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Control by Pyragas method with variable delay: from simple models to experiments

Aleksandar Gjurchinovski†, Thomas Jüngling‡ and Viktor Urumov†

†Institute of Physics, Faculty of Natural Sciences and Mathematics, Saints Cyril and Methodius University
P. O. Box 162, 1000 Skopje, Macedonia
‡Institute for Theoretical Physics, University of Würzburg, Am Hubland, D-97074, Würzburg, Germany
Email: agjurcin@pmf.ukim.mk, thomas.juengling@physik.uni-wuerzburg.de, v.urumov@gmail.com

Abstract—We will report on our findings concerning generalization of the Pyragas feedback technique by introducing variability in the delays. In addition to the theoretical basis for the method, several examples will be provided starting from the simplest case of an unstable steady state of focus type, to stabilization of unstable points and orbits in several standard systems, and finally an experimental realization with an electronic circuit. Variability of the delay, whether temporal or distributed, deterministic or not, leads to significant extension of the domain of successful stabilization and increased robustness.

1. Introduction

Control theory is an important subject of interest in many engineering applications. Beyond the developments based on classical approaches, comparatively recently, new ideas evolved closely related with the studies of deterministic chaos. A prominent place was gained by the proposal of Pyragas [1] to introduce in the equations of the system under consideration a feedback proportional to the difference of the current state of the system at instant t and its state at some instant in the past $t-\tau$, where τ is the time-delay, hence the name of the method time-delayed feedback control (TDFC). This approach was intended to achieve stabilization of unstable periodic orbits which are abundant in chaotic systems, but proved to be useful also for stabilization of unstable fixed points. Detailed exposition of the field is available in the voluminous Handbook of Chaos Control [2]. The method obtained experimental verification and was applied to various theoretical models. Successful extensions of the method were based on the introduction of additional feedback terms with the same structure but with different delays. In the following we shall deal with variable delays (VTDFC).

2. Stabilization of unstable focus

Firstly, we consider the generic case [3] of an unstable focus at the origin described with the following equations

$$\begin{aligned} \dot{x} &= \lambda x + \omega y, \\ \dot{y} &= -\omega x + \lambda y, \end{aligned} \qquad (1)$$

where λ and ω are positive constants. Eigenvalues of the system are $\Lambda = \lambda \pm i\omega$, which is characteristic for an unstable focus. We assume that the above system is a linearization in the vicinity of the fixed point $\mathbf{x}^*$ given by $\mathbf{f}(\mathbf{x}^*) = 0$ of some general two-dimensional dynamical system $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$, where $\mathbf{x}(t) = \mathrm{Col}[x(t), y(t)]$.

In order to stabilize the system, we introduce an additional force which transforms equations (1) into

$$\begin{aligned} \dot{x} &= \lambda x + \omega y + K[x(t-\tau(t)) - x(t)], \\ \dot{y} &= -\omega x + \lambda y + K[y(t-\tau(t)) - y(t)], \end{aligned} \qquad (2)$$

where K is the feedback gain. Instead of the more common constant delay, we have variable delay given by some function $\tau(t)$. The additional terms in (2) do not change the position of the fixed point, but the additional parameters, K and those characterizing $\tau(t)$, if properly chosen could change the stability properties of $\mathbf{x}^*$ in a desirable way.

In presence of variable delay $\tau(t)$, the usual exponential ansatz $x(t) \sim \exp(\Lambda t)$, $y(t) \sim \exp(\Lambda t)$, does not produce an outright equation for the eigenvalues Λ. However, assuming that $\tau(t)$ is periodic and changing with high frequency, we can replace $\exp(\Lambda\tau(t))$ with its average value. Taking as an example that $\tau(t)$ changes linearly and periodically between the limiting values $T_0 \pm \varepsilon$, where T_0 is the average value of the delay and ε is the amplitude of the delay variation, one finds the following equation for the eigenvalues Λ,

$$\lambda \pm i\omega = \Lambda + K\left(1 - \frac{\sinh(\Lambda\varepsilon)}{\Lambda\varepsilon}\exp(-\Lambda T_0)\right). \qquad (3)$$

One arrives at the same equations by applying the theorem obtained by Michiels et al. [4], which connects asymptotic stability properties of systems with variable time-delay with some related system with distributed delays and equivalent stability.

Eq. (3) has an infinite number of complex roots. The steady state at $\mathbf{x}^*$ is stable only if all the roots have negative real parts. Significant enlargements of domains of successful control obtained by VTDFC are shown in Fig. 1. Similar results are obtained when VTDFC is applied to the Lorenz system [5]. When more than one feedback term is applied [6], analogous extensions [7] with variable delay tend to enlarge the stabilization domains. Amplitude

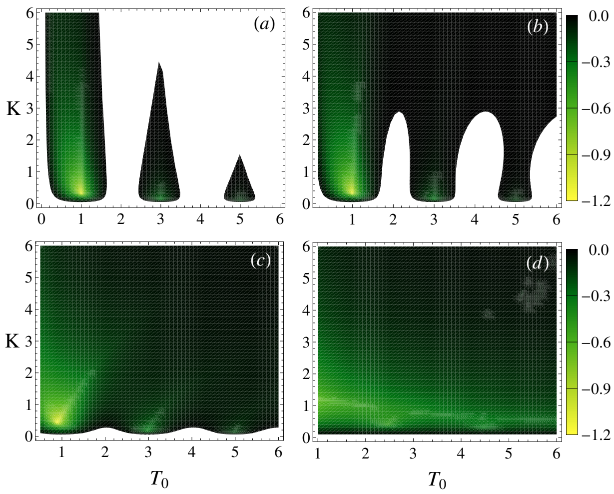

Figure 1: Domains of control in the (K, T_0)-plane for $\varepsilon = 0$ (a), $\varepsilon = 0.15$ (b), $\varepsilon = 0.5$ (c) and $\varepsilon = 1$ (d). Time-delay changes as a sawtooth wave with $\nu = 10$. The parameters are $\lambda = 0.1$ and $\omega = \pi$.

death of coupled oscillators under influence of variable and distributed delays was studied in references [8, 9].

3. Stabilization of unstable periodic orbits

One of the merits of the Pyragas method is that the control is noninvasive, that is the feedback force vanishes when stabilization is achieved. This requirement is more demanding when it comes to stabilization of unstable periodic orbits. Therefore the delay has to be a multiple of the period of the unstable orbit. Therefore, if we want to introduce variability in the delay, it has to jump between integer multiples of the basic period.

Now we consider the chaotic Rössler model [10] defined with the equations

$$\begin{aligned} \dot{x}(t) &= -y(t) - z(t), \\ \dot{y}(t) &= x(t) + 0.2y(t) + K(t)[y(t-(t)) - y(t)], \\ \dot{z}(t) &= 0.2 + z(t)[x(t) - 5.7]. \end{aligned} \tag{4}$$

The last term in the second equation represents the feedback, where as an additional feature, variability of the gain factor $K(t)$ is introduced. The shortest unstable periodic orbit for the Rössler system has a period $T_1 = 5.88$.

The simplest choice for the varying time delay is to have delays equal to T_1 and $2T_1$ for consecutive duration of time interval T_p each, which are repeated alternatively. Keeping the gain K constant, numerical simulations show that there is an increase of the control interval for the gain by having variable time delay. The control interval for the gain is largest when $T_p = 2T_1$, that is when the period of modulation is twice the period of the unstable orbit. Additional enlargement of the control interval can be achieved by introducing variations in the control gain $K(t)$, for example by changes between two constant values K and $K/2$, simultaneous with the changes of the delay [7, 11].

4. Desynchronization of Hindmarsh-Rose oscillators

It is thought that the reason behind some pathological disorders in patients with brain related diseases is due to synchronized neural activity in specific locations of the brain. As an appropriate model to describe the characteristic firing of neuronal cells in the brain, it is common to use, among others, the Hindmarsh-Rose (HR) oscillators [12]. Here we shall follow in the footsteps of Rosenblum and Pikovsky [13] who considered a large number of identical HR-oscillators coupled through the mean field $X(t) = \frac{1}{N}\sum_{i=1}^{N} x_i(t)$ created by one of the oscillators components. The equations of motion for the system of HR-oscillators are the following

$$\begin{aligned} \dot{x}_i &= y_i - x_i^3 + 3x_i^2 - z_i + 3 + F_1(t) + F_2(t), \\ \dot{y}_i &= 1 - 5x_i^2 - y_i, \\ \dot{z}_i &= 0.006[4(x_i + 1.56) - z_i]. \end{aligned} \tag{5}$$

Here $F_1(t) = K_{MF}X(t)$ defines the interaction of each of the oscillators with the other oscillators through their mean field, where K_{MF} is the coupling strength. For a sufficiently strong coupling, the system, starting from random initial positions for the individual oscillators, evolves towards a synchronized state with non-zero order parameter.

To eliminate this undesirable situation, control feedback in the Pyragas form is applied, represented with the last term of the first of equations (5). It is expressed in terms of the mean field

$$F_2(t) = K[X(t - \tau(t)) - X(t)], \tag{6}$$

and contains variable delay. To quantify the influence of the feedback, a suppression factor S is introduced [13], defined as

$$S = [var(X)/var(X_F)]^{1/2}, \tag{7}$$

where $var(X)$ and $var(X_F)$ are the variances of the mean field, without and with feedback, respectively. The numerical calculations were performed [14] for a set of 1000 HR-oscillators and the mean field coupling was set to $K_{MF} = 0.08$. Without feedback one observes oscillations of the mean field with an average period $T = 175$. In Fig. 2 we represent by shaded scale the suppression factor S as a function of the control gain K and the delay time τ normalized by T. The panel at the top shows the results for the case of constant delay τ, while the panel at the bottom provides results from the calculations with variable delay defined by $\tau(t) = \tau + \varepsilon \sin(\nu t)$, where $\varepsilon = 40$, $\nu = 10$ and constant $\tau \geq \varepsilon$. It is evident that the domains of suppression of the mean field oscillations are significantly enlarged and have become more profound. It was also observed that the desynchronization is achieved in shorter time. The drawback is in the need to increase the minimal gain to obtain desynchronization, which could be reduced by proper

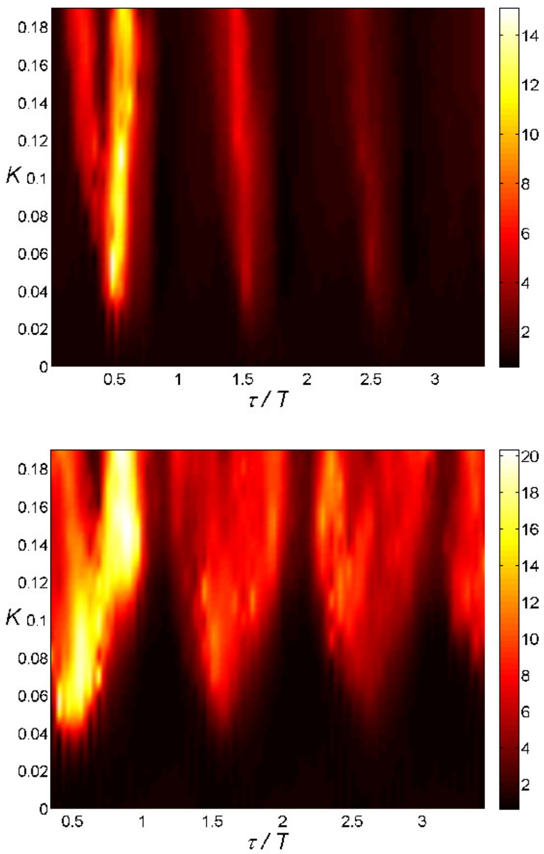

Figure 2: Suppression factor of the mean oscillations in the plane parametrized by the feedback gain K and constant (panel at the top) and average time delay τ (panel at the bottom). Parameters used in the calculation are provided in the text. Color codes of suppression factor are given on the right side of the panels.

choice of the type of time-delay modulation. Even more substantial enlargement of the domains of efficient suppression is obtained with application of two feedback terms (6) with different average delay times $\tau_{1,2}$.

5. Experiment

The effect of variable delay on the Pyragas control method was recently demonstrated in an electronic circuit experiment. The nonlinear element of the oscillator was a diode with a characteristic, which is well modelled by a piecewise linear function with two branches. The setup was already used in other context [15], where it has been described in detail. There are three differential equations governing the dynamics of the circuit, one of them containing the nonlinearity. In the voltage range used in the experiment, only the fixed point at the origin plays a role. The control parameter in the equations is denoted by the letter a.

The device for accomplishing variability of the delays has a digital storage for signals received from the electronic circuit. They are sampled with a clock frequency, which can be changed in a prescribed manner as a function of time $f(t)$ and therefore changing the delay-time. The information is moved through the device on the first-in-first-out basis (FIFO). Denoting by N the capacity of the storage, one can derive the relationship

$$N = \int_{t-\tau(t)}^{t} f(t')\, dt' \,, \qquad (8)$$

where $\tau(t)$ is the current delay-time. From there one can find that a sawtooth wave for $\tau(t)$ arises if $f(t)$ changes periodically between two constant values f_1 and f_2.

In the experiment, it was assumed that the stabilization of the fixed point has been successful if the oscillations are below 0.1 V, which is about two orders of magnitude less than the characteristic amplitudes for the uncontrolled oscillations. The measurements have clearly indicated considerable enlargements of the domain of stabilization. This was also the case when distributed delays were applied [16].

In another experiment, similar improvements are demonstrated in the stabilization of periodic orbits. We consider the same electronic circuit in the domain of parameters without stable periodic orbits and use two delay lines with different constant delays, $\tau_1 = 3T_p$ and $\tau_2 = 4T_p$, where T_p is the period of the basic unstable periodic orbit. The control term is chosen in the form

$$F(t) = K[(X(t-\tau_1) + X(t-\tau_2))/2 - X(t)], \qquad (9)$$

where $X(t)$ is one of the components of the system. This can be viewed as distributed delay defined with two δ-functions concentrated at $\tau = \tau_1$ and $\tau = \tau_2$ or alternatively as variable time-delay with very fast switching between two delay times. The results are shown on the right panel in Fig. 3 in the (a, K)-plane. For comparison, on the left panel in Fig. 3 are depicted the experimental results for the conventional Pyragas control with only a single delay line with constant time-delay $\tau_1 = 3T_p$. The panel in the middle of Fig. 3 provides results obtained in a similar experiment using the method of extended time-delay feedback control (ETDFC) [17] with memory parameter $R = 0.5$.

6. Conclusions

Our analytical and numerical studies were confirmed by performing experiments and have shown considerable improvements in the control of unstable steady states and unstable orbits when the time-delay feedback technique is extended to variable or distributed delays. The main advantage of the variability is in the significant enlargement of the domains of successful control and in the disposal of the need for fine-tuning of the control parameters. At the same time the robustness also has favorable gains. It was demonstrated that the method is applicable for different systems

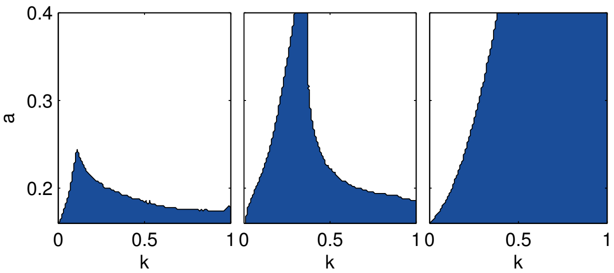

Figure 3: Successful control of periodic orbit is defined by standard deviation of $X(t - T_p) - X(t)$ smaller than 0.1 V. Results obtained by TDFC (left panel), ETDFC (middle panel), VTDFC (right panel).

described with few equations and complex systems of interacting oscillators. The method is efficient even in the case when other methods do not work, for example when delay times are large.

A characteristic feature of the Pyragas method is its non-invasiveness, which remains valid in the current extension. The control feedback tends to vanish with the approach to stabilized state of the system, whether steady or periodic. Prior knowledge of the unstable steady state is not required, while in the case of unstable orbits, one needs to know the period of the orbit, but not the orbit itself.

The influence of the modulation of the delay-times depends on the particular system under consideration. It is not a monotonic function of the amplitude of modulation. This opens up the question for optimal choice of the modulation in eventual applications. The same applies for distributed delays.

Both resonance effects and interference play a role in the observed phenomena. Signs of resonance are visible when there is interplay between some internal frequency (of the periodic orbit or the torsion of the fixed point) and the modulation frequency of the time-delayed feedback. Examination of the equations for the eigenvalues shows analogy with interference experiments from single and double slit [16].

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