Toward Interactive Image Communications System
with Invariants of Lie Algebra

Mitsunori Makino[†]

†Department of Information and System Engineering, Chuo University
1-13-27 Kasuga, Bunkyo-ku, Tokyo 112-8551, Japan
Email: makino@m.ieice.org

Abstract—In this paper, a concept of interactive image communications is presented, in which an image itself is not sent but only small set of invariants for each object is sent to other terminals. Generating meshes from the invariants, the objects are represented in suitable resolution for observation at receiver's side. This concept is available even for very narrow-band communication channel. This concept(paper) is practically followed by two papers, Sagara and Kaneko.

1. Introduction

Multimedia data has been recently used in intelligent communication system such as tele-existence, tele-immersive and tele-collaboration, as well as they have been used in the internet. Among the data, image is essential information to the system, since we obtain much information through eyes.

When we send a raw image to a receiver, its quality and quantity should be considered for good communication whether the image has necessary and sufficient (neither short nor redundant) information to be played at the receiver. Furthermore, we have to take performance of a communication path into account, which connects the sender with the receiver. When the path has low capacity (narrow-band), transmission of images with high quality and large quantity is very difficult. By contraries, when the path has too enough capacity (wide-band), a receiver is forced to satisfy images images with lower quality and smaller quantity than transmittable images. Therefore image and video compression technologies are not absolute solution in a case beyond expectations of the sender, although they are very useful in some forecastable cases.

We can see many examples in the internet that a sender provides an image/video with a variety of resolusions and bit rates, for their comfortable transmission and replay at a receiver. Since performance of communication path and environment (equipment) differ very much, such variety works limitedly effectively. If a sender add another resolusion and/or bit rate, it causes a raise in cost.

While the above handles raw image/video, there are communication systems such as VRML, which do not send/receive raw information but only information on generating image/animation. The sender in the system provides unique information for each image, which is used for generation of an image by a receiver, in consideration of the receiver's environment. Since an image itself has usually larger size than its information (parameters) which can generate the same image in any resolusion, such system are not sensitive to performance of communication path. Alternatively, it requires efficient (high speed) generation of images from a set of parameters at a receiver. Such system supposes that terminal (computer) of communication channel is developed faster than communication channel with a variety of capacity. Computer graphics (CG) technology must play an important role in this respect of fast image generation.

In this paper, a concept of interactive image communications is presented, in which an image itself is not sent but only small set of invariants defined by Lie algebra for each object is sent to other terminals. Generating meshes from a surface determined by the invariants, the objects are represented in suitable resolution for observation at receiver's side. This concept is available for very narrow-band communication channel as well as for wide-band channel. In the following sections, we clarify requirements for such communication and consider some kinds of surface models under the requirements.

2. Requirements

Shapes and sizes of objects are the first priority for generating an image in a scene, and, texture and color should be discussed later. For interactive communication systems, it is important that generation of an image at a receiver consists with definition of parameters at a sender. In a case of shape generation/definition, a shape (surface) model requires the followings (also see Fig.1):

1. the smallest possible number of parameters, which can define shape of a surface,

2. adaptive image generation including the surface given by 1. with CG technology, which does not corresponds to the sender's environment but to a receiver's environment,

3. easy manipulation (e.g., movement, rotation and deformation) of the surface at the receiver,

4. easy re-definition of the manipulated surface by a new set of parameters, which enables us to send the updated parameters to another side.

Furthermore, shape parameters of a surface should be separated from position and orientation parameters, in order to enable us to reuse smoothly the manipulated surface in 4, as well as to identify a surface having the same shape.
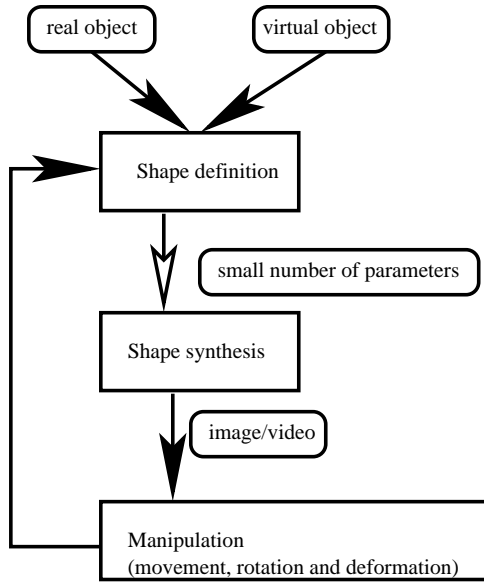
```
#VRML V2.0 utf8 # header
Shape{          # definition of shape
  appearance Appearance{
    material Material{}
  }
  geometry Sphere{ # definition of sphere
    radius 2.0 # radius
  }
}
```

Figure 2: Description of a sphere in VRML

```
#VRML V2.0 utf8
Shape{
  appearance Appearance{
    material Material{}
  }
  geometry IndexedFaceSet{
    coord Coordinate{
      point[           # position of vertex
        1.0 0.0 0.0,# no.0
        0.0 1.0 0.0,# no.1
        -1.0 0.0 0.0,   # no.2
        0.0 -1.0 0.0    # no.3
      ]
    }
    coordIndex[          # triangles
      0,1,2,    -1,# vertex 0,1,2
      0,2,3,    -1 # vertex 0,2,3
    ]
  }
}
```


Figure 1: Flow of supposed interactive communication system. White arrowhead shows communication path.

Figure 3: Description of two triangles in VRML

3. Consistency/inconsistency of Major Models with Requirements

3.1. VRML

The VRML (Virtual Reality Modeling Language) is a standard file format, which represents three dimensional interactive graphics, designed particularly with WWW. The current version is VRML97 standardized by ISO (ISE/IEC 14772-1: 1997), following by X3D.

Figure 2 shows an example of a sphere with radius 2. By the VRML shape information is separated from size, position and orientation. Such feature matches the requirements described in the previous section. However, the format has some restriction of definition of shape that only simple shape can be defined by a set of parameters and complex shape is defined by meshes (see Fig.3. When we use meshes, the VRML does not satisfy the requirement 1.

3.2. Free Form Surface

Since the so-called free form surface can define smooth surface, it is applicable to CAD (computer aided design), CAM (computer aided manifacuturing) and CAE (computer aided engineering) as well as computer graphics. In a case of bicubic Bézier model, a surface $S(u, v)$ is represented by the following formula:

$$S(u, v) = \sum_{i=0}^{3} \sum_{j=0}^{3} P_{ij} B_i(u) B_j(v), \quad 0 \le u, v \le 1, \quad (1)$$

where $P_{ij}, i = 0, 1, 2, 3, j = 0, 1, 2, 3$, are control points, which defines shape of the surface, and $B_k(w), k = 0, 1, 2, 3$, are blending functions such that

$$B_k(w) = \frac{6}{(3-k)!k!} w^k (1-w)^{3-k}, \quad 0 \le w \le 1. \quad (2)$$

In the above case, information on position of the 16 control points is enough to form shape (see Fig.4). Although this formation matches partially the requirements, information

on size, position and orientation is involved in the parameters (16 points). Therefore, there are some difficulties in reuse and identification of surfaces.
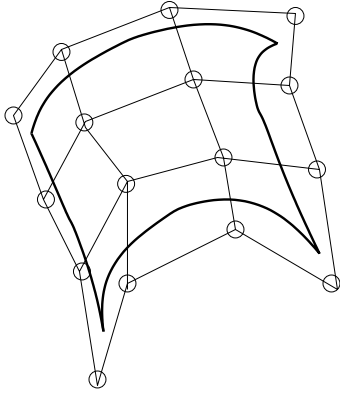

Figure 4: An example of a bicubic Bézier surface: 16 small circles are control points, and surrounded area by thick lines is a surface generated by the points.

4. Surface Model defined by Lie Algebra

A surface model defined by Lie algebra (*L surface model*) has been studied both in computer vision[1][2] and computer graphics field[3][4]. It is noted that characteristics of a shape of surface, defined by the L surface model, are represented by a few parameters called as "invariants." As a result, a certain complicated and smooth objects can be defined by a set of the invariants. Since position information is not involved in the invariants, its property benefits in identification of shape. Since both of extraction from real objects and representation by computer graphics have been being studied, it is expected to connect virtual world with real world.

For example, in a case of linear Lie algebra, the L surface model can be described as follows:

$$v = Ax, \tag{3}$$

where v is a normal vector at a point x on the surface. A is called as a representation matrix such that:

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}. \tag{4}$$

A can be decomposed by product of four matrices:

$$A = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix} P_\theta Q_\phi R_\psi. \tag{5}$$

Here P_θ, Q_ϕ and R_ψ are rotation matrices determined by angles θ, ϕ and ψ with respect to X, Y and Z, respectively.

Here a set of λ_i, $i = 1, 2, 3$ and θ, ϕ and ψ is called as invariants. Adding an initial point on a surface to the invariants, we can obtain points on a surface by solving Eq.(3). The similar formulation can be discussed in other type of the L surface model, e.g., Hamiltonian Lie algebra and fibre bundle. From the points a smooth surface can be defined.

However, the model has a disadvantage that exact representation of a surface may require huge computational cost because of complicated and large calculation of many integral calculus curves on the surface. In such case, meshes of triangular patches usually substitutes for the original surface.

For an initial point (position) x_0 on a L surface, a normal vector v_0 can be obtained by Eq.(3), and tangent lines at x_0 can be defined. Here $w_{j,0}$, $j = 1, 2, \ldots, m$, is assumed to be tangent vectors selected under a certain condition. Then a series of points $\{x_{j,i}\}$ for each $w_{j,0}$ can be calculated as the followings:

$$x_{j,i+1} = x_{j,i} + \Delta t w_{j,i}, \quad i = 0, 1, \ldots, n, \tag{6}$$

where $w_{j,i}$ is a tangent vector at $x_{j,i}$ calculated by Eq.(3). Interlacing three neighbor points, we can define an approximated L surface as a set of meshes.

In order to establish interactive communication with the meshes, meshing technique with high quality in low computational cost is required. Therefore appropriate determination of Δt to local shape of the surface should be discussed. Also effecient extraction of invariants and position from deformed meshes should be considered for interactive communication such as tele-collaboration.

Adaptive and efficient generation of meshes has been proposed in order not to generate meshes redundantly[3][4]. The proposals focus on local variation on a surface. Therefore more efficient meshing technique should be developed with respect to level of detail (LOD) and view-dependence, which are current major techniques for fast representation of surfaces in CG field. Sagara et al.[5] propose an adaptive meshing method, which is combined with the LOD and view-dependence. Furthermore efficient manipulation should be developed for the L surface model. Kaneko et al.[6] propose an interactive manipulation system, in which a kind of the L surface model can be deformed and invariants and position are extracted from the deformed surface.

5. Conclusion

In this paper, a concept of interactive image communications is presented, in which an image itself is not sent but only small set of invariants for each object is sent to other terminals. Generating meshes from the invariants, the objects are represented in suitable resolution for observation at receiver's side. This concept is applicable from very narrow-band to wide-band communication channel. Also application to three dimensional interactive shape database via the network is expected. In such case separation of the

invariants (shape information) from position is useful for manipulation.

More detailed discussion and consideration will be shown in other papers, which focus matters from appropriate meshing to some applications.

References

[1] J. Chao, A. Karasudani, T. Shimada, K. Minowa, "Invariant object representation and recognition using Lie algebras of perceptuation vector fields," *IEICE Trans.(DII)*, vol.J83-DII, no.9, pp.1870–1878, 2000.

[2] M. Suzuki, J. Chao, "Invariant extraction and segmentation of 3D objects using linear Lie algebra models," *IEICE Trans. Inf.& Syst.*, vol.E85-D, no.8, pp.1306–1313, 2002.

[3] S. Kawano, M. Makino, "A high quality triangular meshing for suraface defined by linear Lie algebra," *IEICE Trans.(DII)*, vol.J83-DII, no.12, pp.2707–2715, 2000.

[4] Y. Yamamoto, H. Sano, M. Makino, "A high quality mesh generation with automatic diffentiation for surface defined by Lie algebra," *Simulation, J. of Japan Society for Simulation Technology*, vol.20, no.3, pp.251–258, 2001.

[5] N. Sagara, M. Makino, J. Chao, "An adaptive mesh generation for surface model based on a fibre bundle of 1-Parameter groups," *Prof. of NOLTA2005*, 2005 (CD-ROM).

[6] M. Kaneko, M. Makino, J. Chao, "A visually interactive deformation of surfaces defined by linear Lie algebra with extraction of invariants," *Prof. of NOLTA2005*, 2005 (CD-ROM).