Numerical Solution for Transient Scattering by Uniformly Moving Targets

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***Abstract*-A numerical method for transient electromagnetic scattering by uniformly moving target is proposed. Two reference systems are adopted: one is attached on the radar platform while the other is fixed on the target. Finite Difference Time Domain (FDTD) method is employed to analyze the scattering in the target system. Lorentz transform is exploited to transfer the electromagnetic fields between the two reference frames. A perfectly conducting sphere is used to examine the method. A simply missile model is calculated using the proposed scheme.**

I. INTRODUCTION

Analytical methods for electromagnetic wave scattering by moving bodies has been developed principally for one-, two-, and three-dimensional structures [1-5]. Canonical problems considered include planar conducting and dielectric interfaces in uniform translation or vibration [6], uniformly moving random rough surfaces [7], uniformly moving or vibrating cylindrical and spherical shapes [8-10], and simple rotating bodies [11].

Numerical methods for electromagnetic scattering by moving target with arbitrary shapes were reported [12-14]. In [12], FDTD method with relativistic electromagnetic field boundary conditions was exploited. The solution directly in the laboratory frame was given using this approach. In [13], frequency domain method using Lorentz transform was adopted. In [14], a method with combined relativistic boundary condition and characteristic variable boundary condition was explored.

In this work, an alternate scheme is presented. Scattering analysis is performed in the target system, while Lorentz transform is applied to transfer electromagnetic fields between the radar and target reference frames.

II. ALGORITHM

Two reference frames are set: Radar reference frame (S) and target reference frame (S'). Initial time is $t = t' = 0$, when the origin of S' in S is $\boldsymbol{r}_0 = (x_0, y_0, z_0)$. The relative velocity of S' to S is $\boldsymbol{v}$. We define

$$\boldsymbol{\beta} = \frac{\boldsymbol{v}}{c},\ \gamma = \frac{1}{\sqrt{1-\beta^2}},\ \boldsymbol{\alpha} = \boldsymbol{I} + (\gamma - 1)\frac{\boldsymbol{v}\boldsymbol{v}}{v^2} \tag{1}$$

In S, a Gauss pulse wave is transmitted. The expressions of incident wave are

$$\boldsymbol{E}^i(\boldsymbol{r},t) = \boldsymbol{E}_0 \exp\left[-\frac{4\pi\left(t - t_0 - \hat{\boldsymbol{k}}\cdot\boldsymbol{r}/c\right)^2}{\tau^2}\right] \tag{2}$$

$$c\boldsymbol{B}^i(\boldsymbol{r},t) = \hat{\boldsymbol{k}} \times \boldsymbol{E}_0 \exp\left[-\frac{4\pi\left(t - t_0 - \hat{\boldsymbol{k}}\cdot\boldsymbol{r}/c\right)^2}{\tau^2}\right] \tag{3}$$

where $\hat{\boldsymbol{k}}$ is the unit direction vector of incident wave, τ is pulse width. Transformed to S' frame, the incident wave become [15]

$$\boldsymbol{E}'^i(\boldsymbol{r}',t') = \gamma\left[\boldsymbol{E}^i(\boldsymbol{r},t) + \boldsymbol{\beta}\times c\boldsymbol{B}^i(\boldsymbol{r},t) - \frac{\gamma}{1+\gamma}\left(\boldsymbol{\beta}\cdot\boldsymbol{E}^i(\boldsymbol{r},t)\right)\boldsymbol{\beta}\right] \tag{4}$$

$$c\boldsymbol{B}'^i(\boldsymbol{r}',t') = \gamma\left[c\boldsymbol{B}^i(\boldsymbol{r},t) - \boldsymbol{\beta}\times\boldsymbol{E}^i(\boldsymbol{r},t) - \frac{\gamma}{1+\gamma}\left(\boldsymbol{\beta}\cdot c\boldsymbol{B}^i(\boldsymbol{r},t)\right)\boldsymbol{\beta}\right] \tag{5}$$

with $\boldsymbol{r}$ and t on the right-hand sides to be replaced by

$$\boldsymbol{r}-\boldsymbol{r}_0 = \boldsymbol{\alpha}\cdot\boldsymbol{r}' + \gamma\boldsymbol{v}t', \quad t = \gamma\left(t' + \frac{\boldsymbol{v}\cdot\boldsymbol{r}'}{c^2}\right) \tag{6}$$

In S' system, Finite Difference Time Domain (FDTD) is exploited to perform the scattering analysis and to obtain the equivalent currents and charges on the output boundary. Then the retarded potentials are calculated, and the scattering far-fields are found through

$$\begin{aligned}\boldsymbol{E}'(\boldsymbol{r}',t') &= -\frac{1}{\varepsilon_0}\nabla'\times\boldsymbol{A}_m(\boldsymbol{r}',t') - \frac{\partial\boldsymbol{A}(\boldsymbol{r}',t')}{\partial t'} - \nabla'\varphi(\boldsymbol{r}',t') \\ &\approx \eta_0\hat{r}'\times\frac{\partial\boldsymbol{A}_m(\boldsymbol{r}',t')}{\partial t'} + \hat{r}'\times\hat{r}'\times\frac{\partial\boldsymbol{A}(\boldsymbol{r}',t')}{\partial t'}\end{aligned} \tag{7}$$

$$\begin{aligned}\boldsymbol{H}'(\boldsymbol{r}',t') &= \frac{1}{\mu_0}\nabla'\times\boldsymbol{A}(\boldsymbol{r}',t') - \frac{\partial\boldsymbol{A}_m(\boldsymbol{r}',t')}{\partial t'} - \nabla'\varphi_m(\boldsymbol{r}',t') \\ &\approx -\frac{1}{\eta_0}\hat{r}'\times\frac{\partial\boldsymbol{A}(\boldsymbol{r}',t')}{\partial t'} + \hat{r}'\times\hat{r}'\times\frac{\partial\boldsymbol{A}_m(\boldsymbol{r}',t')}{\partial t'}\end{aligned} \tag{8}$$

where $\eta_0 = \sqrt{\mu_0/\varepsilon_0}$. The scattered fields in S system are achieved through the inverse transform like (4)-(5), i.e.

$$\boldsymbol{E}^s(\boldsymbol{r},t) = \gamma\left[\boldsymbol{E}'(\boldsymbol{r}',t') - \boldsymbol{\beta}\times c\boldsymbol{B}'(\boldsymbol{r}',t') - \frac{\gamma}{1+\gamma}\left(\boldsymbol{\beta}\cdot\boldsymbol{E}'(\boldsymbol{r}',t')\right)\boldsymbol{\beta}\right] \tag{9}$$

$$c\boldsymbol{B}^s(\boldsymbol{r},t) = \gamma\left[c\boldsymbol{B}'(\boldsymbol{r}',t') + \boldsymbol{\beta}\times\boldsymbol{E}'(\boldsymbol{r}',t') - \frac{\gamma}{1+\gamma}\left(\boldsymbol{\beta}\cdot c\boldsymbol{B}'(\boldsymbol{r}',t')\right)\boldsymbol{\beta}\right] \tag{10}$$

where $\boldsymbol{B}' = \mu_0\boldsymbol{H}'$, and $\boldsymbol{r}'$ and t' on the right-hand sides to be replaced by the inverse of (6), i.e.

$$\boldsymbol{r}' = \boldsymbol{\alpha}\cdot(\boldsymbol{r}-\boldsymbol{r}_0) - \gamma\boldsymbol{v}t, \quad t' = \gamma\left(t - \frac{\boldsymbol{v}\cdot(\boldsymbol{r}-\boldsymbol{r}_0)}{c^2}\right) \tag{11}$$

III. NUMERICAL RESULTS

First, a perfect electrically conducting (PEC) sphere is used to verify the numerical method. The radius of sphere is 1m. Its initial position is $\boldsymbol{r}_0$ =(0,0,1×10^{7}m). The PEC sphere is moving at a velocity of $\boldsymbol{v}$ =(2.121×10^{7}m/s,0, 2.121×10^{7}m/s). The radar is located at the origin in the radar system, and we only consider mono-static scattering, i.e. the radar is used as both transmitter and receiver. The pulse width is set to be τ =0.45×10^{-8}s. Fig.1 shows the configuration of the scattering problem. The receiving pulse electric fields of Ex and Ez components are predicted by using the procedure described above, which are found in good agreements with the Inverse Fourier Transform (IFT) results of the analytical MIE series solution, as shown in Fig. 2 (a) and (b). The analytical solutions are obtained by solving the scattering problem in the target system in frequency domain and transforming the data into time domain, and then apply the Lorentz Transform to convert the results to the radar system.

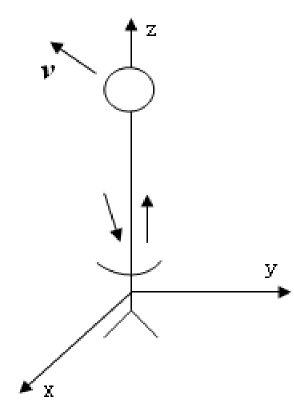
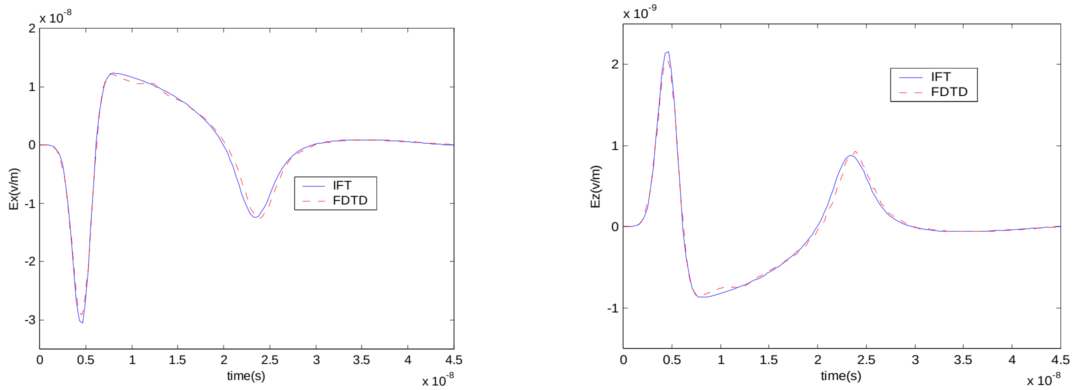


Fig. 1 Transient scattering by a moving PEC sphere

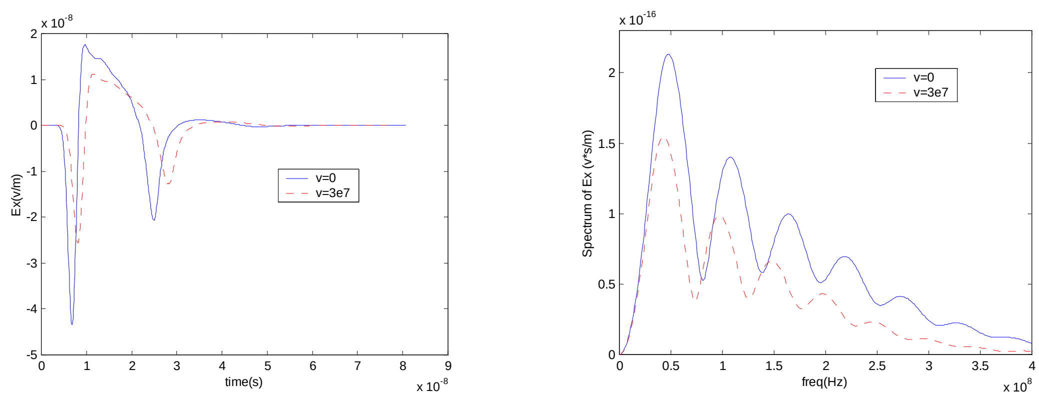
If the sphere is moving at $\boldsymbol{v}$ =(0,0,3×10^{7}m/s), the received pulse calculated by using the proposed numerical scheme is shown in Fig. 3 (a) and compared with that when sphere is at rest. The spectrum of Fig. 3 (a) is displayed in Fig. 3 (b), from which frequency shift property or

Doppler effects is illustrated: the higher the frequency, the larger the frequency shifts. To look it closer, the incident wave is set as a sine wave with frequency 300MHz. The received wave is shown in Fig. 4, from which we can extract the frequency 2.448×10^{8}Hz. According to Lorentz transformation, the frequency should be 2.4545×10^{8}Hz, so the relative error is only 0.27%.



(a) (b)

Fig. 2 Transient responses of a moving PEC sphere to a Gaussian impulse,
(a) Ex component and (b) Ez component.



(a) (b)

Fig.3 Comparisons of responses of a moving sphere with a rest sphere,
(a) in time domain, and (b) in frequency domain.

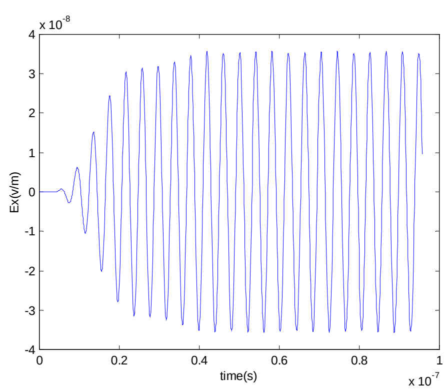


Fig. 4 Received sine wave for a moving PEC sphere

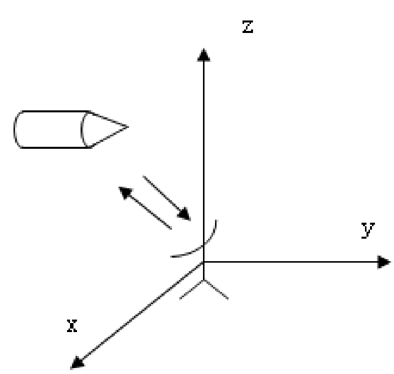


Fig. 5 Scattering geometry of a missile model

Finally, as an example, a simple missile model is considered as a moving target with arbitrary shape, as shown Fig. 7. The model is a combination of a cylinder and a cone. The radius of cylinder and cone is 1m, while the lengths of cylinder and cone are 4m and 2m, respectively. The missile model is flying in the direction of y axis at a velocity of 3×10^7m/s. Its initial location is at $\boldsymbol{r}_0 = (0, -1\times10^7\text{m}, 1\times10^7\text{m})$. The incident Gaussian impulse with $\tau = 3.6\times10^{-9}$s is directed at the missile model. The responses are shown in Fig. 6 (a) in time domain and Fig. 6 (b) in frequency domain, along with comparisons when the model is at rest (v=0). As the target is approaching, the frequency shift is

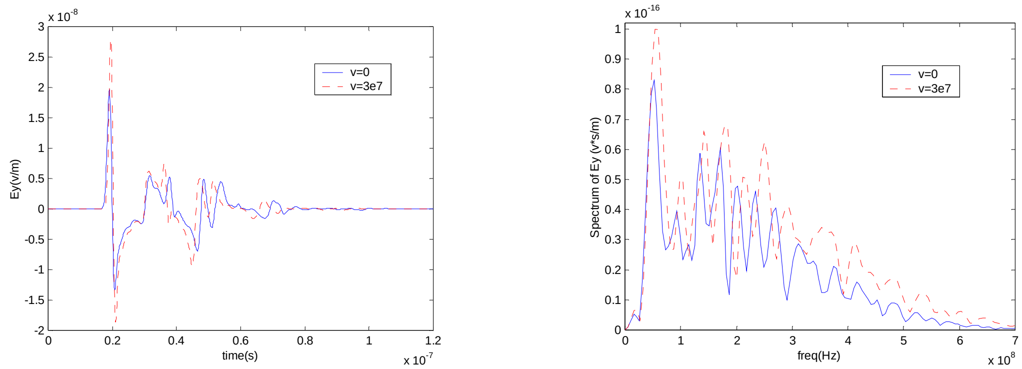


Fig. 6 Responses of a missile model to a Gaussian impulse,
(a) in time domain, and (b) in frequency domain.

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