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Vehicle platoons formed through self-regularisation

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Abstract—In this paper we design a novel control strategy for a string of vehicles. The vehicles are coupled at the control level such that each vehicle influences the behavior of another vehicle resulting in self-organization of the vehicle string: through this interaction a cooperative behavior emerges and a platoon is formed. This contrasts with the traditional control where an independent leader vehicle is to be followed by the other vehicles.

The coupling structure under consideration is unidirectional-ring coupling. In the unidirectional ring, each vehicle regulates the distance with its immediate forward neighbor to a given constant, and the first vehicle in the platoon keeps its distance to the last vehicle constant.

The resulting behavior of the system is a platoon of vehicles moving at a constant velocity with constant distances between each pair of consecutive vehicles. Stability properties of the system are established and the concept of string stability of a platoon is discussed and applied to the proposed interconnection.

1. Introduction

In this paper the systems under study are vehicular platoons. Such systems have gained importance over the years, since they might offer a solution to the congestion of highways in urban areas. The objective being a capacity increase of the highway, these intelligent vehicle/highway systems (IVHS) form strings of vehicles (so-called *platoons*) moving at a desired speed with desired distances between the vehicles. Several algorithms controlling a string of vehicles have been proposed in the literature.

Ref. [3], [4] and [8] rank among the first to investigate this problem and used an LQR approach. Contrary to [3] and [4], most control strategies use tuning of parameters in order to optimize some proposed controller. In most cases the control is of leader-follower type: the leading vehicle of the platoon moves at a desired speed; the other vehicles receive information from the leading vehicle (position, velocity, acceleration) either directly or indirectly through other vehicles in the platoon. Flow of information is usually directed from the head of the platoon towards its tail [6], [1]. In reference [2] one considers a platoon where each vehicle only measures its distance with its immediate forward neighbor and tries to obtain and maintain a desired value for this distance. The leader vehicle drives at a desired speed.

The present paper presents a novel interconnection topology using identical controllers. As in [2], only separation distances are measured. The key property of the interconnection is the absence of a master/leader vehicle that determines the overall behavior of the platoon. All vehicles interact with each other trying to satisfy their individual control objective. As a result of this cooperation a platoon formation emerges. This resulting behavior and its stability properties are investigated.

An important concept regarding the formation of vehicle platoons is *string stability*. A platoon is called string stable if the transient error in the separation distance between vehicles does not grow as one proceeds down the line of vehicles [7]. It is proved in [2] that the system cannot be string stable when identical controllers are applied. Simulations suggest that the control discussed in the present paper does yield a string stable system in practice.

2. System dynamics

2.1. System equations and equilibrium solution

Each vehicle is represented as a moving mass with second order dynamics:

$$\ddot{x}_i + p\dot{x}_i = u_i, \quad i \in \mathcal{N}, \tag{1}$$

where x_i represents the position of the i-th vehicle, u_i is the input to the i-th vehicle and $p \geq 0$ is a parameter representing the friction/drag coefficient per unit mass. The mass of each vehicle is taken equal to one. We propose the following control:

$$u_i = \omega_i + K(x_{i-1} - x_i - L_i), \quad i \in \mathcal{N}, \tag{2}$$

with $K > 0$ the coupling strength and $\omega_i > 0$, $L_1 \leq 0$, $L_i \geq 0$, $i = 2, \dots, N$, real constants. Each vehicle attempts to keep the distance between itself and its immediate forward vehicle as close as possible to the set point L_i. The lead vehicle tries to obtain a desired distance $|L_1|$ between itself and the last vehicle of the platoon. At the same time, each vehicle aims to drive at an imposed reference speed $v_i \triangleq \omega_i/p$.

Theorem 1 *Each function $\varphi : \mathbb{R} \to \mathbb{R}^N; t \mapsto \varphi(t)$ defined by*

$$\varphi_i(t) = \alpha t + \beta_i, \quad \forall i \in \mathcal{N}, \tag{3}$$

where

$$\alpha = \frac{\omega_m - KL_m}{p}, \quad (4)$$

$$\beta_i - \beta_{i-1} = \frac{\omega_i - \omega_m}{K} + L_m - L_i, \quad (5)$$

with $\omega_m \triangleq \frac{1}{N}\sum_{j=1}^{N}\omega_j$ *and* $L_m \triangleq \frac{1}{N}\sum_{j=1}^{N} L_j$*, is a solution of system (1)-(2).*

Assume each vehicle is given a reference speed close to a value v_0. In the ideal case each vehicle would receive a reference speed perfectly equal to v_0 rendering the coupling structure redundant. In a practical situation however, the reference speeds will differ slightly from each other. If the vehicles remain uncoupled in this situation, the group of vehicles becomes dispersed. The coupling structure's aim is to keep the vehicles moving together in a highly structured way through self-organization. Each vehicle abandons its goal to drive at the set speed v_i in order to fall into step with its forward and backward neighbors which move at a speed different from v_i. The more the values v_i differ from each other, the larger the coupling strength has to be to avoid collisions between vehicles when forming a coherent platoon.

The coupling also allows control of the speed of the platoon by varying the set points L_i: by deliberately choosing the set points L_i such that their mean value is different from zero, the string of vehicles starts to move at a constant velocity which depends on this mean value and the mean reference speed. Once the set points have been fixed, the self-regulatory property of the system ensures the resulting motion is a platoon moving at the desired velocity. This contrasts with the supervisory type of control of classical look-ahead interconnections where a driver controls the lead vehicle and the consecutive vehicles try to follow.

2.2. Remarks

The function $t \mapsto \varphi_i(t)$ represents the evolution of the position of the i-th vehicle. Each solution φ represents a string of vehicles moving at a constant velocity given by (4) with distances between consecutive vehicles defined by (5). The system equations are invariant under the change of coordinates

$$x \to x + \gamma\begin{bmatrix}1 & 0 & 1 & 0 & \cdots & 1 & 0\end{bmatrix}^T, \quad \forall\gamma \in \mathbb{R}. \quad (6)$$

In other words, the dynamics are invariant under translations of the origin in the physical space. This invariance is reflected in the spectrum of the system matrix A describing the system (1)-(2): the matrix A possesses at least one zero-eigenvalue, independent of the parameter values.

The solutions of (1)-(2) have two undesirable properties. First, the separation distances between consecutive vehicles do not converge to the set points L_i. However, by (5) it is possible to compute the distances which the platoon converges to. Vice versa, if desired values $\delta_i \triangleq \beta_{i-1} - \beta_i$ for the separation distances are given, equation (5) allows us to calculate the necessary set points L_i. The second disadvantage of system (1)-(2) is that the equilibrium solutions are only stable for sufficiently small coupling strengths (see Section 3). This results in a very slow decay of the transient behavior.

3. Stability analysis

In this section the stability of the equilibrium solution as a function of the coupling strength is investigated. In order to establish the stability properties of (3), the following change of coordinates is performed:

$$x_i = \alpha t + \beta_i + z_i,$$

where α and β_i are defined by (4) and (5). This results in a set of system equations of the form

$$\dot{z} = Az. \quad (7)$$

It can be shown that the system (7) possesses the same translation invariance (6) as the original system. Asymptotic stability of the system is determined by the location of the eigenvalues of A, which leads to the following theorem.

Theorem 2 *If and only if*

$$K < \frac{p^2}{2\cos^2(\pi/N)} \quad (8)$$

the system (1)-(2) is asymptotically stable.

If the number of vehicles tends to infinity, the upper bound on K for stability determined by (8) decreases and converges to the value $p^2/2$. This yields a sufficient condition for asymptotic stability.

Theorem 3 *If* $0 < K < p^2/2$*, system (1)-(2) is asymptotically stable, irrespective of the number of vehicles in the system.*

Example:

Consider system (1)-(2) with 3 vehicles and drag coefficient $p = 2$. The eigenvalues of the corresponding system matrix A are plotted in Figure 1 as a function of K. When the vehicles are uncoupled, three eigenvalues are located at $-p$; the remaining three eigenvalues are located at the origin. When K increases two of the latter eigenvalues move into the open left half plane while two of the eigenvalues located in $-p$ start to move towards the imaginary axis. The sum of all eigenvalues is $-3p$, irrespective of the value K. For all values $K > 0$ there is one eigenvalue at the origin and one in $-p$. When the coupling strength exceeds the value $p^2/4$ the two rightmost eigenvalues different from zero start to move towards the imaginary axis until at $K = 2p^2$ they cross the imaginary axis simultaneously, rendering the system unstable.

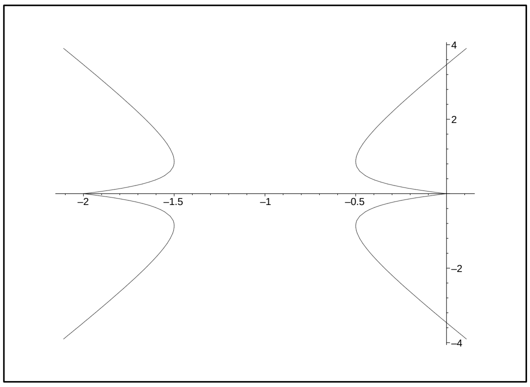

Figure 1: The spectrum of the system consisting of three vehicles

4. String stability

In this section *string stability* is discussed with respect to the proposed interconnection structure. For simplicity, it is assumed that $\omega_i = 0, \forall i \in \mathcal{N}$. For a general treatment of the concept of string stability the reader is referred to [7], where infinite interconnections of a class of nonlinear systems are considered. In practice, however, a vehicle platoon always consists of a finite number of vehicles. A string of vehicles is called string stable if disturbances are attenuated as they propagate down the string [5].

Definition 1 *Let e_i be the distance error between the i-th and $(i-1)$-th vehicle: $e_i(t) = x_i(t) - x_{i-1}(t) - L_m + L_i$. The platoon is called string stable if*

$$\|e_i(t)\|_\infty < \|e_{i-1}(t)\|_\infty, \quad \forall i > 1,$$

where $\|e_i(t)\|_\infty$ denotes $\sup_{t\geq 0} |e_i(t)|$.

The initial values of the distance errors satisfy some restrictions:

- At first, from the definition of e_i it follows that

$$\sum_{j=1}^{N} e_j(t) = 0, \quad \forall t \in \mathbb{R}.$$

- Secondly, each equilibrium solution of (6) has the property that $x_{i-1}(t) > x_i(t),\ i \neq 1$. In order to avoid collisions, only initial conditions are considered with vehicle positions satisfying $x_{i-1}(0) > x_i(0),\ i \neq 1$.

- Thirdly, we impose the extra assumption that changing the values L_i is applied to a platoon driving at constant velocity, implying $\dot{e}_i(0) = 0, \forall i \in \mathcal{N}$. By adjusting the parameters L_i the platoon is steered from one equilibrium solution to another. The manoeuvres included are speeding up, slowing down and starting from a standstill.

Assume that for time $t < 0$ the platoon is at an equilibrium solution, i.e. $x_i(t) - x_{i-1}(t) = \left(\frac{1}{N}\sum_{j=1}^{N} L_j\right) - L_i,\ \forall i \in \mathcal{N},\ \forall t < 0$. At $t = 0$ the set point L_1 is replaced by $\tilde{L}_1$. For positive time the platoon behavior can be described by

$$\ddot{e}_i + p\dot{e}_i = K(e_{i-1} - e_i), \quad \forall i \in \mathcal{N}. \tag{9}$$

with

$$e_i \triangleq x_i - x_{i-1} - \left(\frac{1}{N}\sum_{j=1}^{N} \tilde{L}_j\right) + \tilde{L}_i, \quad i \in \mathcal{N},$$

where $\tilde{L}_i = L_i, \forall i \in \mathcal{N} \setminus \{1\}$. The corresponding initial condition is given by

$$\begin{cases} e_1(0) = \dfrac{N-1}{N}(\tilde{L}_1 - L_1), \\ e_i(0) = \dfrac{L_1 - \tilde{L}_1}{N}, \quad \forall i \in \mathcal{N} \setminus \{1\}, \\ \dot{e}_i(0) = 0,\ \forall i \in \mathcal{N}. \end{cases}$$

In Figure 2, system (1)-(2) is simulated with $N = 39$, $L_1 - \tilde{L}_1 = -5$, $p = 10$, $K = 10$. This corresponds to a slowing down manoeuvre. The figure presents the evolution of the distance errors $e_i(t)$ over time. For reasons of clarity of the picture, half of the distance errors, namely those with even index, are omitted from the picture. The figure suggests that the maximum distance error between pairs of consecutive vehicles does not grow when proceeding towards the tail of the platoon. This is made explicit by the separate plots of Figure 3, where the first 4 separation distance errors are displayed.

Figure 4 shows the distance error e_4 over a longer time period compared to Figure 3 and illustrates a typical feature of the interconnection topology: each distance error rises quickly to its maximum value and then decreases to a value close to zero, but, contrary to leader follower control, after some time each distance error starts to rise *again*. It decreases again to some value near zero. This rising and decreasing is repeated periodically over time. As time evolves, the time it takes for an error to rise and fall down again increases, while the peak value decreases. One could interpret this as if there was a Mexican wave in the error value moving around in the platoon: when the wave reaches the tail of the platoon, it reappears at the leader vehicle. Notice that the Mexican wave continually decreases in amplitude while moving around in the platoon.

5. Robustness

Assume that one of the vehicles starts to malfunction and cannot reach the velocity required by the platoon at that moment. In the case of leader-follower control this causes the leading group of vehicles to abandon the group with the malfunctioning vehicle as first vehicle, and therefore a splitting of the platoon. The distance between both groups increases without bound.

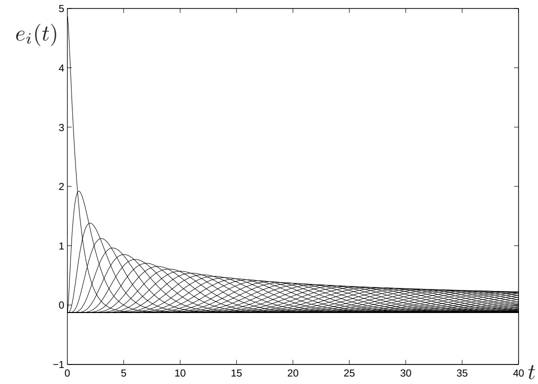

Figure 2: Evolution of the separation distance errors for a platoon of 39 vehicles.

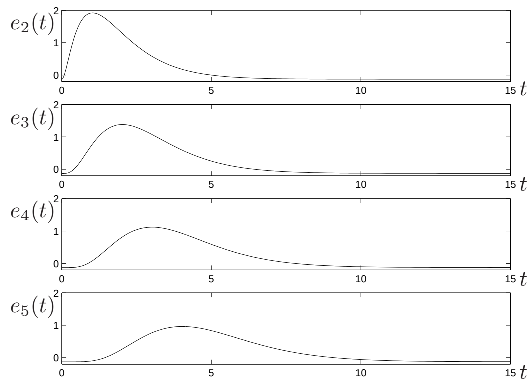

Figure 3: Separation distance errors of the first 5 vehicles.

With the interconnection topology of the present paper all vehicles adapt to the “weakest link” and the platoon starts to drive at the maximum velocity feasible by the malfunctioning vehicle. This is illustrated on the right handside plot of Figure 5: at $t = 80$ s the speed of one of the vehicles becomes bounded by 0.3 m/s. The distance between the first and the second group remains bounded. There is a splitting of the platoon but no abandoning. The left handside plot shows the evolution of the platoon without malfunctions. For reasons of clarity, only the positions of the vehicles with an odd index are plotted.

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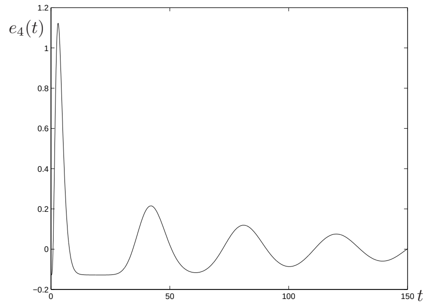

Figure 4: Separation distance error $e_4(t)$.

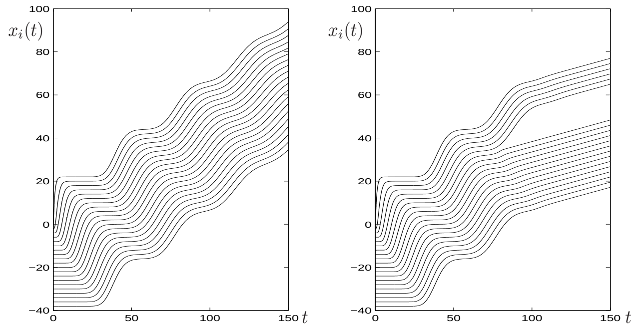

Figure 5: Evolution of the position for a platoon of 39 vehicles. Left handside figure: no malfunctions. Right handside figure: at $t = 80$ s, the 12th vehicle starts malfunctioning and cannot drive faster than 0.3 m/s

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