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Complicated superstable periodic behavior in piecewise constant circuits with impulsive excitation

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Abstract—This paper studies piecewise constant circuits with an impulsive switch. Since vector field of the circuit equation is piecewise constant, the trajectories are piecewise linear : it is well suited for precise analysis. First, we consider the autonomous case. The switch is controlled by a state and the circuits can exhibit chaotic behavior. Second, we consider the non-autonomous case. The switch is controlled by time and the circuits can exhibit rich super stable periodic behavior. We have confirmed the behavior in numerical simulations and embedded return maps have a flat part. Typical phenomena can be confirmed experimentally.

1. Introduction

Simple chaotic circuits are important real physical systems to investigate interesting nonlinear phenomena [1]-[3]. Also, the chaotic systems may be developed into engineering applications including pulse-coupled neural networks and chaos-based communications [3], [4]. On the other hand, switched dynamical systems have been studied for analysis of nonlinear phenomena. The systems have rich bifurcation phenomena and the typical examples are power converters [5]. We have selected an impulsive switch as a key of nonlinear element. The switch can cause interesting phenomena including rich bifurcation phenomena by external excitation [6].

This paper presents simple piecewise constant (ab. PWC) circuits consisting of two capacitors, two voltage-controlled current sources (ab. VCCSs) and an impulsive switch. The VCCSs have signum characteristics and the vector field of the system is piecewise constant : it is well suited for precise analysis [7]-[9]. First, we consider the autonomous case where the switch is controlled by the capacitor voltage. The circuits exhibit chaotic and equilibrium attractors. We have obtained evidence for chaos in [8]. Second, we consider the non-autonomous case where the switch is controlled by a periodic pulse-train input. In this case, the circuits exhibit various interesting phenomena including rich super stable periodic orbits (ab. SSPOs). We have confirmed a variety of SSPOs in numerical simulations. The return maps are piecewise linear and have a flat segment that causes the superstability. Using the map, we show interesting bifurcation diagram. We have confirmed typical phenomena in laboratory experiments.

Motivations for studying such circuits include the following. First, the circuit equation has piecewise constant vector fields and the trajectories are piecewise linear. It is well suited for theoretical analysis of nonlinear phenomena. Second, the impulsive switch can cause various interesting phenomena and the circuits model is very simple. Third, the rich SSPOs have appeared various dynamical systems [10], [11]. The transient time to a SSPO is very short. That is, the SSPOs relate response speed of the system and may be developed into controlled system of the power converter.

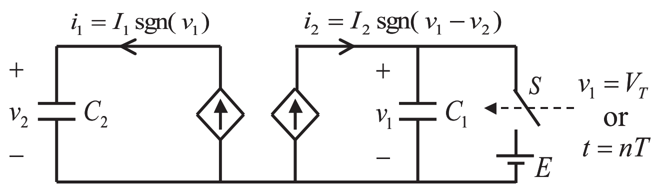

Figure 1: Piecewise constant circuit model.

2. Basic dynamics in piecewise constant circuits

Fig. 1 shows the PWC circuits model. The circuits consist of two capacitors, two VCCSs and an impulsive switch S. The VCCSs have signum characteristics:

$$\begin{cases} i_1 = I_1 \mathrm{sgn}(v_1) \\ i_2 = I_2 \mathrm{sgn}(v_1 - v_2) \end{cases} \quad \mathrm{sgn}(v_1) = \begin{cases} 1 & \text{for } v_1 > 0 \\ 0 & \text{for } v_1 = 0 \\ -1 & \text{for } v_1 < 0. \end{cases} \tag{1}$$

When the switch is open all the time, the circuit dynamics is described by

$$\begin{cases} C_1 \dfrac{dv_1}{dt} = I_2 \mathrm{sgn}(v_1 - v_2), \\ C_2 \dfrac{dv_2}{dt} = I_1 \mathrm{sgn}(v_1), \end{cases} \quad \text{for S=off.} \tag{2}$$

Since the vector fields of Eq. (2) is piecewise constant, the trajectories are piecewise linear. Operation of S is defined afterward. Using the following dimensionless variables and parameters, Eq. (2) is transformed into Eq. (3).

$$\tau = \frac{|I_2|}{C_1|E|}t,\ x = \frac{1}{|E|}v_1\ \left(\dot{x} \equiv \frac{dx}{d\tau}\right),\ y = \frac{1}{a|E|}v_2,$$
$$a = \frac{C_1|I_1|}{C_2|I_2|},\ \gamma_1 = \frac{I_1}{|I_1|},\ \gamma_2 = \frac{I_2}{|I_2|}.$$

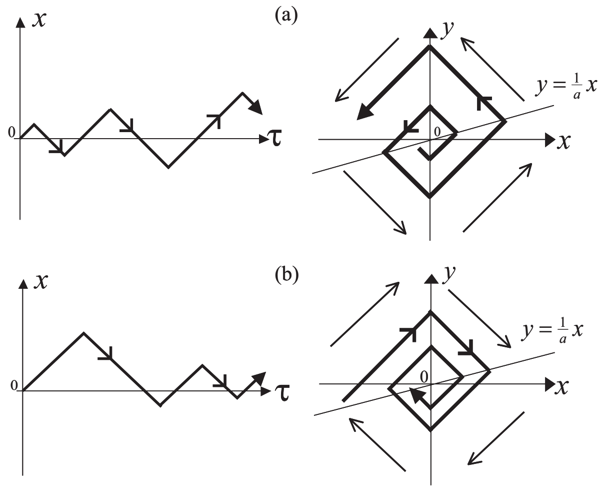

Figure 2: Basic dynamics in piecewise constant circuit for $a > 1$. (a) $(\gamma_1, \gamma_2) = (1, 1)$, (b) $(\gamma_1, \gamma_2) = (-1, -1)$.

$$\begin{cases} \dot{x} = \gamma_2 \mathrm{sgn}(x - ay) \\ \dot{y} = \gamma_1 \mathrm{sgn}(x) \end{cases} \quad \text{for S=off,} \tag{3}$$

where $E < 0$ is the constant base voltage. Note that parameters γ_1 and γ_2 have bipolar value: $\gamma_1 \in \{1, -1\}$, $\gamma_2 \in \{1, -1\}$. For simplicity, we assume the following condition.

$$a > 1. \tag{4}$$

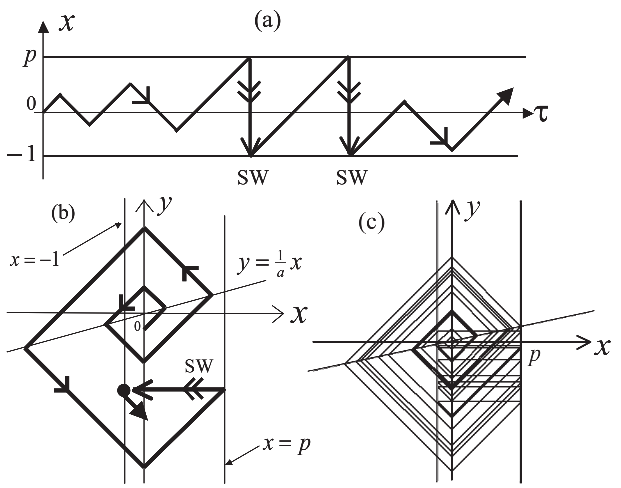

Figure 3: Dynamics and typical phenomena of autonomous PWC circuit for $(\gamma_1, \gamma_2) = (1, 1)$. (a) Time domain, (b) Phase plane, (c) Chaotic behavior for $a = 4.7$, $p = 5.0$.

In this case the vector fields are classified into four cases corresponding to signs of γ_1 and γ_2. If $(\gamma_1, \gamma_2) = (1, 1)$, state x and y can vibrate divergently and the vector field is unstable rect-spiral as shown in Fig. 2 (a). If $(\gamma_1, \gamma_2) = (-1, -1)$, x and y convergent and the vector field is stable rect-spiral as shown in Fig. 2 (b). Hereafter we focus on two cases. The cases $(\gamma_1, \gamma_2) = (1, -1)$ and $(\gamma_1, \gamma_2) = (-1, 1)$ are shown in [9].

3. Autonomous piecewise constant circuits

We consider PWC circuits in autonomous case and define the operation of S: If v_1 reaches the threshold V_T, S is closed and v_1 is reset to the base E. We assume that the switching is instantaneous without delay and continuity property of v_2 is held. The switching rule is described by

$$(v_1(t+), v_2(t+)) = (E, v_2(t)) \ \text{ if } \ v_1(t) = V_T. \tag{5}$$

For simplicity, we assume $V_T > |E|$. Using a parameter $p = \frac{V_T}{|E|} > 1$, Eq. (5) is transformed into Eq. (6).

$$(x(\tau+), y(\tau+)) = (-1, y(\tau)) \ \text{ if } \ x(\tau) = p. \tag{6}$$

When the vector field is unstable rect-spiral, the trajectory moves as shown in Figs. 3(a) and (b). The circuit exhibits chaotic behavior for $(\gamma_1, \gamma_2) = (1, 1)$ as shown in Fig. 3(c). We have obtained that the trajectories reach the equilibrium point at origin for $(\gamma_1, \gamma_2) = (-1, -1)$ in [9].

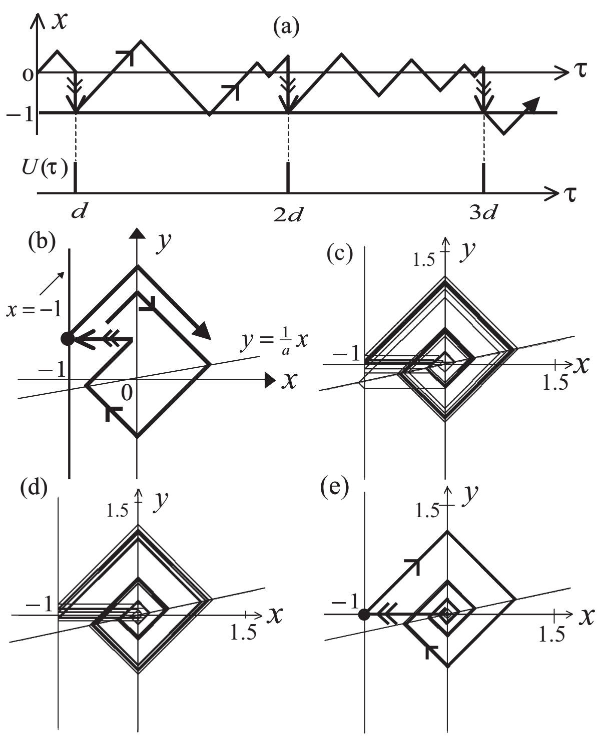

Figure 4: Dynamics and typical phenomena of non-autonomous PWC circuit for $(\gamma_1, \gamma_2) = (-1, -1)$. (a) Time domain, (b) Phase plane, (c) Complicated SSPO for $a = 4.7$, $d = 4.8$, (d) Complicated SSPO for $a = 4.7$, $d = 5.4$, (e) Basic SSPO for $a = 4.7$, $d = 6.0$.

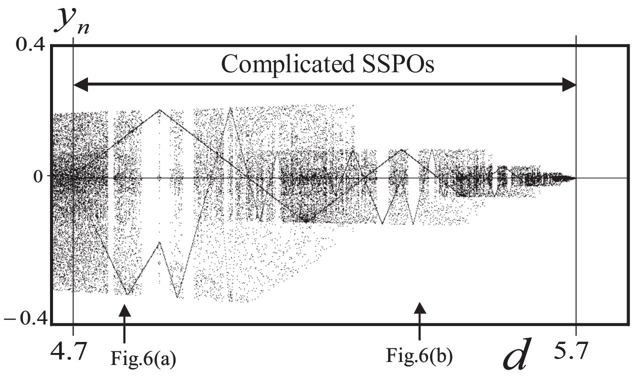

Figure 5: Bifurcation diagram of non-autonomous PWC circuit for $a = 4.7$, $\gamma_1 = \gamma_2 = -1$.

4. Non-autonomous piecewise constant circuits

In the non-autonomous case, the switch is controlled by periodic impulse-train input $U(t)$ with period T. When a pulse $U(t)$ arrives, S is closed and v_1 is reset to E:

$$(v_1(t+), v_2(t+)) = (E, v_2(t)) \;\text{ at }\; t = nT, \tag{7}$$

where n is a nonnegative integer. Using a parameter $d = \frac{|I_2|}{C_1|E|}T$, the switching rule is represented by

$$(x(\tau+), y(\tau+)) = (-1, y(\tau)) \;\text{ at }\; \tau = nd. \tag{8}$$

The switching depends on only time. When $(\gamma_1, \gamma_2) = (1, 1)$, The circuit exhibits complicated chaotic behavior in [9].

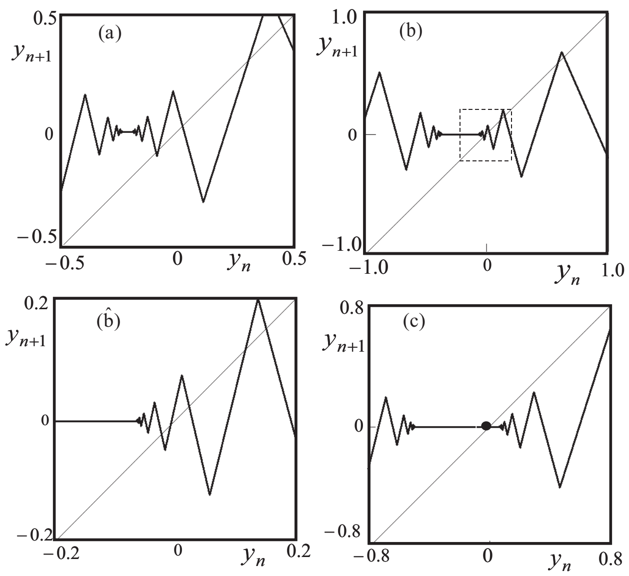

Figure 6: Return maps of non-autonomous PWC circuit for $a = 4.7$, $(\gamma_1, \gamma_2) = (-1, -1)$. (a) $d = 4.8$, (b) $d = 5.4$, (d) $d = 6.0$. ($\hat{\text{b}}$) is enlargement of the dotted box in (b). (a) to (c) correspond to Fig. 4 (c) to (e).

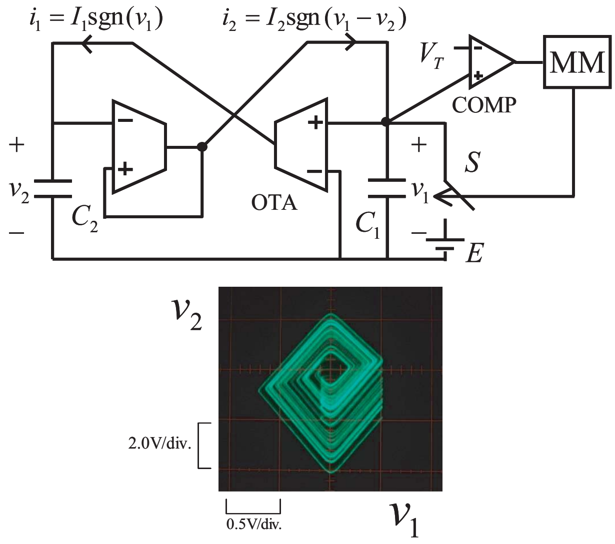

Figure 7: Simple test circuit in autonomous case and laboratory measurement. $I_1 = I_2 \doteq 0.1$mA, $C_1 \doteq 47$nF, $C_2 \doteq 10$nF ($a = 4.7$), $V_T = 0.5$V, $E = -0.1$V ($p = 5.0$). The data corresponds to the Fig. 3 (c) .

When $(\gamma_1, \gamma_2) = (-1, -1)$, the trajectory moves as shown in Fig. 4 (a) and (b). The circuit exhibits various phenomena including the SSPOs as shown in Figs. 4 (c) to (e). Fig. 5 shows bifurcation diagram for $a = 4.7$, $(\gamma_1, \gamma_2) = (-1, -1)$, where $y_n \equiv y(nd)$. In numerical simulations, we can confirm that complicated SSPOs appear for $a < d < a+1$. In range of d it seems to be chaotic orbit, but all the orbits are SSPOs. If $d > a + 1$, the circuit exhibits basic SSPO.

Here we define one dimensional return map for analysis. In order to derive the map, let $L_D = \{(x, y, \tau) \mid x = -1, \text{and } \tau = nd\}$ and a point on L_D be represented by its y coordinate. $F : L_D \longrightarrow L_D$, $y_{n+1} = F(y_n)$. Fig. 6 shows typical return maps corresponding to various SSPOs. If $d > a + 1$, a flat part of the maps intersects 45 degrees line and super stable fixed point exists on $y_n = 0$ as shown in Fig. 6 (c). If $a < d < a + 1$, however, a flat part of the maps do not have super stable fixed point and complicated SSPOs can exist (see Figs. 6 (a) to ($\hat{\text{b}}$)).

5. Experiments

Figs. 7 and 8 show a simple test circuits and laboratory measurements. The VCCSs are realized by operational transconductance amplifiers (OTAs). In autonomous case, the impulsive switch is implemented using a comparator (COMP), a monstable multivibrator (MM), an analog switch S. When v_1 reaches V_T, the COMP triggers the MM to close the S and v_1 is reset to the base E.

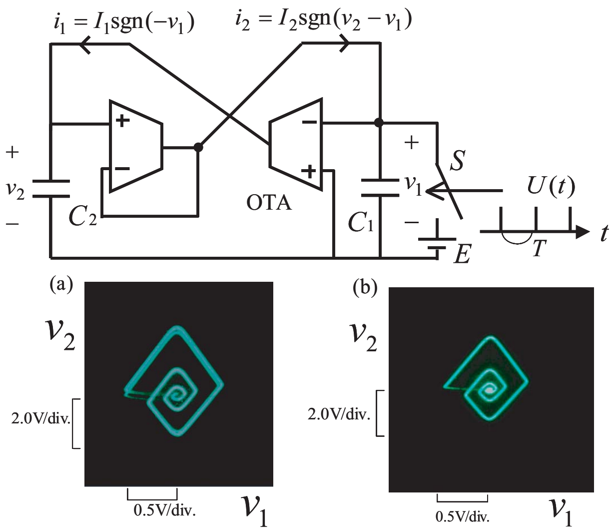

Figure 8: Simple test circuit in non-autonomous case and laboratory measurements. $I_1 = I_2 \doteq 0.12$mA, $C_1 \doteq 47$nF, $C_2 \doteq 10$nF ($a = 4.7$), $E = -0.5$V. (a) Complicated SSPO with $T \doteq 1.05$ms ($d = 5.4$). (b) Basic SSPO with $T \doteq 1.18$ms ($d = 6.0$). Figs. (a) and (b) correspond to Figs. 4 (d) and (e).

6. Conclusions

We considered piecewise constant circuits with an impulsive switch. The switch depends on state or time. A basic classification of vector field is given and typical phenomena are shown. Roughly speaking, the phenomena are classified into the following.

Trajectory for S =off	Autonomous case	Non-autonomous case
Unstable rect-spiral	Chaos	Complicated phenomena
Stable rect-spiral	Equilibrium point	Rich SSPOs, chaos etc

Future problems include detailed analysis of bifurcation phenomena for SSPOs and consideration of engineering applications.

References

[1] L. O. Chua, C. W. Wu, A. Huang and G. -Q. Zhong, A universal circuit for studying and generating chaos, IEEE Trans. Circuits Systs. I, 40, pp. 732-761, 1993.

[2] A. S. Ekwakil and M. P. Kennedy, Construction of classes of circuit-independent chaotic oscillators using passive-only nonlinear devices, IEEE Trans. Circuits Systs. I, 48, pp. 289-307, 2001.

[3] H. Nakano and T. Saito, Grouping synchronization in a pulse-coupled network of chaotic spiking oscillators, IEEE Trans. Neural Networks, 15, 5, pp. 1018-1026, 2004

[4] M. Sushchik, N. Rulkov, L. Larson, L.Tsimring, H. Abarbanel, K. Yao and A. Volkovskii, Chaotic pulse position modulation: a robust method of communicating with chaos, IEEE Comm. Lett., 4, pp. 128-130, 2000.

[5] S. Banerjee, S. Parui, Dynamical Effects of Missed Switching in Current-Mode Controlled dc-dc Converters, IEEE Trans. Circuits Syst. II, 51, pp. 649-654, 2004.

[6] K. Miyachi, H. Nakano and T. Saito, Response of a simple dependent switched capacitor circuit to a pulse-train input, IEEE Trans. Circuits Syst. I, 50, 9, pp. 1180-1187, 2003.

[7] T. Tsubone and T. Saito, Manifold piecewise constant systems and chaos, IEICE Trans. Fundamentals, E82-A, 8, pp.1619-1626, 1999.

[8] Y. Matsuoka and T. Saito, Simple chaotic spiking oscillators having piecewise constant characteristics, Proc. NOLTA, 2, pp. 521-524, 2004.

[9] Y. Matsuoka, T. Saito and H. Torikai, A piecewise constant switched chaotic circuit with rect-rippling return maps, Proc. ISCAS, pp3411-3414. 2005.

[10] C. Wagner, R. Stoop, Renormalization Approach to Optimal Limiter Control in 1-D Chaotic Systems, Journal of Statistical Physics, 106, pp. 97-107, 2002.

[11] Y. Kon'no, T. Saito and H. Torikai, Rich dynamics of pulse-coupled spiking neurons with a triangular base signal, Neural Networks, accepted, 2005.