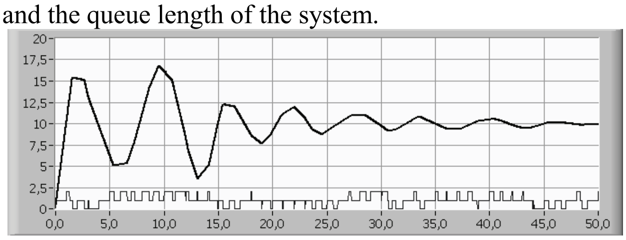
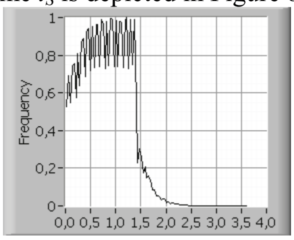
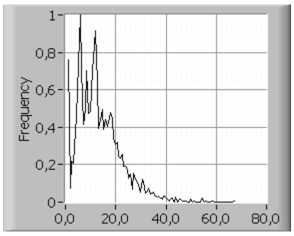


Figure 7: Step response and queue length for example 1

The histogram of the delay times T_{dn} and the settling time t_S is depicted in Figure 8.



a) Delay time



b) Settling time

Figure 8: Histograms for example 1

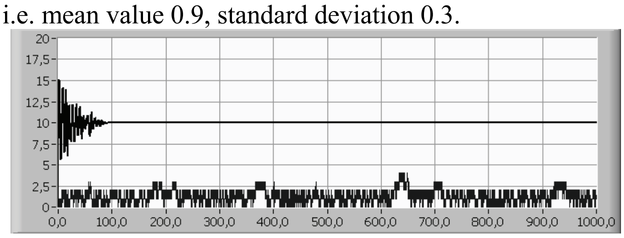
In this example the average service time (0.7) is obviously smaller than the sampling time. The delay time in principle is distributed over the interval (0, 1.5) which is approximately the interval of the service times. The queue length remains small (1 or 2). If it is 2 over a longer time the performance of the system gets worse. The settling time reaches values up to 40 sampling periods.

Example 2:

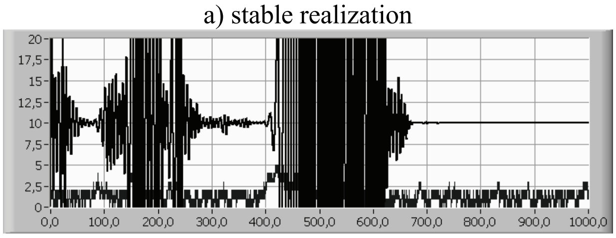
Plant: PT1 with $\tau_p = 20$

Controller: PI with $k_P = 20$ and $k_I = 0.67$

Service time intervals: uniformly distributed in (0.4, 1.4), i.e. mean value 0.9, standard deviation 0.3.



a) stable realization

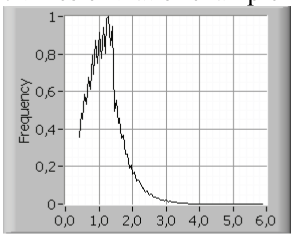


b) Realization with oscillation intervals

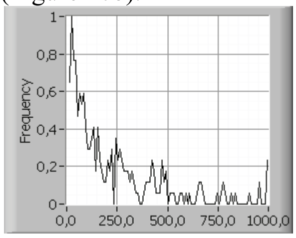
Figure 9: Realizations of the step response in example 2

In this example the average service time (0.9) is close to the sampling interval time (1.0). The step response is stable in the mean (Figure 9a) but needs a longer time to settle. It can contain periods of heavy oscillations caused by longer queue lengths and thus longer delay times (Figure 9b).

This also can be seen in the delay time histogram (Figure 10a). The settling time reaches values more than 20 times of that of example 1 (Figure 10b).



a) Delay time



b) Settling time

Figure 10: Histograms for example 2

6. Conclusions

A model-based simulation method for the performance analysis of networked control systems has been presented. Whereas the controller and the plant are modeled as classical continuous-time systems, the delay mechanism caused by the data transfer over the network is represented by a queuing system which allows a more realistic modeling. An analysis tool has been developed, and application examples of behavioral analysis have been demonstrated.

References

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