2005 International Symposium on Nonlinear
Theory and its Applications (NOLTA2005)
Bruges, Belgium, October 18-21, 2005

An Adaptive Mesh Generation for Surface Model Based on a Fibre Bundle of 1-Parameter Groups

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Abstract—In this paper, an adaptive mesh generation algorithm will be proposed for the surface model based on 1-parameter groups of linear Lie algebra. Considering apparently local variation of a surface from a viewpoint, a set of adaptive meshes can be obtained directly from its invariants. The proposed method is expected to be used in an information network system, which sends/receives only small size of information on objects and represent their shapes according to specification of each terminal.

1. Introduction

Since technology of computer graphics (CG) can represent more complicated shape (surface/solid), CG has been used recently in various engineering fields. However, current CG technology can not always satisfy needs in the engineering field. Therefore research on surface/ solid modeling has been studied, which can represent more complex shape. Namely in the field of three dimensional object recognition and representation, a representation in the use of Lie algebra has been studied and developed[1]. Co-authors proposed a fibre bundle model of 1-parametr groups, which uses either or both fibre and base curves as 1-parameter groups[2]. This model represents a surface locally (i.e. at a neighborhood of points) as a direct product between a base curve and a fibre. Since such curves can be exactly represented by 6 invariants of linear Lie algebra, the model can be described by a base curve plus 6 invariants or a base point plus 12 invariants. Therefore, the model can represent an object (surface) from small number of parameters, so that it is also expected to be useful in the field of intelligent communication system[3].

However in general, it is difficult to visualize the complicated shape defined by such model, because it needs huge computational cost in exact representation. Therefore, an approximation by a set of meshes is often used. It is obviously seen that the quality of approximation depends on suitable size of meshes for the shape of the object. However, the method needs more data(meshes) than parameters of a certain of free form surfaces including the Lie algebra model. In other words, managing such massive mesh data causes uselessness and inefficiency, while we can get any size of meshes from the parameters. Therefore in this paper, an adaptive mesh generation algorithm will be proposed for the surface model based on 1-parameter groups of linear Lie algebra. The interval between two points are determined on the base curve and the fibre bundle curve. In addition, in order to represent the shape more interactively, we shall propose an adaptive meshing for a surface model based on a fibre bundle of 1-parameter groups, with accuracy and level of details in consideration with viewpoints, which determine priority of production and visualization.

2. Lie algebra[1]

Lie group is a C^∞ class differentiable manifold, and normal/tangent vector field on the Lie group is called as Lie algebra. Since Lie group has a one-to-one correspondence to Lie algebra, we can get a shape defined by global information (Lie group) from normal/tangent vector field formed from local information (Lie algebra).

For a given point $\boldsymbol{p}$ on a surface by linear Lie algebra, its normal vector $\boldsymbol{v}$ is defined as follows:

$$\boldsymbol{v} = A\boldsymbol{p}, \tag{1}$$

where A is a representation matrix such that

$$A = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix} \boldsymbol{P}_\theta \boldsymbol{Q}_\phi \boldsymbol{P}_\psi. \tag{2}$$

Here, $\boldsymbol{P}_\theta$, $\boldsymbol{Q}_\phi$ and $\boldsymbol{P}_\psi$ are rotation matrices with respect to X, Y and Z axis, respectively, and a set of λ_1, λ_2, λ_3, θ, ϕ and ψ is called as invariant.

3. Fibre bundle model of 1-parameter groups of linear Lie algebra

First we briefly review the fibre bundle surface model shown in [2], which uses a fibre curve in the fibre bundle model as 1-parameter groups of linear Lie algebra.

Let $\boldsymbol{b} = \{\boldsymbol{b}(v), v \in R\}$ be a base curve, and, $\boldsymbol{g}_v = \{\boldsymbol{g}_v(u) = e^{uA}b(v), u \in R\}$ be a fibre curve as 1-parameter Lie group. A surface is defined as

$$F := x(u,v) = e^{uA}\boldsymbol{b}(v) \qquad u, v \in R. \tag{3}$$

The points $\boldsymbol{b}(v)$ on the base curve $\boldsymbol{b}$ are initial points for the integral curve of 1-parameter groups $\boldsymbol{g}_v$. In fact, the base curve needs not to be in a form of parameterized curve $\boldsymbol{b}(v)$(see Fig.1).

More common model uses both of the base curves and the fibre curves as 1-parameter groups:

$$x(u, v) = e^{uA+vB}x_0 + d. \tag{4}$$

The Lie algebra consisits of the following two linear algebras.

$$\dot{x}_u = Ax, \qquad \dot{x}_v = Bx. \tag{5}$$

In this case, the information to descibe the fibre bundle model is a base point $x(0, 0) = \{\boldsymbol{b}(0)$ and twelve invariants of matrices A and B.

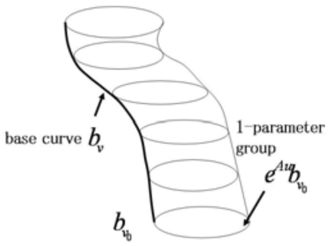

Figure 1: fibre bundle of 1-parametr group

4. Synthesis by inverse Laplacian transformation

Another advantage for the surface model using fibre bundle of 1-parameter groups is that shape synthesis needs only calculation of elementary functions without numerical errors of integration. In fact, the integration can be obtained using Laplacian transform as follows.

For a curve with fixed v

$$x(u, v) = (x_1(u, v), x_2(u, v), x_3(u, v))^T. \tag{6}$$

define the Laplacian transformation with respect to u as $\boldsymbol{X}(p, v)$:

$$\boldsymbol{X}(p, v) = (X_1(p, v), X_2(p, v), X_3(p, v))^T. \tag{7}$$

Since the fibre has a linear Lie algebra $\dot{x}_u = Ax$

$$(pI - A)\boldsymbol{X}(p, v) = x(0_+, v). \tag{8}$$

The fibre curve is obtained by the inverse Laplacian transform

$$\boldsymbol{X}(p, v) = (pI - A)^{-1}x(0_+, v). \tag{9}$$

where $x(0_+, v)$ is the initial point on the base curve.

Here $(pI-A)^{-1}$ is a three dimensional square matrix with entries of rational functions in p. The denominators are polynomials with degrees no more than three. By the inverse Laplacian transformation of partial expansions, the fibre can be expressed by elementary functions such as trigonmetric functions and exponentials. In other words, the shape can be synthesized fast and error free without numerical integration.

For fibres with affine Lie algebras

$$(pI - A)\boldsymbol{X}(p, v) = x(0_+, v) + \frac{1}{p}d_v \tag{10}$$

Thus, the fibres can be obtained from the inverse Laplacian transformation of

$$\boldsymbol{X}(p, v) = (pI - A)^{-1}x(0_+, v) + \frac{1}{p}(pI - A)^{-1}d_v. \tag{11}$$

again using elementary functions.

In the model with both the base curve and the fibres generated by 1-parameter groups

$$x(u, v) = e^{uA+vB}x_0 + d. \tag{12}$$

$$\frac{\partial x}{\partial u} = Ax, \frac{\partial x}{\partial u} = Bx \tag{13}$$

The tangent plane is spanned by these two basis.

$$t_x = a\frac{\partial x}{\partial u} + b\frac{\partial x}{\partial u} = (aA + bB)x. \tag{14}$$

The surface can be generated by double Laplacian transformation

$$\boldsymbol{X}(p, v) = (pI - A)^{-1}(x(0, v) + \frac{1}{p}d). \tag{15}$$

$$\boldsymbol{X}(u, q) = (qI - B)^{-1}(x(u, 0) + \frac{1}{p}d). \tag{16}$$

5. Choice of fibres and generation matrix A

In this section, we show a general form of the matrix A when the fibres are chosen as 1-parameter groups on a known surface S.

Theorem 1. *Assume a normal vector field n (of planes) of a surface S forms a linear Lie algebra*

$$n = Mx = \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix}. \tag{17}$$

Let P be an arbitrary three dimensional skew-symmetric square matrix such as

$$P = \begin{pmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{pmatrix}. \tag{18}$$

The tangent vector t of the fibre at an initial point $x_0 \in S$ can be expressed by $t = PMx_0$. Here the direction of t on the tangent plane Tx_0S can be controlled by matrix P.

$$t = PMx = at_1 + bt_2 + ct_3 \tag{19}$$

Then any 1-parameter groups on S through the initial point $x(0)$ can be generated by e^{tA} with

$$A = PM \tag{20}$$

A is generation matirix

6. Adaptive meshing

In this section, we propose an adaptive meshing for the surface model on a fibre bundle of 1-paremeter groups.

6.1. Generation of a base curve and a fibre curve

Suppose that a base curve is given as initial data, which is equation or generation matrix A. In generation matrix case, the base curve $\boldsymbol{b}(v)$ can be given by inverse laplacian transformation of Eq.(16). Therefore, the fibre curve $x(u, v)$ can be given by inverse laplacian transformation of Eq.(15). Here, length of interval of neighbor point p_{i+1} of p_i on fibre bundle is Δt. We descibe the next subsection about decision of Δt.

6.2. Decision of Δt

In this section an adaptive decision of Δt is proposed in consideration of the variation of tangent vector around the given point.

Since each pixel can represent information only on a point of a surface, information on two or more points is redundant for a pixel(see Fig.2). Therefore we define the smallest $dist_{min}$ of Δt by resolution of an image to be generated, and Δt is enlarged by the shape. $dist_{min}$ is defined as following:

$$dist_{min} = \frac{dist_{p_i} h}{d}, \tag{21}$$

where $dist_{p_i}$ is distance between p_i and the viewpoint, d is distance between the pixel, where p_i is projected, and the viewpoint, and h is size of each pixel.

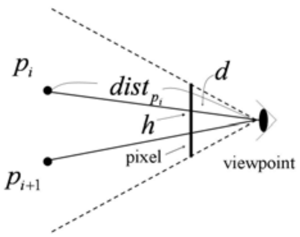

Figure 2: Range of a pixel

We here assume that a representation matrix and a point p_i on the surface are given. Also let β be feasible angle such as:

$$\begin{aligned} \beta = \quad & \beta_{max} - c \cdot dist_{p_i}, \\ & (\text{if} \quad \beta \leq \beta_{min}, \text{then} \quad \beta = \beta_{min}) \end{aligned} \tag{22}$$

where β_{max} and β_{min} are input in advance.

p_{i+1} is calculated with an initial length Δt. Then an angle α between the tangent vector w_i on p_i and w_{i+1} on p_{i+1} is calculated as the following:

$$\alpha = \cos^{-1} \frac{w_{i+1} \cdot w_i}{|w_{i+1}||w_i|} \tag{23}$$

w_i calculated from Eq.(17) by following:

$$w_i = Ap_i, \tag{24}$$

where A is generation matrix.

If $\alpha < \beta$ then it is concluded that Δt is too small to visualize in suitable computational cost. In this case Δt is enlarged until the revised p_{i+1} and w_{i+1} are feasible.

6.3. Generating a set of mesh

Here let us consider a case of generation of meshes between integral curves $(S+1)$ and $(S+2)$, where the curve $(S+2)$ has more points than $(S+1)$. For adjunct points $(S+2)p_i$ and $(S+2)p_{i+1}$, we determine a point on $(S+1)$, which can form an equilateral triangle or the nearest shape to an equilateral triangle with $(S+2)p_i$ and $(S+2)p_{i+1}$. Repeating this formation, we can obtain a set of featured triangles between $(S+1)$ and $(S+2)$ (see Fig.3).

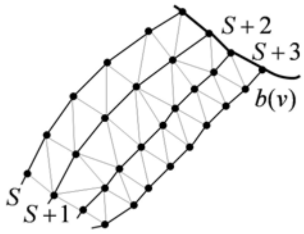

Figure 3: Generation of the meshes from the points

6.4. Algorithm

Here, we show an outline of the proposed algorithm.

Step1: Input a set of invariants, a base curve $\boldsymbol{b}(v)$ or a generation matrix B, a generation matrix of a fibre curve A and a viewpoint.

Step2: Calculate a base curve $\boldsymbol{b}(v)$ and a fibre curve $x(u, v)$ by the generation matrix B and A.

Step3: Determine an interval for generating the mesh on a base curve. Determine p_0 on the base curve $\boldsymbol{b}(v)$, and if p_0 does not exist within the interval, go to **Step 12**.

Step4: Calculate the tangent vector w_0 on p_0 by Eq.(24).

Step5: Determine an initial Δt by the distance between a the viewpoint and a point p_i on the surface.

Step6: Calculate a tangent vector w_i on p_i and a tangent vector w_{i+1} on p_{i+1} by Eq.(24),.

Step7: Calculate an angle α between a tangent vector w_i and w_{i+1} by Eq.(23).

Step8: Determine a feasible angle β by Eq.(22).

Step9: If $\alpha < \beta$, return to **Step 3** with $\Delta t \leftarrow \delta \Delta t (\delta > 1)$.

Step10: If $\alpha > \beta$ and if a condition described is not satisfied, return to **Step 4** with $i = i + 1$.

Step11: If the condition is satisfied, return to **Step 3** with $v = v + j$.

Step12: Generate a set of mesh from the points.

7. Simulation

We here show some results of the proposed algorithm. Tables 1-2 show system sepecification and initial data for all cases. Table 3 show number of generated meshes, computational cost, and distance between viewpoint and starting point of base curve.

In Figs.4-7, the left figures show wireframe model while the right figures show shaded images. Figure 4(Fig.5) and Fig.6(Fig.7) show the images generated by the proposed method at different viewpoints near/far from the object, respectively. Comparing these figures, we note that the meshes are generated adaptively with respect to the distance between the viewpoint and the object.

Table 1: System specification

OS	Micrisoft Windows XP
CPU	Pentium(R) 4 CPU 2.40GHz
Memory	512 GB RAM
Video Bord	nVIDIA GeForece4 Ti4200

Table 2: Invariants on the each simulation

	λ_1	λ_2	λ_3	θ	ϕ	ψ
Base curve	$\sqrt{3}$	$\sqrt{3}$	0.0	0.0	0.0	$-\pi/2$
Fibre bundle	1.0	0.0	1.0	0.0	0.0	$\pi/2$

Table 3: Result of meshes generation

	meshes	cost[s]	distance
Fig.4, 5	12540	2.687	332
Fig.6, 7	3190	0.453	1338

8. Conclusion

In this paper, we have proposed an adaptive meshing algorithm with the guaranteed quality for the surface model based on a fibre bundle of 1-parameter groups.

In the proposed method, the size of mesh is determined adaptively and automatically in consideration of local variation of tangent vectors around the point on the surface and

Figure 4: Meshes from near view

Figure 5: Surface from near view

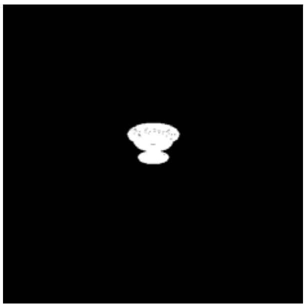

Figure 6: Meshes from far view

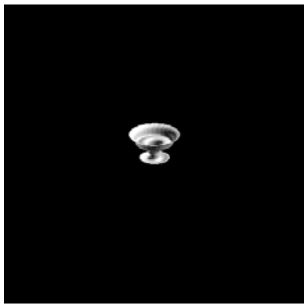

Figure 7: Surface from far view

the interval between the viewpoint and the object. Futuremore since the size is enlarged and decreased, it is seen that the computational cost for visualization becomes relatively low.

Acknowledgments

This paper is partially supported by Chuo University 21COE Program and Grant for Special Research, and Collaborative Research Grant of the Institute of Science and Engineering, Chuo University.

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