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Phase-Model Analysis of Supply Stability in Power Grid of Eastern Japan

Motoki Nagata†, Isao Nishikawa‡, Naoya Fujiwara$,‡, Gouhei Tanaka‡,$$,
Hideyuki Suzuki†,‡,$$, and Kazuyuki Aihara†,‡

†Graduate School of Information Science and Technology, The University of Tokyo
Hongo, Bunkyo, Tokyo 113–8656, Japan
‡Institute of Industrial Science, The University of Tokyo, Komaba, Meguro, Tokyo 153-8505, Japan
$ FIRST, Aihara Innovative Mathematical Modelling Project, Japan Science and Technology Agency
Komaba, Meguro, Tokyo 153-8505, Japan
$$ CREST, JST, Honcho, Kawaguchi, Saitama 332-0012, Japan

Abstract—It is becoming very important to ensure a stable supply of energy because of the recent developments in renewable and decentralized energy. In order to stabilize the supply of power grids, it is useful to analyze mathematical models. In the field of nonlinear science, power grids have been described by phase models and the minimum coupling strength for global frequency synchronization has been used as a measure of stability in power grids. Such studies regard frequency synchronization as the hallmark of a stable supply. In fact, however, if the coupling strength is near the synchronization transition point, the supply of energy is sometimes destabilized. Therefore, a new stability criterion is required. In this paper, we take into account the fact that the phase difference between generators and substations in a model of real power grids is very small. We define a modified minimum coupling strength (MMCS) and calculate it by using the topology of the power grid in eastern Japan. Further, we identify important links for a stable supply by the MMCS.

1. Introduction

In order to reduce CO_2 emissions, renewable and decentralized energy such as wind and photovoltaic (PV) energy plays an essential role. Particularly in Japan, the movement for denuclearization has received a fresh impetus after the 2011 Tohoku earthquake and tsunami. Therefore, renewable and decentralized energy is becoming increasingly important.

The supply of such energy is more unstable than that of nuclear/thermal energy. Hence, it is very important to ensure a stable supply of energy. In the field of electrical power engineering, the balance between demand and supply has been analyzed [1]. Since frequency synchronization of voltage is necessary for a stable energy supply, various mathematical models have been used to analyze the frequency stability of generators [2].

However, such approaches are too complicated to analyze the global stability. Therefore, in the field of nonlinear sciences, simple models that extract crucial properties have been adopted. In early studies, various aspects of power grids were investigated [3–5]. Among others, the Kuramoto-like phase oscillator model as an explanation of power grids is a breakthrough [6–9]. In Refs. [7–9], the minimum coupling strength for global frequency synchronization was used as a measure of the power grids' stability. Recently, the robustness of power grids against perturbation was analyzed, using the Kuramoto-like model [10].

Such studies assumed that a stable supply is achieved if frequency synchronization is globally stable. In fact, however, if the coupling strength is near the synchronization transition point, the supply of energy is sometimes destabilized, because the phase difference between power grids and substations is too large for power grids to supply electricity stably. Since this destabilization mechanism cannot be captured by the conventional framework based on the synchronization transition, a new stability criterion is required.

In this paper, we take into account the fact that the phase difference between generators and substations in real power grids is very small. We define a modified minimum coupling strength (MMCS) and calculate it by using the topology of the power grid in eastern Japan. Further, we identify important links for a stable supply by the MMCS.

The remainder of this paper is organized as follows. First, in Section 2, we introduce a phenomenological phase model for power grids. Section 3 describes the model settings of the power grid in eastern Japan that we replicated. Then, in Section 4, we clarify that frequency synchronization is not sufficient for a stable supply and introduce the MMCS. Section 5 is devoted to showing the influence of removing links and identifying important links for stable supply. The conclusions are summerized in Section 6.

2. Phase Model of Power Grids

It is necessary to regulate voltage and electrical power in power grids. One of the crucial dynamical properties of power grids is their frequency synchronization of voltage. If the phase difference among nodes is constant, that is, their frequency is synchronized, the power flow from one node to another is constant. Normally, the frequency of the voltage is constant. The amplitude of the voltage can be considered to be the same in any place while keeping track of the power flow [6].

We denote the phase of voltage by θ and regard the generators and substations as phase oscillators. The thermal or mechanical power generated in a generator, P_s, is divided into the following three components:

- Dissipated power owing to the rotation of the turbines, $P_d = \gamma\dot{\theta}^2$ (γ: the damping coefficient)
- Accumulated kinetic energy per unit time, $P_a = I\dot{\theta}\ddot{\theta}$ (I: the inertia moment)
- Transmitted power from element i to j, $P_t = -P_{ij}^{MAX}\sin(\theta_j - \theta_i)$

If element i to j is connected, we set $P_{ij}^{MAX} = P^{MAX}$. If element i to j is not connected, we set $P_{ij}^{MAX} = 0$. From the conservation law of energy [6, 8],

$$P_s = P_d + P_a + P_t, \tag{1}$$

we can represent their behavior with swing equations. In element i, Eq. (1) is rewritten as

$$P_i = \gamma_i\dot{\theta}_i^2 + I_i\dot{\theta}_i\ddot{\theta}_i - \sum_j P_{ij}^{MAX}\sin(\theta_j - \theta_i), \tag{2}$$

If element i is a substation, P_i is negative. Turbines produce electrical power with a frequency that is close to the standard frequency Ω (=50/60 Hz). Therefore, if we write

$$\theta_i = \Omega t + \phi_i, \tag{3}$$

we can assume $\dot{\phi}_i \ll \Omega$. By inserting Eq. (3) into Eq. (2) and using this assumption, we obtain

$$\dot{\phi}_i = \left[\frac{P_i}{2\gamma_i\Omega} - \frac{\Omega}{2}\right] - \frac{I_i}{2\gamma_i}\ddot{\phi}_i^2 + \frac{1}{2\gamma_i\Omega}\sum_j P_{ij}^{MAX}\sin(\phi_j - \phi_i). \tag{4}$$

We assume γ_i and I_i to be the same for all nodes and denote them by γ and I, respectively. By variable transformation, we obtain

$$\dot{\phi}_i = \omega_i - \alpha\ddot{\phi}_i + \sigma\sum_j a_{ij}\sin(\phi_j - \phi_i), \tag{5}$$

where a_{ij} is an element of the adjacency matrix, i.e., $a_{ij} = 1$ iff elements i and j are connected; otherwise, $a_{ij} = 0$. Here, $\omega_i = \frac{P_i}{2\gamma\Omega} - \frac{\Omega}{2}$ is the natural frequency of element i, and $\sigma = \frac{P^{MAX}}{2\gamma\Omega}$ is the coupling strength.

3. Model Settings of the Power Grid in Eastern Japan

We replicated the topology of the power grid from maps published by Tokyo Electric Power Company Inc.[1] and Tohoku Electric Power Company Inc.[2]. Figure 1 shows the topology. It was produced with the Pajek software.[3] We regard generators and substations as nodes of the graph structure. We define N^+, N^0, N^-, and $N = N^+ + N^0 + N^-$ as the number of generators, branch points of power lines, substations, and nodes, respectively. In the eastern Japan network, $N^+ = 47$, $N^0 = 67$, $N^- = 120$ and thus $N = 234$. We normalize the natural frequency ω_i as follows:

$$\omega_i = \begin{cases} \frac{1}{N^+}, & \text{node } i \text{ is a generator} \\ 0, & \text{node } i \text{ is a branch point} \\ -\frac{1}{N^-}. & \text{node } i \text{ is a substation} \end{cases} \tag{6}$$

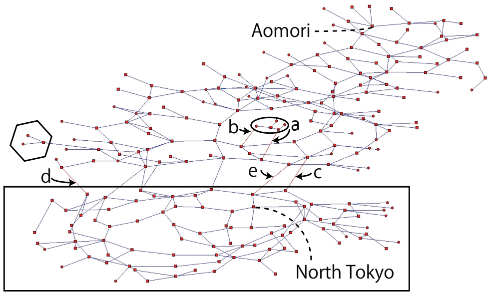

Figure 1: Topology of the power grid in eastern Japan that we replicated.

4. Minimum Coupling Strength for Stable Supply

In order for the frequency of all the nodes to be synchronized, $\dot{\theta}_i = \Omega$, that is, $\dot{\phi}_i = \ddot{\phi}_i = 0$ must be satisfied for all i. Therefore, iff σ is so large that

$$0 = \omega_i + \sigma\sum_j a_{ij}\sin(\phi_j - \phi_i), \quad \text{for all } i \tag{7}$$

can hold, all the nodes can be synchronized. There is a critical value of the coupling strength σ_C. Iff $\sigma \geq \sigma_C$, frequency synchronization comes into existence. It is called minimum coupling strength, which has been used as a measure of the power grids' stability [8]. If σ is near σ_C, the phase difference among the nodes is too large for power grids to supply electricity stably. In fact, however, the phase difference among the nodes is very small in real power grids [11]. In this paper, we introduce a modified minimum coupling strength (MMCS). If the coupling strength is larger than the MMCS, the phase difference is small enough to supply electricity stably.

4.1. Minimum Coupling Strength for Frequency Synchronization

We numerically test the critical value by calculating the effective frequency dispersion defined as follows:

$$r = \sqrt{\frac{1}{N}\sum_{i=1}^{N}[\dot{\phi}_i - \langle\omega\rangle]^2}, \tag{8}$$

where $\langle\omega\rangle$ is the mean value of the natural frequency ω_i. In this case, $\langle\omega\rangle = 0$. We define the critical value σ_C as the minimum value satisfying $r = 0$. In the eastern Japan network, we numerically obtained $\sigma_C \approx 0.047$.

[1] http://www.tepco.co.jp/ir/tool/annual/index-j.html
[2] http://www.tohoku-epco.co.jp/ir/report/annual_report/index.html
[3] http://pajek.imfm.si/doku.php?id=pajek

4.2. Frequency Synchronization is NOT Sufficient For Stable Supply

Previous studies that use σ_C [6–9] have regarded frequency synchronization as the hallmark of a stable supply. In fact, however, if the coupling strength is near σ_C, the supply of energy is sometimes destabilized, because the phase difference among the connected nodes is too large. It is empirically known that if the phase difference is large, frequency synchronization collapses when a drastic voltage drop occurs. This mechanism is analyzed by the equal area method [11].

4.3. Modified Minimum Coupling Strength

As discussed above, in order to supply electricity stably, not only frequency synchronization but also a sufficiently low phase difference is required. Therefore, we denote by σ_M an MMCS, which is the minimum value of the coupling strength satisfying both frequency synchronization and the condition that the phase difference should be less than 10°. From now on, we regard σ_M as a measure of the power grids' stability.

For the eastern Japan network, we obtained $\sigma_M \approx 0.313$.

5. Identification of Important Links in Power Grids in Eastern Japan

5.1. Identification Using the MMCS

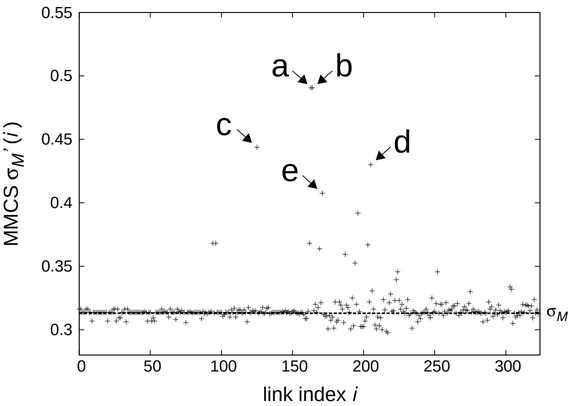

Figure 2: The effect of the link removal. The horizontal axis represents the removed link index i. The vertical axis represents $\sigma'_M(i)$, which is the MMCS after the removal of a link i. $\sigma_M \approx 0.313$ denotes the MMCS before the removal.

Many studies have analyzed the robustness of power grids [8,12–14]. In these studies, the impact of the removal of links or vertices has been studied. We define the link index as i. In the eastern Japan network, the number of the links is 324, i.e., $i \in \{1, 2, \ldots, 324\}$. We remove one link i and recalculate the MMCS $\sigma'_M(i)$. $\sigma'_M(i)$ is the MMCS after the removal of a link i. As the MMCS increases, maintaining a stable supply becomes difficult. Therefore, the larger $\sigma'_M(i)$ is, the more important the link i is for a stable supply. Hence, by calculating $\sigma'_M(i)$, the importance of each link within the power grid will be clarified.

Figure 2 shows the influence of the link removal. The horizontal axis represents the removed link index i. The vertical axis represents $\sigma'_M(i)$. Now we denote links a, b, c, d, and e in Fig. 2 by i_a, i_b, i_c, i_d, and i_e, respectively. Labels a-e correspond to the links in Fig. 1, respectively. $\sigma'_M(i_t)$ $(t = a, b, c, d, e)$ are larger than the other $\sigma'_M(i)$. This implies that the impact of the removal of these links is higher than the impact of the removal of the other links. Therefore, these links are more important than the other links with respect to a stable supply.

5.2. Comparison of the MMCS and the AMCS

From a practical point of view, σ_M can be used to identify which links are important for a stable supply. On the other hand, the previous work [8] suggests the approximate minimum coupling strength (AMCS) σ_A and asserts that σ_A can be used to identify which links are important for a stable supply. In this subsection, we compare the MMCS and the AMCS.

The AMCS can be calculated by considering all the possible divisions of the noes in a power grid into two non-overlapping sets S and $\bar{S}$. The definition of σ_A is given by

$$\sigma_A = \max_S \frac{|\sum_{i\in S} \omega_i|}{\sum_{i\in S, j\in\bar{S}} a_{ij}}. \tag{9}$$

The AMCS σ_A is expected to be close to the critical value for frequency synchronization σ_C. In fact, this was shown in European networks [8]. We confirmed this fact also in the eastern Japan network. We calculated σ_A in the eastern Japan network and the result was $\sigma_A = 0.0426$. This is near the real minimum coupling strength $\sigma_C \approx 0.047$ as previously explained.

We remove one link i and recalculate both the AMCS $\sigma'_A(i)$ and the MMCS $\sigma'_M(i)$. $\sigma'_A(i)$ is the AMCS after the removal of a link i. Figure 3 shows a scatter plot of $\sigma'_A(i)$ and $\sigma'_M(i)$. The parameter is the link index i.

In region B in Fig. 3, the AMCS underestimates the importance of the links. Although $\sigma'_A(i) \approx \sigma_A$, $\sigma'_M(i)$ is considerably larger than σ_M. On the other hand, in region C in Fig. 3, the AMCS overestimates the importance of the links. Although $\sigma'_M(i) \approx \sigma_M$, $\sigma'_A(i)$ is larger than σ_A. Therefore, for the purpose of identifying the importance of the links, the MMCS is more appropriate than the AMCS.

In region A in Fig. 3, both the AMCS and the MMCS can identify the importance of the links. In this case, the increases in $\sigma'_M(i)$ and $\sigma'_A(i)$ are positively correlated. If one removes the link a, b, c, d, or e, $\sigma'_A(i)$ changes from σ_A as $\sigma'_M(i)$ changes from σ_M. The reason why $\sigma'_A(i)$ changes by the link removal is explained as follows. We define an element of the adjacency matrix after the removal of a link

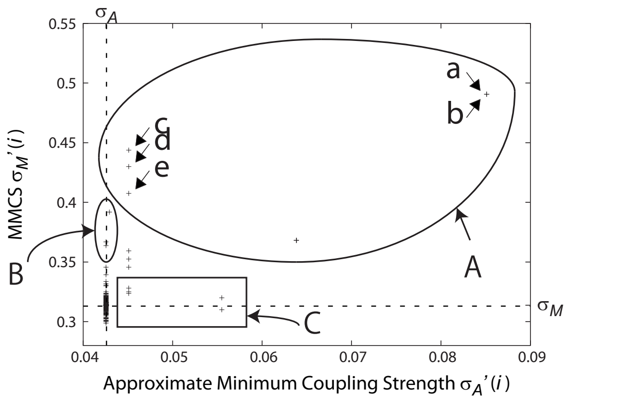

Figure 3: Correlation between the AMCS and the MMCS. The parameter is the link index i. The horizontal axis represents $\sigma'_A(i)$. The vertical axis represents $\sigma'_M(i)$. The vertical and horizontal lines at $\sigma_M \approx 0.313$ and $\sigma_A \approx 0.0425$ denote the MMCS and the AMCS before the removal, respectively.

i as

$$a'_{kj}(i) = \begin{cases} 0, & \text{corresponding link index is } i \\ a_{kj}. & \text{otherwise} \end{cases} \qquad (10)$$

We also define $a'_{kj}(0) = a_{kj}$. If one sets $S'^*(i)$ as

$$S'^*(i) = \arg\max_S \frac{|\sum_{k\in S} \omega_k|}{\sum_{k\in S, j\in \bar{S}} a'_{kj}(i)}, \qquad (11)$$

$\sigma'_A(i)$ is expressed as

$$\sigma'_A(i) = \frac{|\sum_{k\in S'^*(i)} \omega_k|}{\sum_{k\in S'^*(i), j\in \bar{S}'^*(i)} a'_{kj}(i)}. \qquad (12)$$

Now $S'^*(0) = S^*$ and $\sigma'_A(0) = \sigma_A$ hold. The essential meaning of S^* and $S'^*(i)$ is the following. They contain either many generators or many substations ($|\sum_{i\in S} \omega_i|$). Moreover, the number of links connecting $S'^*(i)(S^*)$ and $\bar{S}'^*(i)(\bar{S}^*)$ is small ($\sum_{k\in S, j\in \bar{S}} a'_{kj}(i)$). The latter condition is called minimum cut. The rectangle in Fig. 1 shows the set S^* for the original power grid network. If one removes either a or b from the network, $S'^*(i)$ turns into the nodes indicated by the ellipse. Similarly, if one removes one of the nodes c, d, and e, $S'^*(i)$ turns into the nodes indicated by the hexagon. In summary, the links that change $\sigma'_M(i)$ from σ_M dramatically also change $S'^*(i)$ from S^* and the shift in $S'^*(i)$ causes the change in $\sigma'_A(i)$. Therefore, a drastic change in $\sigma'_M(i)$ is closely related to $\sigma'_A(i)$.

6. Conclusion

In this paper, we have taken into account the fact that the phase difference between generators and substations in real power grids is very small. We have defined a modified minimum coupling strength (MMCS) and have calculated it by using the topology of the power grid in eastern Japan. Further, we have identified important links for a stable supply by calculating the MMCS.

We have found that the MMCS is more appropriate than the AMCS for the purpose of identifying the importance of links, although the AMCS is also useful in some cases.

Acknowledgments

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