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## Performance of Chaotic CDMA System implemented on Software-Defined Radio System

Hisashi Watanabe<sup>†</sup>, Yuichi Omori<sup>‡</sup>, Mikio Hasegawa<sup>†,††</sup> and Kazuyuki Aihara<sup>‡</sup>

<sup>†</sup>Department of Electrical Engineering, Tokyo University of Science,  
 1-14-6 Kudankita, Chiyoda-ku, Tokyo, 102-0073 Japan

<sup>‡</sup>Wireless Network Research Institute, National Institute of Information and Communications Technology  
 3-4 Hikarino-oka, Yokosuka, Kanagawa, Japan

<sup>††</sup>Institute of Industrial Science, The University of Tokyo  
 4-6-1 Komaba, Meguro-ku, Tokyo, 153-8505 Japan

E-mail: { watanabe, omori }@haselab.ee.kagu.tus.ac.jp, hasegawa@ee.kagu.tus.ac.jp, aihara@sat.t.u-tokyo.ac.jp

**Abstract**– We implement the chaotic code division multiple access (CDMA) system using the software-defined radio (SDR) technology and evaluate the performance of the implemented system. In the researches on the chaotic CDMA, it has been shown theoretically that the bit error rate (BER) can be reduced by using chaotic sequences with negative autocorrelation. Furthermore, by more realistic analysis including the band-limiting filter, it has been clarified that the BER can be improved by using the Gaussian filters better than the general case with the Nyquist filter, when we use the ideal chaotic spreading codes with negative autocorrelation. The experimental results using our implemented system on the SDR clarify that the Gaussian filter and the spreading sequence with a specific negative autocorrelation leads to the lowest BER in band-limited asynchronous CDMA system.

### 1. Introduction

Direct sequence code division multiple access (DS/CDMA) using the chaotic spreading sequences has better performance than the conventional sequences in chip-asynchronous DS/CDMA [1]. The spreading spectrum (SS) codes with negative autocorrelation generated by chaotic maps have been shown effective to reduce the interference among the spreading sequences by numerical simulations [2], [3]. Asynchronous CDMA using the chaotic SS code has been shown to make a 15% gain of the number of simultaneous accessible users at the same bit error rate (BER) compared to the conventional Gold and the random SS codes [4]. These analyses has been done under the assumptions that the sequences have ideal rectangular shaped pulses, but the real CDMA systems have to include band-limiting filter and the performance of chaotic CDMA system depends on the chip waveform as well as spread spectrum codes. Therefore, Ref. [5] showed that the Gaussian filtered pulse significantly reduces BER with same excess bandwidth energy as general root raised cosine (RRC) filter.

In our research, we implement such a chaotic CDMA system by using the software-defined radio (SDR) technologies to investigate the performance of the chaotic SS code with negative autocorrelation and each chip waveform. We develop the chaotic CDMA using the GNU Radio [6], which is a free and open source software

development toolkit. The GNU Radio works on the Universal Software Radio Peripheral (USRP) [7], which is hardware for the SDR system. This paper shows the experimental results of the chaotic CDMA with each pulse shape using the implemented system.

### 2. Optimal Chaotic Codes for Chip-Asynchronous DS/CDMA System

#### 2.1. Interference in Asynchronous DS/CDMA System

The BER in asynchronous DS/CDMA can be given by the average interference, which is a variance of multiple access interference (MAI) with respect to code symbols. Ref. [5] defines MAI of chip-asynchronous DS/CDMA system with  $K$  users as shown in Fig. 1. The data symbol  $d_p^{(i)}$  and the spreading code  $x_n^{(i)}$  are given by the following equations,

$$d^{(i)}(t) = \sum_{p=-\infty}^{\infty} d_p^{(i)} \delta(t - pT_d), \quad (1)$$

$$x^{(i)}(t) = \sum_{n=0}^{N-1} x_n^{(i)} \delta(t - nT_c), \quad (2)$$

where  $T_d$  is the data to be sent and  $T_c = T_d/N$  is the chip duration,  $N$  is the spreading factor, respectively.

Here, we assume that transmitter and receiver have pulse shaping filters and they are common for all users and normalized. Then, MAI caused from  $j$ -th transmitter to  $i$ -th receiver can be calculated by the following equation,

$$\text{MAI}^{(i,j)}(t) = x^{(j)}(t) * h_T(t) * \overline{x^{(i)}(-t)} * h_R(t), \quad (3)$$

where  $h_T(t)$  is the pulse shaping filter of the transmitter side,  $h_R(t)$  is that of the receiver side, an asterisk sign denotes a convolution and  $\bar{z}$  denotes the complex conjugate of  $z$ , respectively. Thus, the variance of the MAI can be obtained as follows,

$$\{\sigma_{\text{MAI}}^{(i,j)}\}^2 = \frac{1}{N} \sum_{k=2-2N}^{2N-2} |\text{MAI}^{(i,j)}(t_{ij} - pT_d)|^2, \quad (4)$$

where  $t_{ij} = t_i - t_j$  is the relative time delay in the asynchronous CDMA. Since the time delay  $t_{ij}$  is not known to the receiver, we take the expectation on MAI with respect to the time delay as follows,

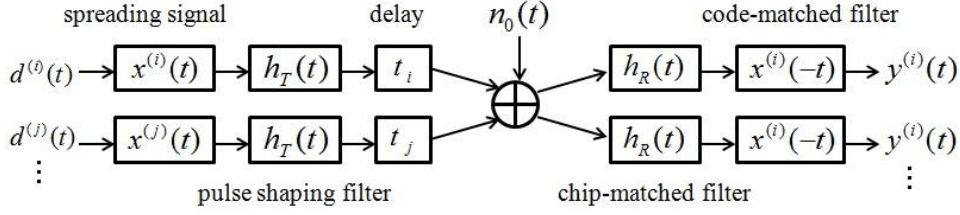


Figure 1: The baseband equivalent model of a chip-asynchronous DS/CDMA system with  $K$  users.

$$\langle \{\sigma_{\text{MAI}}^{(i,j)}\}^2 \rangle = \frac{1}{NT_d} \sum_{k=2-2N}^{2N-2} [x^{(i)} \bullet x^{(j)} \bullet x^{(i)} \bullet x^{(j)}]_k h_{\text{TR}} * h_{\text{TR}}(kT_c), \quad (5)$$

where  $[x^{(i)} \bullet x^{(j)}]_k$  is Pursley's aperiodic cross-correlation function [8] defined as

$$[x^{(i)} \bullet x^{(j)}]_k = \begin{cases} \sum_{n=0}^{N-l-1} x_n^{(i)} \bar{x}_n^{(j)} & \text{for } l \geq 0 \\ \sum_{n=0}^{N-l-1} x_n^{(i)} \bar{x}_n^{(j)} & \text{for } l < 0 \end{cases}. \quad (6)$$

## 2.2. Generation of SS codes by the Markov chain

Let  $\mathbf{X} = \{X_n\}_{n=0}^{N-1}$  and  $\mathbf{Y} = \{Y_n\}_{n=0}^{N-1}$  be sequences of  $\{-1, 1\}$  valued stationary random variables. Suppose that  $\mathbf{X}$  and  $\mathbf{Y}$  are stationary two-state Markov chains with two-dimensional transition matrix  $\mathbf{P}$  and are mutually independent. Let their probabilities be  $\text{Prob}\{X_n = 1\} = \text{Prob}\{X_n = -1\} = \text{Prob}\{Y_n = 1\} = \text{Prob}\{Y_n = -1\} = 1/2$ , and transition matrix  $\mathbf{P}$  can be defined as

$$\mathbf{P}(\lambda) = \begin{bmatrix} \frac{1+\lambda}{2} & \frac{1-\lambda}{2} \\ \frac{1-\lambda}{2} & \frac{1+\lambda}{2} \end{bmatrix}. \quad (7)$$

where  $\lambda$  is one of the eigenvalues of  $\mathbf{P}$  except 1. Fig. 2 depicts the state transition diagram of  $\mathbf{P}$ . According to this condition, we have

$$E_X[X_n X_{n+l}] = \lambda^l \quad (8)$$

where  $E_{P_z}[\cdot]$  denotes the expected value with respect to the distribution of a random variables  $Z$ .

Equation (8) gives

$$\frac{1}{N} E_X[[X \bullet X]_l] = \lambda^{|l|}, \quad (9)$$

$$\frac{1}{N^2} E_{XY}[[X \bullet X \bullet Y \bullet Y]_l] = \left(|l| + \frac{1+\lambda^2}{1-\lambda^2}\right) \lambda^{|l|}. \quad (10)$$

Equation (10) is used to evaluate (5). See [9] for derivation and expression for finite  $N$ .

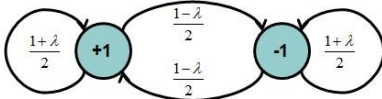


Figure 2: Two-state Markov chain with two-dimensional transition matrix for generating  $\{-1, 1\}$  valued sequences used for cross-correlation analysis.

## 2.3. Optimal spreading codes considering the pulse shaping filter

We investigate the pulse shaping filter suitable for the chaotic CDMA as already been analyzed in Ref. [10]. In

the real wireless systems, the bandwidth have to be limited by such filters.

This paper compares interferences in a DS/CDMA system employing the rectangular pulse, the RRC pulse and the Gaussian pulse. For the comparisons, we use the implemented chaotic DS/CDMA system on the SDR in order to obtain real performance results.

A. Rectangular Pulse: For a rectangular pulse, we have

$$h_{\text{TR}}^{(r)} = \begin{cases} 1 - \frac{|t|}{T_c} & \text{for } |t| < T_c \\ 0 & \text{for } |t| \geq 2T_c \end{cases}, \quad (11)$$

$$E_X \left[ \langle \{\sigma_{\text{MAI}}^{(i,j)}\}^2 \rangle \right] = \frac{2}{3} \frac{1 + \lambda + \lambda^2}{1 - \lambda^2}. \quad (12)$$

Thus, the eigenvalue  $\lambda = -2 + \sqrt{3}$  minimizes the variance of the MAI, and therefore the BER can be minimized [4].

B. Root Raised Cosine (RRC) Pulse: For the RRC pulse, we have

$$h_{\text{TR}}^{(rc)} = \text{sinc}(t/T_c) \frac{\cos(\pi\beta t/T_c)}{1 - 4(\beta t/T_c)^2}, \quad (13)$$

where  $\beta$  is the roll-off factor. Many practical systems employ nonzero excess bandwidth. In this paper,  $\beta = 0.22$ .

C. Gaussian Pulse: The Gaussian chip pulse sacrifices signal component, however, they give lower BER than RRC pulses. The Gaussian pulse with a parameter  $\alpha > 0$  is defined as

$$h_{\text{TR}}^{(G)}(t, \alpha) = \frac{1}{\sqrt[4]{2}} \text{gauss}\left(\frac{\alpha t}{T_c}\right), \quad (13)$$

where  $\text{gauss}(x) = \sqrt[4]{2} \exp(-\pi x^2)$ . The energy of this pulse is concentrated with 92.4% when  $\alpha = 1$ . We evaluate average signal-to-interference ratio (SIR), to compare each pulses,

$$\text{SIR} = \frac{E_X \left[ |\text{MAI}^{(i,i)}(0)|^2 \right]}{N \cdot E_{XY} \left[ \langle \{\sigma_{\text{MAI}}^{(i,j)}\}^2 \rangle \right]}. \quad (14)$$

Ref. [5] shows that the Markovian SS codes with negative eigenvalues are superior to i.i.d. codes in terms of SIR. The SIR for the Gaussian pulse is maximized by  $\lambda \approx -0.371$ . The Gaussian chip pulse gives higher SIR than the RRC pulses.

### 3. Implementation of Chaotic CDMA using Software Defined Radio

We implement the asynchronous chaotic CDMA system using the GNU Radio and the USRP, which are software-defined radio (SDR) systems. Fig. 3 shows the overall architecture of the communication system using the GNU Radio and the USRP. The high-sample-rate processes take place in a FPGA on the USRP. The low-sample-rate operations, which are most parts of the digital signal processes provided by the GNU Radio, can be performed on the host computer, or in the FPGA. The USRP and the host computer are connected with a Gigabit Ethernet interface that enables simultaneous sending up to 50 MHz of the RF bandwidth between them.

In the GNU Radio, the design and the implementation of the chaos-based communication system are realized by constructing a flowchart, which is composed of the signal processing blocks described by C++ and arrows described

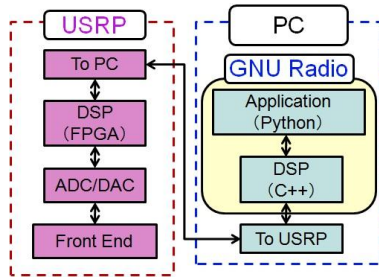


Figure 3: Overall architecture of the communication system using the GNU Radio and the USRP.

by Python to connect them. Figures 4 and 5 show the flowcharts of the transmitter and the receiver of the asynchronous chaotic CDMA system, respectively. In the scramble and descramble blocks, the input signals are spread or despread by multiplying chaotic sequences having optimal autocorrelation. Each terminal is connected with the coaxial cables and the mixer, as shown in Fig. 6.

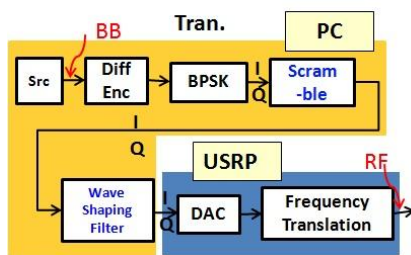


Figure 4: Flowchart of the chaotic CDMA transmitter.

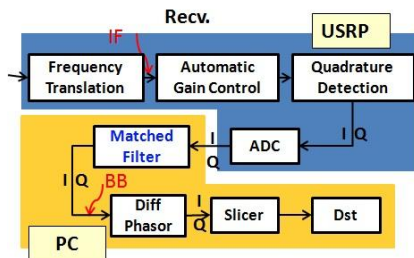


Figure 5: Flowchart of the chaotic CDMA receiver.



Figure 6: Implemented chaotic CDMA system on three USRPs, which is used for the experiments.

### 4. Experimental Results

The flow of the experiment in this study is as follows. First, in the communication system simulation, we compare the BER of asynchronous CDMA using each of the rectangular wave, the RRC wave and the Gaussian wave, and investigate the effectiveness by changing the parameter  $\lambda$ , which controls the autocorrelation of the Markov chain. Second, we investigate them using our implemented chaotic CDMA system.

#### 4.1. Results of Communication System Simulation

First of all, we analyze the performance of asynchronous CDMA communication system using each wave form and investigate the optimal Markov sequences with  $\lambda$ , by communication system simulations. The simulation results of 20 users system are shown in Fig. 7. The result shows that there are the lowest BER point for each pulses, when  $\lambda = -0.25, -0.15, -0.40$  for the rectangular, the RRC and the Gaussian pulse, respectively. This  $\lambda$  is the optimal autocorrelation for each pulse.

We also analyze the performance of the CDMA system using the optimal Markov sequence for each wave form by varying the number of users as shown in Fig. 9. The simulation results shows that the Gaussian wave form provides lower BER than the RRC wave form at the optimal Markov sequence for each.

#### 4.2. Results of implemented chaotic CDMA system

We evaluate the chaotic CDMA using the implemented communication system with two users asynchronous CDMA. The signals from two transmitters are mixed in the mixer through the coaxial cables. Another terminal receives mixed signals and reconstructs transmitted symbols using the same sequences allocated to each transmitter. The parameters of the implemented CDMA system are shown in Table 1. The experimental results are shown in Fig. 9. Fig. 9 shows the BER of the implemented CDMA system with the RRC pulse and the Gaussian pulse. The result shows that the implemented chaotic CDMA system using the Gaussian pulse with its optimal parameter  $\lambda = -0.35$  is lower BER than the RRC pulse with its optimal parameter  $\lambda = -0.15$ . From these results, the analytical results shown in the conventional researches could be confirmed by our real implemented system, that the optimal  $\lambda$  takes negative value and the Gaussian filter provides the best performance.

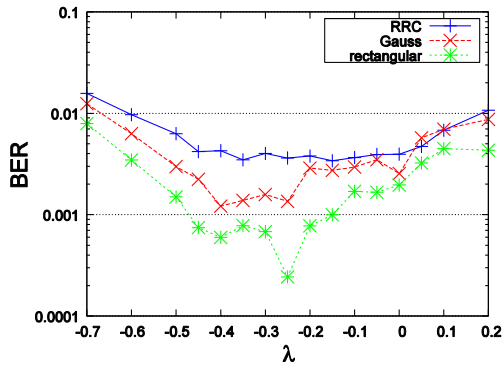


Figure 7: Simulation results of the BER performance on the 20 user asynchronous CDMA system for different  $\lambda$ , which is the damping factor of the autocorrelation of the sequences. The length of spreading sequences is 127.

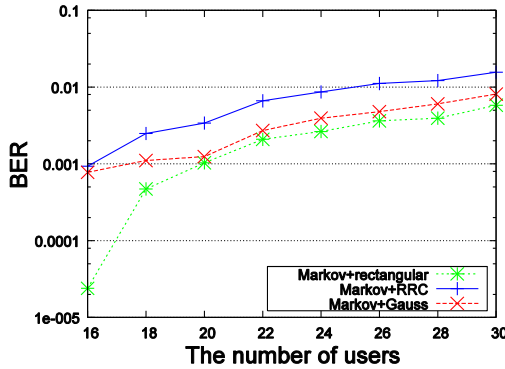


Figure 8: Simulation results of BER performance on the asynchronous CDMA system with optimal Markov sequence for each pulse for different the number of users. The length of spreading sequences is 127.

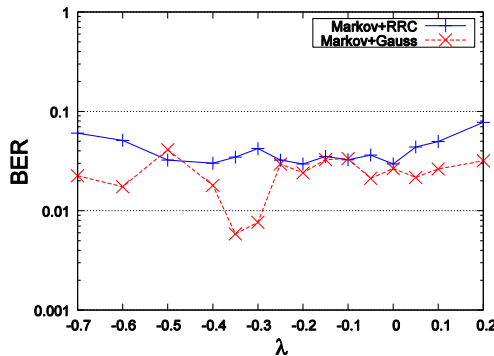


Figure 9: BER of the implemented chaotic CDMA system using SDR technology. The parameter  $\lambda$  controls the autocorrelation of the Markov SS code. The number of spreading sequences is 31.

Table 1: Parameters used in the implemented CDMA system in the experiments.

Data Modulation	DBPSK
Chip Rate	500[kchip/sec]
Carrier Frequency	2.45[GHz]
Spreading Factor	31

## 5. Conclusion

We investigated the effectiveness of the chip-asynchronous chaotic CDMA system by a real system implemented on the SDR technologies. The experimental results using our implemented system clarify that the Gaussian filter and the SS code with a specific negative autocorrelation leads to the lower BER in band-limited asynchronous CDMA system. We could confirm the effectiveness of the chaotic CDMA shown in the analytical studies of the conventional researches, by using our real implemented system.

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