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Bifurcation in Injection-locked Class-E_M Oscillator

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Abstract—The main purpose of this paper is to understand how the bifurcation phenomena affect the circuit dynamics and to identify the circuit's stable period-1 oscillation parameter based on the bifurcation analysis in the injection-locked class-E_M oscillator. First, we explain the circuit model and its dynamics of the injection-locked class-E_M oscillator. Then, the dynamic behavior of the circuit is investigated based on the bifurcation diagrams. Finally, we show the power-conversion efficiency are verified by the numerical results.

1. Introduction

Nonlinear dynamics in the power electronic circuits have been analyzed since decades ago. In particular, bifurcation phenomena in the simple class of power electronic circuits have been rigorously studied both of mathematical and experimental points of view [1–3]. It is important to analyze dynamic behaviors of the power electronic circuits via the bifurcation theory; analytical results contribute to understand the circuit's stable period-1 oscillation parameter.

The class-E_M oscillator [4–6] is an improved version of the class-E oscillator [7, 8], which satisfies zero-voltage switching (ZVS), zero-voltage-derivative switching (ZVDS), zero-current switching (ZCS), and zero-current-derivative switching (ZCDS). Therefore, it achieves high power-conversion efficiency at high frequencies. The previous works for the class-E_M oscillator focused on circuit designs for achieving the class-E_M ZVS/ZVDS/ZCS/ZCDS conditions, e.g., the circuit parameters for nominal conditions are obtained numerically in [5]. At the nominal conditions, jumpless switch voltage and current waveforms and the maximum power-conversion efficiency are obtained. Therefore, it is important to derive the circuit parameters at the nominal conditions. In real circuits, however, there are component tolerances, which make the class-E_M oscillator behave outside the nominal conditions. What happen in the circuit dynamics outside of the nominal conditions? As the first step to study the circuit dynamics, we have investigated the dynamic behavior of the injection-locked class-E_M oscillator outside the nominal conditions [6]. It was confirmed that the non-periodic solution is observed outside the nominal conditions in [6]. Therefore, it can be predicted that the bifurcation phenomena may occur in the circuit dynamics and the stable period-1 solution changes to the non-periodic solution against the parameter variations. The detailed analysis for that, however, has not been reported at all. Research interest is to understand how the bifurcation phenomena affect the circuit dynamics and to identify the circuit's stable period-1 oscillation parameters based on the bifurcation analysis.

In this paper, we investigate the bifurcation phenomena in the injection-locked class-E_M oscillator, which is operated outside the nominal conditions. First, we explain the circuit model and its dynamics. Then, the dynamic behavior of the injection-locked class-E_M oscillator is investigated based on the bifurcation diagrams. In particular, it is seen from the bifurcation diagram that the injection-locked class-E_M oscillator generates the stable one-periodic output in the range of −0.63% to 1.62% from the nominal conditions. Finally, we focus on the power-conversion efficiency from the numerical investigation.

2. Circuit dynamics

2.1. Class-E_M oscillator

Figure 1 shows the injection-locked class-E_M oscillator, which consists of the class-E free-running oscillator (main circuit) and the injection circuit. The main circuit has dc-supply voltage source V_{DD1}, dc-feed inductor L_{C1}, switch element S_1, shunt capacitance C_{S1} and L_1-C_1-R series-resonant filter. The injection circuit has dc-supply voltage source V_{DD2}, dc-feed inductor L_{C2}, switch element S_2, shunt capacitance C_{S2} and L_2-C_2 series-resonant filter. The

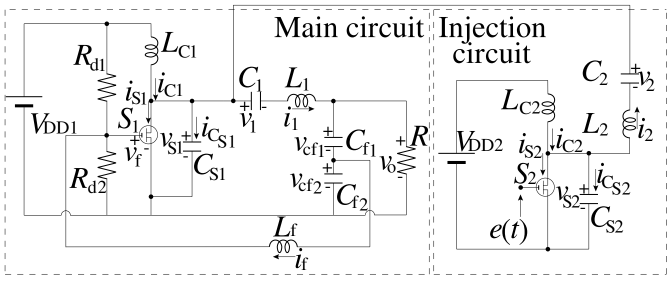
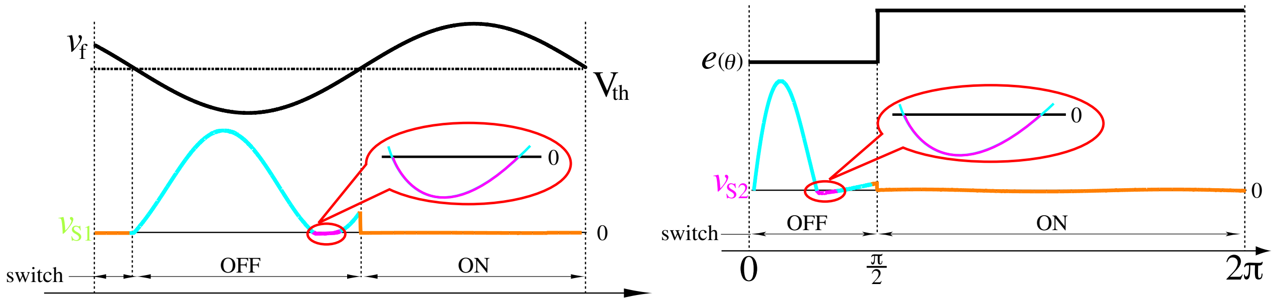


Figure 1: Circuit model.



(a) Switch S_1 (b) Switch S_2

Figure 2: Switching rule.

main circuit is driven by the feedback voltage v_f. On the other hand, the injection circuit is driven by the input signal $e(\theta)$. By injecting the current with the second-harmonic frequency component, the switch voltage and current of the main circuit achieves ZVS/ZVDS/ZCS/ZCDS conditions [4, 5], which are called nominal conditions. When we use the class-E frequency doubler as the injection circuit, the output voltage is locked by the input voltage of the injection circuit. In [5], only the design method and circuit operation at the nominal conditions were discussed. It is, however, quite important to comprehend the circuit behaviors outside the nominal conditions, for example, locking range of the injection-locked class-E_M oscillator. For this purpose, we believe that the bifurcation theory is one of the strongest tools. This is because the unstable oscillation may occur via bifurcation from the stable oscillation. In other words, the stable region is obtained when the bifurcation diagrams can be illustrated.

2.2. Parameters and Circuit Equations

First, the parameters are defined as follows.

1. $f = \omega/2\pi$: The operating frequency of the main circuit.
2. $f_k = 1/2\pi\sqrt{L_k C_k}$: The resonant frequency.
3. $f_f = 1/2\pi\sqrt{L_f C_g}$: The resonant frequency in the feedback network.
4. $A_k = f_k/f$: The ratio of the resonant frequency. to the resonant frequency.
5. $B_k = C_k/C_{Sk}$: The ratio of the resonant capacitance to the shunt capacitance.
6. $H_k = L_k/L_{Ck}$: The ratio of the resonant inductance to the dc-feed inductance.
7. $J = C_1/C_{f1}$: The ratio of the resonant capacitance in the main circuit to the feedback network capacitor.
8. $K = C_{f1}/C_{f2}$: The ratio of the capacitance between two capacitors in the feedback network.
9. $M = f_f/f$: The ratio of the resonant frequency in the feedback network to the operating frequency.
10. $Q_k = \omega L_k/(R + r_{Lk})$: The loaded quality factor.
11. $Q_f = \omega L_f/r_g = 1/\omega M^2 C_g r_g$: The loaded quality factor in the feedback network.
12. $\alpha = V_{DD2}/V_{DD1}$: The ratio of the dc-supply voltages.
13. r_{L_k}: The ESR of L_k.
14. C_g: The equivalent gate-to-source capacitance.
15. r_g: The equivalent gate-to-source resistance.

The subscript $k = 1$ and 2 mean the main and injection circuits, respectively. The circuit equations are expressed as follows:

$$\left\{\begin{array}{lcl}
\frac{R}{V_{DD1}}\frac{di_{c1}}{d\theta} & = & \frac{H_1}{Q_1}(1 - \frac{v_{S1}}{V_{DD1}}) \\
\frac{R}{V_{DD1}}\frac{di_1}{d\theta} & = & \frac{1}{Q_1}\frac{(v_{S1} - v_1 - v_{cf1} - v_{cf2} - r_{L1}i_1)}{V_{DD1}} \\
\frac{1}{V_{DD1}}\frac{dv_{S1}}{d\theta} & = & \frac{A_1^2 B_1 Q_1 R}{V_{DD1}}(\frac{i_{C1}}{R} - \frac{v_{S1}}{R_{S1}} - \frac{i_1}{R} + \frac{i_2}{R}) \\
\frac{1}{V_{DD1}}\frac{dv_1}{d\theta} & = & \frac{A_1^2 Q_1}{V_{DD1}} i_1 \\
\frac{R}{V_{DD1}}\frac{di_{C2}}{d\theta} & = & \frac{2H_2}{Q_2}(\alpha - \frac{v_{S2}}{V_{DD1}}) \\
\frac{R}{V_{DD1}}\frac{di_2}{d\theta} & = & \frac{2}{Q_2}\frac{(v_{S2} - v_2 - v_{S1} - r_{L2}i_2)}{V_{DD1}} \\
\frac{1}{V_{DD1}}\frac{dv_{S2}}{d\theta} & = & \frac{A_2^2 B_2 Q_2 R}{2V_{DD1}}(\frac{i_{C2}}{R} - \frac{v_{S2}}{R_{S2}} - \frac{i_2}{R}) \\
\frac{1}{V_{DD1}}\frac{dv_2}{d\theta} & = & \frac{A_2^2 Q_2 R}{2V_{DD1}} i_2 \\
\frac{R}{V_{DD1}}\frac{di_f}{d\theta} & = & \frac{R}{Q_f r_g}\frac{(v_{cf2} - v_g - r_{Lf}i_f - r_g\omega C_g \frac{dv_g}{d\theta})}{V_{DD1}} \\
\frac{1}{V_{DD1}}\frac{dv_{cf1}}{d\theta} & = & \frac{A_1^2 J Q_1 R}{V_{DD1}}\left(i_1 - \frac{v_{cf1} + v_{cf2}}{R}\right) \\
\frac{1}{V_{DD1}}\frac{dv_{cf2}}{d\theta} & = & \frac{A_1^2 J K Q_1}{V_{DD1}}\left(i_1 - \frac{v_{cf1} + v_{cf2}}{R} - i_f\right) \\
\frac{1}{V_{DD1}}\frac{dv_g}{d\theta} & = & \frac{M^2 Q_f r_g}{V_{DD1}}\left(i_f + \frac{V_{DD1} - v_f}{R_{d1}} - \frac{v_f}{R_{d2}}\right) \\
v_f & = & v_g + r_g\omega C_g \frac{dv_g}{d\theta}.
\end{array}\right. , \quad (1)$$

In this paper, the parameters f and R varies in order to investigate the circuit behaviors outside the nominal conditions. This is because the operating frequency f has sensitivity to the circuit behaviors, which was pointed out in [5].

2.3. Switching Rules

The injection-locked class-E_M oscillator has two switches S_1 and S_2. The state of the switch S_1 depends on the feedback voltage and the drain-to-source voltage. When $v_f > 0$, the switch in ON. In case of $v_f \le 0$, the switch is OFF for $v_{S1} > 0$. However, the MOSFET body-diode turns ON when $v_{S1} \le 0$ and $v_f \le 0$. Therefore, we have

$$R_{S1} = \begin{cases} \infty & \text{for} \quad v_{S1} > 0 \text{ and } v_f < V_{th}, \\ r_d & \text{for} \quad v_{S1} < 0 \text{ and } v_f < V_{th}, \\ r_{S1} & \text{for} \quad v_f > V_{th}, \end{cases} \qquad (2)$$

Similarly, the switch S_2 depends on the input voltage and the voltage across the switch. In this paper, we assume that switch-off duty ratio of the input signal is 0.25 and the switch S_2 if OFF during $0 < \theta \le \pi/2$. Therefore,

$$R_{S2} = \begin{cases} \infty & \text{for} \quad v_{S2} > 0 \text{ and } 2n\pi < \theta \le 2n\pi + \frac{\pi}{2}, \\ r_d & \text{for} \quad v_{S2} < 0 \text{ and } 2n\pi < \theta \le 2n\pi + \frac{\pi}{2}, \\ r_{S2} & \text{for} \quad 2n\pi + \frac{\pi}{2} < \theta \le 2(n+1)\pi. \end{cases} \qquad (3)$$

$$(n = 0, 1, 2, 3....)$$

Figure 2 shows the summary of the switching rule. In this figure, blue, orange, and red lines mean that the switch is OFF, switch is ON, and MOSFET body-diode is ON, respectively.

3. Analytical Results

3.1. Numerical Results

For investigating the bifurcation phenomena in the injection-locked class-E_M oscillator, the following specifications are given: $Q_1 = Q_f = 10$, $C_g = 760$ [pF], $r_g = 0.438[\Omega]$, $H_1 = 0.035$, $H_2 = 0.047$ for the nominal conditions. Figure 3 shows an example of the one-parameter bifurcation diagram as a function of f/f_{nom}, where f_{nom} is the operating frequency for the nominal operation. The waveform is mapped by every period of 2π. The bifurcation parameter f/f_{nom} is changed from $f/f_{nom} = 0.98$ to $f/f_{nom} = 1.03$. We can observe the period-1 solution around the nominal conditions. However, if the parameter is changed, the period-1 solution bifurcates to the non-periodic solution. In the following analysis, we consider what happen in the circuit dynamics when the parameter f is changed from the nominal conditions. Figure 4 shows an example of the waveforms. It can be stated that the circuit dynamics satisfies ZVS and ZVDS characteristics and the oscillator achieves minimum amount of the switching loss at the nominal conditions (see Fig. 4 (a)). On the other hand, if the parameter f is slightly changed from the nominal condition, several switching patterns appear. For example, the switch S_1 has a little switch-voltage jump and the switch S_2 achieve only the ZVS condition. In Figure 4 (b) both the S_1 and S_2 do not achieve the ZVS condition. Furthermore, the stable period-1 solution bifurcates to the non-periodic solution if the parameter f/f_{nom} is changed away from the nominal conditions, and bigger switch-voltage jump appears as shown in Figure 4 (c), which causes large switching power losses. Therefore, we conclude that if the circuit parameters are changed away from the nominal conditions, the bifurcation phenomena occur and the switching losses appear. It is seen from the bifurcation diagram that the injection-locked class-E_M oscillator generates the stable one-periodic output in the range of −0.63% to 1.62% from the nominal conditions. Figure 5 shows an example of the two-parameter bifurcation diagram at f/f_{nom}-R/R_{nom} plane. The black and white regions denote the existence region of the period-1 solution and the non-periodic solution, respectively. It can be stated that the bifurcation phenomena occur and the stable period-1 solution bifurcates to the non-periodic solution. It is seen from Figure 5 that the one-periodic-solution region is narrow for the frequency variations. It is, however, wide for the load resistor variation. From these results, it is confirmed that the circuit behavior is very sensitive to the input-signal frequency of the injection circuit.

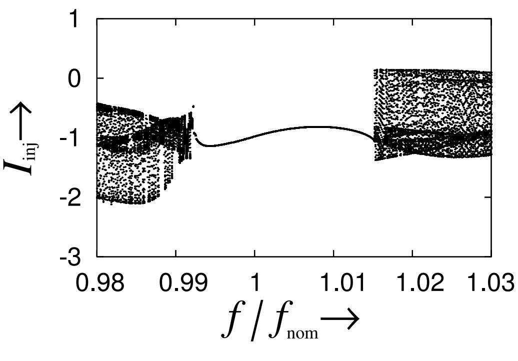


Figure 3: One-parameter bifurcation diagram ($R = 1$).

3.2. Power-Conversion Efficiency

Table I gives the element values for the nominal conditions. Figure 6 shows the numerical result of the power-conversion efficiency η diagram. The frequency f is changed from $f = 3.4$ [MHz] to $f = 3.65$ [MHz]. We can observe that the nominal conditions give the highest power-conversion efficiency, which can be confirmed by numeri-

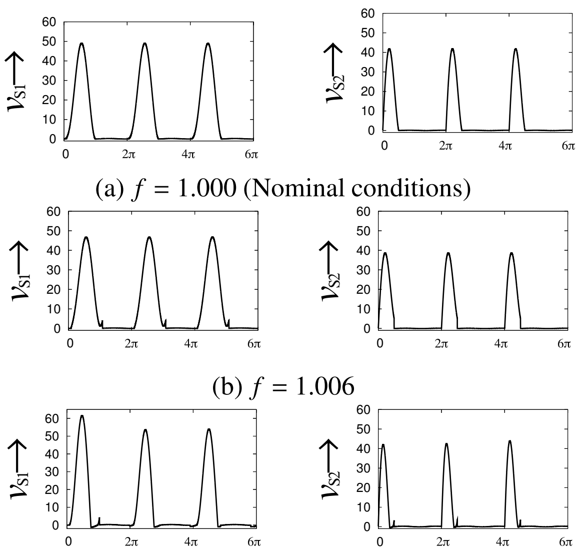
Table 1: parameters.

Parameter	Calculated	Parameter	Calculated
V_{th}	3.0V	C_{S2}	670pF
r_S	0.07Ω	C_2	254pF
C_g	760pF	L_f	2.93μH
r_g	438mΩ	C_{f1}	337pF
V_{fFET}	20V	C_{f2}	16.5nF
L_{C1}	172μH	r_{L1}	208mΩ
L_1	6.14μH	r_{LC1}	13.3mΩ
C_{S1}	1170pF	V_{DD1}	11.5V
C_1	337pF	V_{DD2}	6.28V
L_{C2}	49.3μH	R_{d1}	850kΩ
L_2	2.35μH	R_{d2}	300kΩ
f_{nom}	3.5MHz	R	13.5Ω

cal investigation. On the other hand, if the parameter f is changed away from the nominal condition, the power-conversion efficiency is decreased. In particular, it is very interesting that the power-conversion efficiency is suddenly decreased around $f = 3.46$[MHz] and $f = 3.57$[MHz]. We conclude that the power-conversion efficiency is suddenly decreased, due to the bifurcation phenomena if the circuit parameter is changed away from the nominal conditions. In addition, it might be said that if the circuit behaves as the stable period-1 solution, it is no problem to use the injection-locked class-E_M oscillator outside the nominal conditions, because the power-conversion efficiency is almost same as at the nominal conditions.

4. Conclusion

In this paper, we have investigated the bifurcation phenomena in the injection-locked class-E_M oscillator, which is operated outside the nominal conditions. First, we explained the circuit model and its dynamics. Then, the circuit equation was normalized by using the dimensionless values. Next, the dynamic behavior of the circuit was investigated based on the bifurcation diagrams. Finally, we showed an example of the application of the normalized results. As results, it is seen from the numerical investigation results that (i) the injection-locked class-E_M oscillator generates the stable one-periodic output in the range of −0.63% to 1.62% from the nominal conditions, (ii) the power-conversion efficiency is suddenly decreased effected by the bifurcation phenomena if the circuit parameter is changed away from the nominal conditions, (iii) if the circuit behaves as the stable period-1 solution, it might be no problem to use the injection-locked class-E_M oscillator outside of the nominal conditions, because the power-conversion efficiency is almost same as at the nominal conditions. Stability analysis of the stable period-1 solution is future work.



(a) $f = 1.000$ (Nominal conditions)

(b) $f = 1.006$

(c) $f = 0.971$

Figure 4: Waveforms of the numerical result.

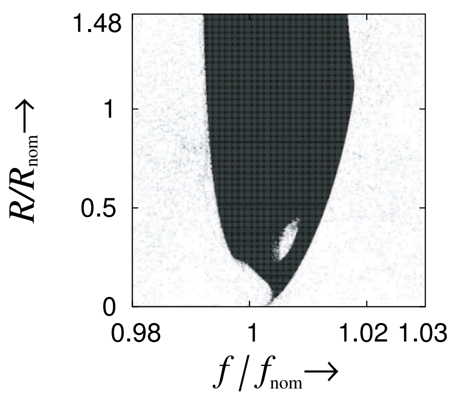


Figure 5: Two-parameter bifurcation diagram in f - R plane.

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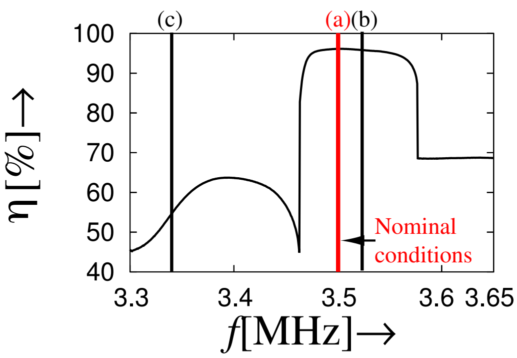

Figure 6: Power-Conversion efficiency of numerical result.