2005 International Symposium on Nonlinear Theory and its Applications (NOLTA2005)
Bruges, Belgium, October 18-21, 2005

Stability of Dissipative Interconnections

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Abstract—This paper is concerned with the stability analysis of the interconnection of two systems. The idea is that, under suitable assumptions, the interconnection of dissipative systems is Lyapunov stable. This is the basis of the classical stability criteria such as the small gain theorem, the positive operator theorem, IQC's, etc. In terms of quadratic differential forms, this paper derives stability conditions for the interconnection consisting of a linear time-invariant subsystem and a general (nonlinear, time-varying) subsystem in the general setting where the supply rate is induced by a polynomial or rational matrix.

1. Introduction

The main principle that underlies the stability results that have emerged in the control literature in the last decades is the observation that under suitable conditions the interconnection of dissipative systems is Lyapunov stable. This principle allows to prove stability of a closed system by viewing it as the interconnection of two dissipative open systems. This is the basis of the small gain theorem, the positive operator theorem [10],[6], IQC's [2], etc. There have been several attempts to extend these results to the behavioral context [3],[5]. Usually, in these results, the supply rate used is assumed to be a memoryless function of the system variables. However, by using quadratic differential forms, this restriction can be dispensed with. The aim of this paper is to outline the stability theory for interconnected systems in which one of the subsystems is linear time-invariant, and the other is a general nonlinear and/or time-varying system. The results generalize the classical multiplier techniques to the behavioral framework.

2. Preliminaries

2.1. Dynamical systems

In the behavioral approach, a dynamical system is characterized by its *behavior*. The behavior is the set of trajectories which meet the dynamic laws of the system. Throughout the paper, we identify a dynamical system with its behavior. In the continuous-time setting, the behavior of a dynamical system is typically defined by the set of all solutions to a system of differential(-algebraic) equations

$$f(w, \frac{dw}{dt}, \cdots, \frac{d^L w}{dt^L}, t) = 0$$

Note that we make no *a priori* assumptions on the choice of inputs and outputs among the elements of w. The behavior is defined by

$$\mathcal{B} = \left\{ w \in \mathfrak{C}^\infty(\mathbb{R}, \mathbb{R}^{\mathtt{w}}) \;\middle|\; f(w(t), \ldots, \frac{d^L w}{dt^L}(t), t) = 0 \;\; \forall t \in \mathbb{R} \right\}.$$

When the system is linear time-invariant (LTI), the above differential-algebraic equation becomes

$$R(\frac{d}{dt})w = R_0 w + R_1 \frac{dw}{dt} + \cdots + R_L \frac{d^L w}{dt^L} = 0,$$

where $R(\xi) = R_0 + R_1\xi + \cdots + R_L\xi^L \in \mathbb{R}^{\bullet\times\mathtt{w}}[\xi]$. We define $\mathcal{L}^{\mathtt{w}}$ as the set of such LTI differential behaviors with $\mathtt{w}$ variables.

A behavior $\mathcal{B}$ is said to be *controllable* if, for any w_1, $w_2 \in \mathcal{B}$ and $t_0 \in \mathbb{R}$, there exist $w \in \mathcal{B}$ and $T > 0$ such that

$$w(t) = \begin{cases} w_1(t) & \text{for } t < t_0, \\ w_2(t) & \text{for } t \geq t_0 + T \end{cases}$$

We denote by $\mathcal{L}^{\mathtt{w}}_{\mathrm{cont}}$ the set of controllable LTI differential behaviors in $\mathcal{L}^{\mathtt{w}}$.

2.2. Quadratic differential forms

A quadratic differential form (QDF) Q_Φ is defined as a quadratic functional of $w \in \mathfrak{C}^\infty(\mathbb{R}, \mathbb{R}^{\mathtt{w}})$ and its derivatives. Namely,

$$Q_\Phi(w) = \sum_{i=0}^{k} \sum_{j=0}^{k} \left(\frac{d^i w}{dt^i}\right)^\top \Phi_{ij} \left(\frac{d^j w}{dt^j}\right)$$

where $\Phi_{ij} \in \mathbb{R}^{\mathtt{w}\times\mathtt{w}}$ and $\Phi_{ji}^\top = \Phi_{ij}$ $(i = 0, 1, \ldots, k)$. We can associate Q_Φ with a symmetric two-variable polynomial matrix

$$\Phi(\zeta, \eta) = \sum_{i=0}^{k} \sum_{j=1}^{k} \Phi_{ij} \zeta^i \eta^j \in \mathbb{R}^{\mathtt{w}\times\mathtt{w}}_{s}[\zeta, \eta].$$

Notice that the indeterminates ζ and η correspond to the differentiations on $w^\top$ and w, respectively. A detailed discussion of QDFs can be found in [8].

A QDF Q_Φ is said to be *nonnegative* if $Q_\Phi(w)(t) \geq 0$ holds for all w and t, and *positive* if in addition $Q_\Phi(w) = 0$ implies $w = 0$. In the same way, we can define the nonnegativity and positivity along the behavior $\mathcal{B}$.

2.3. Asymptotic stability

A behavior $\mathcal{B}$ is said to be *asymptotically stable* if $w(t) \to 0$ for $t \to \infty$ holds for all $w \in \mathcal{B}$. Usually, stability is proven by constructing a *a Lyapunov function*, that is a functional V satisfying for some $\varepsilon > 0$

$$V(w)(t) \geq 0$$
$$\frac{d}{dt}V(w)(t) \leq -\varepsilon\|w(t)\|^2$$

$\forall w \in \mathcal{B}$ and $t \in \mathbb{R}$. It is easy to see that these inequalities imply $\int_0^\infty \|w(t)\|^2\, dt < \infty$ for $w \in \mathcal{B}$. Using some regularity conditions that are implied by $w \in \mathcal{B}$, this then leads to $w(t) \to 0$ for $t \to \infty$. However, in the present informal write-up, we will only be concerned with the above $\mathcal{L}_2$ stability, which we will refer to as stability. The refinements required to obtain asymptotic stability will be dealt with in a sequel article.

3. Dissipation Theory

Definition 1 Let σ be a real-valued functional on $\mathfrak{C}^\infty(\mathbb{R}, \mathbb{R}^{\mathtt{w}})$ with $\sigma(0) = 0$. We call σ *a supply rate*. The behavior $\mathcal{B}$ is said to be *dissipative on* $\mathbb{R}$ *with respect to the supply rate* σ if

$$\int_{-\infty}^{\infty} \sigma(w)(\tau)d\tau \geq 0 \ \ \forall w \in \mathcal{B} \cap \mathcal{D},$$

where $\mathcal{D}$ denotes the family of infinitely often differentiable functions with compact support. The behavior $\mathcal{B}$ is said to be *dissipative on* $\mathbb{R}_-$ *with respect to the supply rate* σ if

$$\int_{-\infty}^{t} \sigma(w)(\tau)d\tau \geq 0 \ \ \forall w \in \mathcal{B} \cap \mathcal{D},\ \forall t \in \mathbb{R}.$$

$\mathcal{B}$ is said to be *strictly dissipative on* $\mathbb{R}_-$ *with respect to* σ if there exists a positive constant ε such that

$$\int_{-\infty}^{t} \sigma(w)(\tau)d\tau \geq \varepsilon \int_{-\infty}^{t} \|w(\tau)\|^2 d\tau \ \ \forall w \in \mathcal{B} \cap \mathcal{D},\ \forall t \in \mathbb{R}$$

Definition 1 is a generalization of the definition of dissipativity on $\mathbb{R}_-$ for LTI systems [1],[8],[9] to general nonlinear and/or time-varying (NTV) systems. If $\mathcal{B}$ is time-invariant, the integral interval in the above definition can be taken to be $(-\infty, 0]$. If $\mathcal{B}$ is (strictly) dissipative on $\mathbb{R}_-$ with respect to the QDF Q_Φ, then we simply say that $\mathcal{B}$ is *(strictly)* Φ-*dissipative on* $\mathbb{R}_-$.

Another characterization of the dissipativity is in terms of a *dissipation inequality*. Namely, $\mathcal{B}$ is dissipative with respect to σ if there exists a a real-valued functional S on $\mathfrak{C}^\infty(\mathbb{R}, \mathbb{R}^{\mathtt{w}})$ satisfying $S(0) = 0$ and

$$\frac{d}{dt}S(w)(t) \leq \sigma(w)(t) \ \ \forall w \in \mathcal{B} \text{ and } t \in \mathbb{R}.$$

This inequality is called the *dissipation inequality*, and S a *storage function*. The next lemma establishes the relation between the dissipativity on $\mathbb{R}_-$ and the dissipation inequality (see [1],[8],[9] for the case where $\mathcal{B} \in \mathcal{L}^{\mathtt{w}}_{\mathrm{cont}}$ and $\sigma = Q_\Phi$.).

Lemma 1 *Let the controllable behavior* $\mathcal{B}$ *and the supply rate* σ *be given. The following statements are equivalent.*

(i) The behavior $\mathcal{B}$ *is dissipative on* $\mathbb{R}_-$ *with respect to* σ.

(ii) There exists a nonnegative storage function S *for* $\mathcal{B}$ *and* σ. *If* $\mathcal{B}$ *belongs to* $\mathcal{L}^{\mathtt{w}}_{\mathrm{cont}}$ *and* σ *is a QDF* Q_Φ, *then there exists a storage function that is also a QDF.*

Idea of Proof: (i)$\Rightarrow$(ii): Define

$$S_+(w)(t) := \inf_{\substack{v \in \mathcal{B} \cap \mathcal{D},\\ v(\tau) = w(\tau)\ (\tau \geq t)}} \int_{-\infty}^{t} \sigma(v)(\tau)d\tau$$

This is well-defined for any $w \in \mathcal{B}$ because $\mathcal{B}$ is controllable. Then, for any $h > 0$, there holds

$$S_+(w)(t) + \int_t^{t+h} \sigma(w)(\tau)d\tau = \inf_{\substack{v \in \mathcal{B} \cap \mathcal{D},\\ v(\tau) = w(\tau)\ (\tau \geq t)}} \int_{-\infty}^{t+h} \sigma(v)(\tau)d\tau$$

$$\geq \inf_{\substack{v \in \mathcal{B} \cap \mathcal{D},\\ v(\tau) = w(\tau)\ (\tau \geq t+h)}} \int_{-\infty}^{t+h} \sigma(v)(\tau)d\tau = S_+(w)(t+h)$$

Hence,

$$S_+(w)(t+h) - S_+(w)(t) \leq \int_t^{t+h} \sigma(w)(\tau)d\tau.$$

Dividing this inequality by h and taking $h \to 0$ yields

$$\frac{d}{dt}S_+(w)(t) \leq \sigma(w)(t) \ \ \forall w \in \mathcal{B},$$

Hence, S_+ is a storage function. The nonnegativity of S_+ is obvious from its definition.

(i)$\Leftarrow$(ii): Let S be a nonnegative storage function. By integrating (3) from t_0 to t, we get

$$\int_{t_0}^{t} \sigma(w)(\tau)d\tau \geq S(w)(t) - S(w)(t_0)$$

Noting $w \in \mathcal{B} \cap \mathcal{D}$, taking $t_0 \to -\infty$ yields

$$\int_{-\infty}^{t} \sigma(w)(\tau)d\tau \geq S(w)(t) \geq 0$$

This completes the proof of (ii)$\Rightarrow$(i).

Remark 1 The storage function S_+ is referred to as the *required supply* for $\mathcal{B}$ and Q_Φ, since it represents the total energy supply that is necessary to realize the future trajectory $w(\tau)$, $\tau \geq t$.

4. Stability Analysis of Dissipative Interconnection

We present two stability theorems for the interconnection of a LTI behavior $\mathcal{B}$ and a general NTV behavior $\mathcal{G}$.

To explain the notion of interconnection, let w and w' be the manifest variables of $\mathcal{B}$ and $\mathcal{G}$, respectively. The interconnection is formed by taking $w = w'$. That is, the interconnection of two systems is defined by $\mathcal{B} \cap \mathcal{G}$, the intersection of their behaviors. This means that the trajectories of the interconnection must satisfy the laws of both systems.

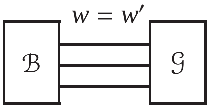

Figure 1: Interconnection

4.1. Stability with polynomial supply rate

The first result provides a stability criterion in terms of a supply rate described by a QDF $Q_\Phi(w)$ with $\Phi(\zeta, \eta)$ polynomial matrix.

Theorem 1 *Let $\Phi(\zeta, \eta) \in \mathbb{R}_s^{\mathtt{w}\times\mathtt{w}}[\zeta, \eta]$ induce the QDF Q_Φ. Assume that $\mathcal{B} \in \mathcal{L}_{\text{cont}}^{\mathtt{w}}$ is strictly Φ-dissipative on $\mathbb{R}_-$, and $\mathcal{G}$ is controllable and $(-\Phi)$-dissipative on $\mathbb{R}_-$. Then, the interconnection $\mathcal{B} \cap \mathcal{G}$ is stable.*

Idea of Proof: It follows from Lemma 1 that there exist nonnegative storage functions Q_Ψ, S, and a positive constant ε such that

$$\frac{d}{dt} Q_\Psi(w)(t) + \varepsilon \|w(t)\|^2 \leq Q_\Phi(w)(t) \quad \forall w \in \mathcal{B}, t \in \mathbb{R},$$
$$\frac{d}{dt} S(w)(t) \leq -Q_\Phi(w)(t) \quad \forall w \in \mathcal{G}, t \in \mathbb{R}.$$

Adding these inequalities together, we obtain

$$\frac{d}{dt}(Q_\Psi + S)(w)(t) \leq -\varepsilon \|w(t)\|^2 \quad \forall w \in \mathcal{B} \cap \mathcal{G}, t \in \mathbb{R}.$$

Since Q_Ψ and S are nonnegative, we obtain $(Q_\Psi + S)(w)(t) \geq 0$ $\forall w \in \mathcal{B} \cap \mathcal{G}, t \in \mathbb{R}$. Hence $Q_\Psi + S$ is a Lyapunov function for $\mathcal{B} \cap \mathcal{G}$, yielding $\int_0^\infty \|w(t)\|^2 \, dt < \infty$. ■

Remark 2 The classical small gain and positive operator theorems are recovered by taking

$$\Phi(\zeta, \eta) = \begin{pmatrix} I_m & 0 \\ 0 & -I_p \end{pmatrix}, \begin{pmatrix} 0 & I_p \\ I_p & 0 \end{pmatrix}.$$

We shall present a typical example where $\Phi(\zeta, \eta)$ is a (two-variable) polynomial matrix.

Example (Popov criterion):

We consider the feedback interconnection of a SISO LTI system $\mathcal{B}$ and a memoryless time-invariant nonlinearity $\mathcal{N}$. Note that, of course, $\mathcal{N}$ is a member of NTV systems. Let $\mathcal{B}$ and $\mathcal{N}$ be described by a stable transfer function $G(\xi)$

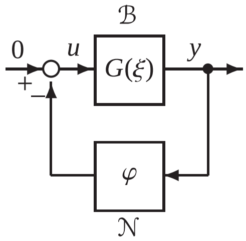

Figure 2: Feedback interconnection

and a nonlinear function φ, respectively. Also, we partition the manifest variable w as $w = (y, u)$ to conform with $y = G(\xi)u$ and $u = -\varphi(y)$ as described in Figure 2. We assume that φ satisfies the so-called sector condition

$$\varphi(0) = 0, \quad 0 \leq \frac{\varphi(y)}{y} \leq k$$

for some $k > 0$. Define

$$\Phi_\alpha(\zeta, \eta) = \frac{1}{2} \begin{pmatrix} 0 & 1 + \alpha\zeta \\ 1 + \alpha\eta & \frac{2}{k} \end{pmatrix}$$

for a positive constant α. It is easy to show from the sector condition that $\mathcal{N}$ is $(-\Phi_\alpha)$-dissipative on $\mathbb{R}_-$. In fact, since we have

$$-Q_{\Phi_\alpha}(w) = -(y + \alpha\frac{dy}{dt})u - \frac{u^2}{k}$$
$$= -\frac{1}{k}\varphi(y)\{\varphi(y) - ky\} + \alpha\varphi(y)\frac{dy}{dt} \geq \alpha\varphi(y)\frac{dy}{dt}$$

for all $(y, u) \in \mathcal{N}$, $\mathcal{N}$ is $(-\Phi_\alpha)$-dissipative on $\mathbb{R}_-$ with a storage function $S(w) = \alpha \int_0^y \varphi(s)ds$. The nonnegativity of $S(w)$ along $\mathcal{N}$ follows from the sector condition. By Theorem 1, the feedback interconnection $\mathcal{B} \cap \mathcal{N}$ is stable if there exists an $\alpha > 0$ such that $\mathcal{B}$ is strictly Φ_α-dissipative on $\mathbb{R}_-$. It is not difficult to show that, for a stable transfer function $G(\xi)$ and $\alpha \geq 0$, the Φ_α-dissipativity of $\mathcal{B}$ on $\mathbb{R}_-$ is guaranteed by the existence of a constant δ satisfying

$$\begin{pmatrix} G(i\omega) \\ 1 \end{pmatrix}^* \Phi_\alpha(-i\omega, i\omega) \begin{pmatrix} G(i\omega) \\ 1 \end{pmatrix}$$
$$= \text{Re}[(1 + i\alpha\omega)G(i\omega)] + \frac{1}{k} \geq \delta > 0, \quad \forall \omega \in \mathbb{R}$$

This stability condition is known as *the Popov criterion*. If this condition is satisfied, $\mathfrak{B}$ admits a nonnegative QDF storage function Q_Ψ. Then we obtain the Luré-type Lyapunov function

$$(Q_\Psi + S)(w) = Q_\Psi(w) + \alpha \int_0^y \varphi(s)ds.$$

4.2. Stability with rational supply rate

We now extend the result in the previous subsection to the case where $\Phi(\zeta, \eta)$ is a rational matrix. For this purpose, we assume that $\Phi(\zeta, \eta)$ is factorized as

$$\Phi(\zeta, \eta) = H^\top(\zeta)\Sigma_\Phi H(\eta), \quad \det H \neq 0$$

where $\Sigma_\Phi \in \mathbb{R}^{w\times w}$ is a nonsingular constant matrix of the form of $\Sigma_\Phi = \mathrm{diag}(I_k, -I_l)$, and $H(\xi)$ is a rational matrix in $\mathbb{R}^{w\times w}(\xi)$. We can assume that $H(\xi)$ has all poles in $\mathrm{Re}\xi < 0$. From the viewpoint of classical stability theory, we can say that $H(\xi)$ serves as *a multiplier* for the interconnection.

Let v denote the output of the system with transfer function $H(\xi)$ and input w. Introduce the left coprime factorization of $H(\xi)$ over the polynomial ring as $H(\xi) = D^{-1}(\xi)N(\xi)$, $D, N \in \mathbb{R}^{w\times w}[\xi]$. Then, we obtain

$$D(\frac{d}{dt})v - N(\frac{d}{dt})w = 0.$$

Define an augmented system composed of $\mathcal{B}$ and the dynamics of $H(\xi)$ by

$$\mathcal{B}^{\mathrm{aug}} := \left\{(v, w) \in \mathfrak{C}^\infty(\mathbb{R}, \mathbb{R}^{2w}) \,\middle|\, w \in \mathcal{B},\ D(\tfrac{d}{dt})v = N(\tfrac{d}{dt})w\right\}$$

Clearly, $\mathcal{B}^{\mathrm{aug}}$ belongs to $\mathcal{L}^{2w}$. Note that, for any $w \in \mathcal{B}$, there exists a $v \in \mathfrak{C}^\infty(\mathbb{R}, \mathbb{R}^w)$ satisfying $(v, w) \in \mathcal{B}^{\mathrm{aug}}$. The controllability of $\mathcal{B}^{\mathrm{aug}}$ does not depend on the choice of coprime factors $D(\xi), N(\xi)$, though they are not unique. In the same manner, we can also define the augmented system $\mathcal{G}^{\mathrm{aug}}$ for the general NTV behavior $\mathcal{G}$.

Moreover, the supply rate Q_Φ is redefined as

$$\begin{pmatrix} v \\ w \end{pmatrix}^\top \begin{pmatrix} \Sigma_\Phi & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} v \\ w \end{pmatrix} =: \begin{pmatrix} v \\ w \end{pmatrix}^\top \Theta \begin{pmatrix} v \\ w \end{pmatrix} = Q_\Theta(v, w),$$

where we abuse the notation "Q_Φ" for the rational $\Phi(\zeta, \eta)$.

From the above observation, $\mathcal{B}$ is Φ-dissipative on $\mathbb{R}_-$ iff $\mathcal{B}^{\mathrm{aug}}$ is dissipative on $\mathbb{R}_-$ with respect to the *memoryless* quadratic supply rate $Q_\Theta(v, w)$. If $\mathcal{B}^{\mathrm{aug}}$ is controllable, strict Φ-dissipativity of $\mathcal{B}$ on $\mathbb{R}_-$ is equivalent to the existence of a nonnegative storage function $Q_\Psi(v, w)$ and a constant $\varepsilon > 0$ such that

$$\frac{d}{dt}Q_\Psi(v, w)(t) + \varepsilon\|w(t)\|^2 \le Q_\Theta(v, w)(t)\ \forall (v, w) \in \mathcal{B}^{\mathrm{aug}}, t \in \mathbb{R}.$$

It should be noted that strict Φ-dissipativity of $\mathcal{B}$ on $\mathbb{R}_-$ is weaker than the strict Θ-dissipativity of $\mathcal{B}^{\mathrm{aug}}$ on $\mathbb{R}_-$ due to the absence of the term $\varepsilon\|v(t)\|^2$ from the left-hand side of the above inequality.

Similarly, the general NTV behavior $\mathcal{G}$ is $(-\Phi)$-dissipative on $\mathbb{R}_-$ iff $\mathcal{G}^{\mathrm{aug}}$ is $(-\Theta)$-dissipative on $\mathbb{R}_-$. If $\mathcal{G}^{\mathrm{aug}}$ is controllable, the latter condition is equivalent to the existence of a nonnegative storage function $S(v, w)$ such that

$$\frac{d}{dt}S(v, w)(t) \le -Q_\Theta(v, w)(t)\ \forall (v, w) \in \mathcal{G}^{\mathrm{aug}}, t \in \mathbb{R}.$$

In the same way as the previous section, we immediately obtain the next theorem.

Theorem 2 *Let a rational matrix* $\Phi(\zeta, \eta) = H^\top(\zeta)\Sigma_\Phi H(\eta) \in \mathbb{R}_s^{w\times w}(\zeta, \eta)$, $\det H \neq 0$, *induce the supply rate* Q_Φ. *Suppose that both* $\mathcal{B}^{\mathrm{aug}}$ *and* $\mathcal{G}^{\mathrm{aug}}$ *are controllable. Then,* $\mathcal{B} \cap \mathcal{G}$ *is stable if* $\mathcal{B} \in \mathcal{L}^w_{\mathrm{cont}}$ *is strictly* Φ*-dissipative on* $\mathbb{R}_-$ *and* $\mathcal{G}$ *is* $(-\Phi)$*-dissipative on* $\mathbb{R}_-$.

5. Conclusion

In the behavioral framework, we have derived the stability conditions of the dissipative interconnection of an LTI behavior $\mathcal{B}$ and a general NTV behavior $\mathcal{G}$ by using QDFs both in the cases where the supply rate is induced by a polynomial matrix and a rational transfer matrix. The present results generalize the classical multiplier theory to the behavioral context where there is no *a priori* assumption on the input-output relation.

Acknowledgments

This research is supported by the Belgian Federal Government under the DWTC program Interuniversity Attraction Poles, Phase V, 2002–2006, Dynamical Systems and Control: Computation, Identification and Modelling, by the KUL Concerted Research Action (GOA) MEFISTO–666, and by several grants en projects from IWT-Flanders and the Flemish Fund for Scientific Research.

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