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Dependence of Sensitive Responses of Chaotic Wandering States on Configuration of Inhibited In-Coming Synaptic Connections

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Abstract— In this paper, we investigate the dependence of the sensitive responses of the chaotic wandering states to memory pattern fragments on the configuration of the inhibited in-coming synaptic connections in Nara & Davis chaotic neural network model. It has been shown that Nara & Davis model can quickly access to the target pattern corresponding to the input memory pattern fragment with almost 3 steps and perfect possibility. However, the potentiality depends on the dynamical property of the chaotic wandering states, which changes with the configuration of the inhibited in-coming synaptic connections. Therefore, the purpose of this paper is to investigate the difference of the sensitivity to memory pattern fragments among the configurations of the inhibited in-coming synaptic connections. From computer experiments, with the configuration which does not include in-coming synaptic connections from the elements of memory pattern fragments, the model has quite higher potentiality in the accessing time and the success ratio than the configuration which includes the connections. In addition, from the viewpoints of Lyapunov dimension and information dimension, the model without in-coming synaptic connections from the fragments reveals higher developed chaos.

1. Introduction

Skarda and Freeman have shown that chaos could play the important roles in a learning process and a recalling process[1]. From the theoretical approach, Nara & Davis investigated complex memory search functions in chaotic wandering states of neural network model with multi-cycle memory patterns [2]. They have shown the interesting results that the chaotic wandering states instantaneously converge into a basin of the attractor of the target memory pattern when a memory pattern fragments is given. Kuroiwa & Nara have revealed that the property of the sensitive response to memory pattern fragments in chaotic wandering state is general even though the mechanism of chaos is different[3]. Thus, the sensitive response to memory pattern fragments with chaotic wandering states could play important roles in realizing the rapid and unbiased access in various chaotic neural networks.

Inspired by the potentiality of chaos as mentioned above, we have investigated dynamical properties of three chaotic neuron model, Aihara model, Nara & Davis model and Kuroiwa & Nara model, related with the sensitive response to memory pattern fragments[4, 5]. In this paper, we focus on Nara & Davis model. It has been shown that Nara & Davis model can quickly access to the target pattern corresponding to the input memory pattern fragment with almost 3 steps and perfect possibility. The potentiality is influenced with the dynamical property of the chaotic wandering states, which depends on the configurations of the inhibited in-coming synaptic connections. Therefore, the purpose of this paper is to investigate the difference of the sensitivity to memory pattern fragments among the configuration of the inhibited in-coming synaptic connections. In addition, we evaluate Lyapunov dimension and information dimension in order to investigate the reason of the difference of the sensitivity.

2. Chaotic Neural Network Model

2.1. Recurrent Neural Network Model with Associative Memory

Let us explain a recurrent neural network model, briefly. The updating rule of the recurrent neural network model is written as follows:

$$u_i(t+1) = \sum_{j=1}^{N} w_{ij} z_j(t), \qquad (1)$$

where $u_i(t)$ represents an internal state of the ith element at discrete time t, $z_i(t)$ describes its output, w_{ij} denotes a synaptic connection between the ith element and the jth element, and N is the total number of elements in the recurrent neural network model.

In the present paper, the output is given by the following output function,

$$z_i(t+1) = f(u_i) = \tanh\left(\beta u_i(t+1)\right), \qquad (2)$$

where β corresponds to the steepness of the output function.

In this paper, the synaptic connection is defined as,

$$w_{ij} = \sum_{a=1}^{L} \sum_{\nu=1}^{P} v_i^{a\,\nu+1} (v_j^{a\,\nu})^{\dagger}, \qquad (3)$$

where $\mathbf{v}^{a\ \mu}$ denotes μth memory pattern in ath cycle, and L and P are the number of cycles and the number of patterns per cycle, respectively. Note that we employ cycle memory patterns, thus $\mathbf{v}^{a\ P+1} = \mathbf{v}^{a\ 1}$. The dagger vector of $(\mathbf{v}^{a\ \mu})^{\dagger}$ is given by,

$$(\mathbf{v}^{a\ \mu})^{\dagger} = \sum_{b=1}^{L}\sum_{\nu=1}^{P}(\mathbf{o}^{-1})_{a\ \mu\ b\ \nu}(\mathbf{v}^{\mathbf{b}\ \nu})^{\mathrm{t}}, \quad (4)$$

where the symbol t denotes the transpose operator, and $\mathbf{o}^{-1}$ is the inverse of $\mathbf{o}$ defined by,

$$(\mathbf{o})_{a\ \mu\ b\ \nu} = \sum_{k=1}^{N} v_k^{a\ \mu} v_k^{b\ \nu}. \quad (5)$$

2.2. Nara and Davis Model

Let us explain Nara and Davis model, briefly. The updating rule of Eq.(1) is rewritten by,

$$u_i(t+1) = \sum_{j=1}^{N} w_{ij}\epsilon_{ij}(d)z_j(t). \quad (6)$$

The output $z_i(t)$ is given by Eq.(2), and $\epsilon_{ij}(d)$ denotes a matrix of binary activity values, that is,

$$\epsilon_{ij}(d) = \begin{cases} 0 & (j \in F_i(d)) \\ 1 & \text{(otherwise)}, \end{cases} \quad (7)$$

where $F_i(d)$ represents a configuration of elements at which the in-coming synaptic connection is inhibited, and is given by randomly. The parameter d represent the number of remaining synaptic connection, that is, $\sum_j \epsilon_{ij}(d) = d$, is referred as the connectivity, hereafter. The connectivity of d and the configuration set of $F_i(d)$ are system parameters in this model. In the computer experiments, we employ the following two types of configurations, (i) the configuration which does not include inhibited in-coming synaptic connections from the elements of memory pattern fragments and (ii) the configuration which includes the connections.

3. Difference of Sensitive Responses

3.1. Purpose and Method of Computer Experiments

The purpose of the computer experiments is to investigate difference of the sensitive response to memory pattern fragments depending on the configurations of the inhibited in-coming connections. In order to investigate the sensitivity, we focus on the following two problems.

1. Success ratio: How many times dose it reach the target basin within 30 iteration steps starting from different points of chaotic wandering state? In other words, is the searching procedure by means of chaotic wandering assured?
2. Accessing time: How many steps does it take to reach the target basin related with a memory fragments? In other words, how short is the "access" time for the memory basin corresponding to the external input of a memory fragments?

It should be noted that the higher success ratio and the shorter accessing time mean practical in memory search functions.

In applying the memory pattern fragments, the updating rule of Eq.(6) is rewritten as follows:

$$u_i(t+1) = \sum_{j=1}^{N} w_{ij}\epsilon_{ij}z_j(t) + \rho I_i \ \text{ if } \ i \in F, \quad (8)$$

where I_i represents a memory pattern fragments, F denotes a configuration of the elements corresponding to the memory pattern fragments, and ρ denotes the strength of the input.

The evaluation procedure is as follows. If the system converge into the pattern corresponding to the memory pattern fragment within 30 steps while memory pattern fragment is applying, we identify the search procedure of memory pattern as success. On the other hand, if the system doesn't converge into the pattern, we regard that the search procedure misses and accessing time takes 30 steps. We evaluate the success ratio and the accessing time with

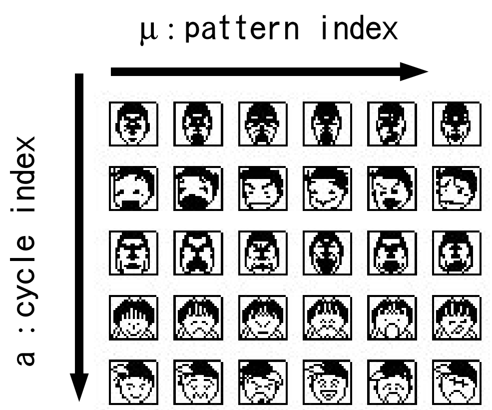

Figure 1: Memory patterns. The number of patterns per cycle, $P = 6$, and each pattern consists of 20×20 pixels which takes 0 or 1.

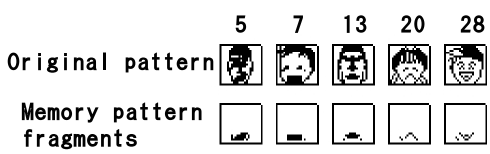

Figure 2: Memory pattern fragments. The below figures show the memory pattern fragments corresponding to each face pattern in the upper figures.

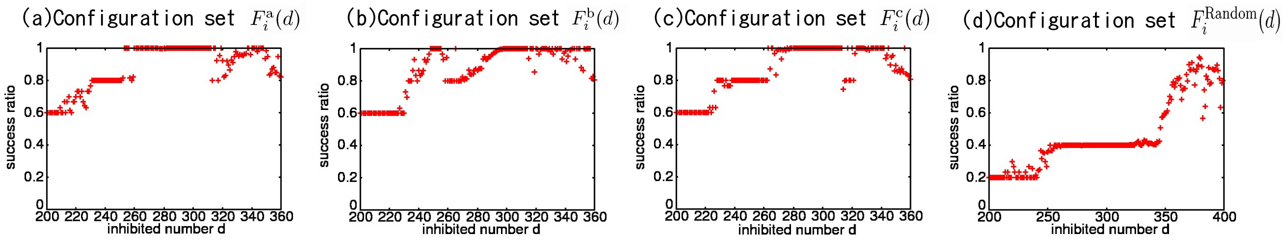

Figure 3: Success ratio to the target memory pattern.

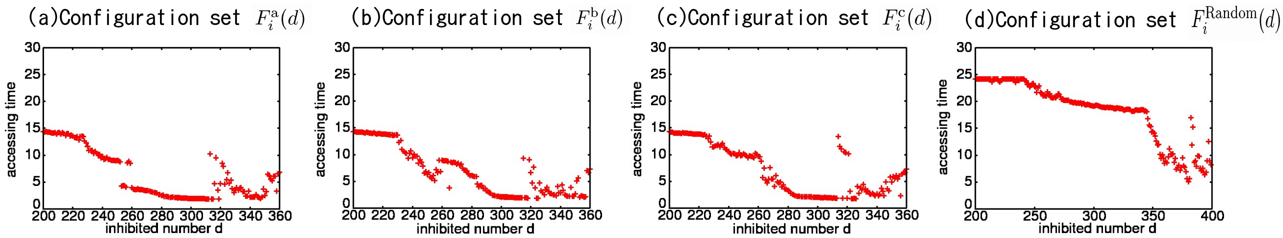

Figure 4: Accessing time to the target memory pattern.

1,000 steps in chaotic wandering state excluding initial 19,997 steps.

We investigate the instability of orbits by Lyapunov dimension and information dimension in order to investigate the reason of the difference of the sensitivity. Lyapunov dimension is calculated from

$$D_{\mathrm{L}} = j + \frac{\sum_{k=1}^{j} \lambda_k}{|\lambda_{j+1}|}, \tag{9}$$

where j is the maximal index satisfying the condition of $\sum_{k=1}^{j} \lambda_k \geq 0$ for descending ordered Lyapunov spectrum. In this paper, if all the Lyapunov exponents takes positive values, the Lyapunov dimension is defined by $D_{\mathrm{L}} = N(= 400)$. Lyapunov spectrum λ_i is given by,

$$\lambda_i = \frac{1}{T} \sum_{t=0}^{T} \log \left| \frac{\delta_i(t+1)}{\delta_i(t)} \right| \tag{10}$$

where $\delta_i(t)$ is a perturbation term defined by,

$$\delta_i(t) = \sigma_i \varepsilon, \tag{11}$$

$$\delta_i(t+1) = \sum_j w_{ij} f(u_j(t) + \delta_j(t)) - u_i(t+1) \tag{12}$$

where the updating rule is Eq.(6), ε is a perturbation parameter and σ_i denotes the normalization parameter which corresponds to the standard deviation of $\{u_i(t)\}$. In this paper, we employ $\varepsilon = 0.1, 0.01$ and 0.001. The normalization factor σ_i is necessary that $\{u_i(t)\}$ takes a different value depending on the connectivity of d.

The information dimension is given by

$$D_{\mathrm{I}} = \lim_{\Delta \to 0} \frac{S(\Delta)}{\ln(1/\Delta)}, \tag{13}$$

$$S(\Delta) = -\frac{1}{N} \sum_{i=1}^{N} \sum_{k=1}^{N(\Delta)} p_k^i \ln p_k^i, \tag{14}$$

where p_k is a stationary probability distribution of ith neuron such that the internal state of a certain element falls into the kth bin with the bin size of $\Delta = \frac{(u_i^{max\,(d)} - u_i^{min\,(d)})}{N(\Delta)}$, where $u_i^{max\,(d)}$ and $u_i^{min\,(d)}$ are the maximum value and the minimum value of $u_i(t)$ in ith neuron with the connectivity of d among 10,000 steps excluding initial 19,997 steps. In computer experiments, we employ $N(\Delta) = 100$, 500 and 1000.

In this paper, we apply 30 memory patterns with 5 cycles, and each cycle consists of 6 bit-patterns, as shown in Figure.1. In addition, we employ the memory pattern fragments with 40 pixels as shown in the below part of Figure.2. We set values of system parameters as follows: $\beta = 100$ and $\rho = 100$. We employ three different configurations which do not include the connections, F_i^{a}, F_i^{b} and F_i^{c}, and one configuration which includes, $F_i^{\mathrm{Random}}(d)$.

3.2. Results

The success ratio and the accessing time are given in Figure 3 and Figure 4. From the results, the success ratio and the accessing time with the configuration $F_i^{\mathrm{a}}(d)$, $F_i^{\mathrm{b}}(d)$ and $F_i(d)$ are quite higher success ratio and quite shorter accessing time than the configuration $F_i^{\mathrm{Random}}(d)$. Thus, the model without in-coming synaptic connections from the fragments reveals quite higher potentiality in the accessing time and the success ratio.

At next, we present results of Lyapunov dimension and information dimension as shown in Figure.5 and Figure.6. Landscape behaviors of Lyapunov dimension and information dimension for various ε and bin size Δ are almost same, meaning that the calculations are correct. From re-

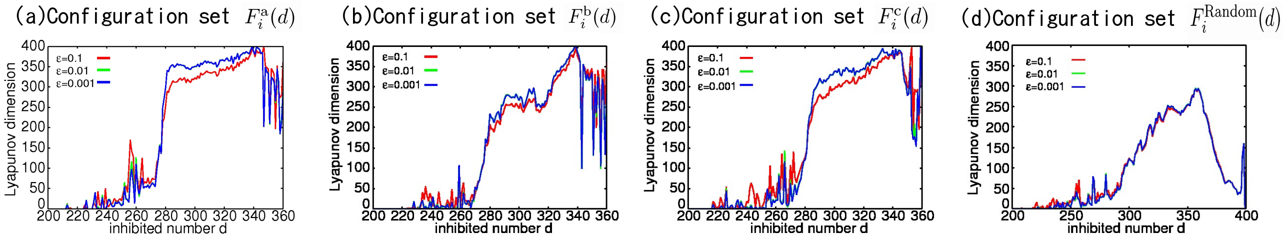

Figure 5: Lyapunov dimension without in-coming connections from the fragments and with connections.

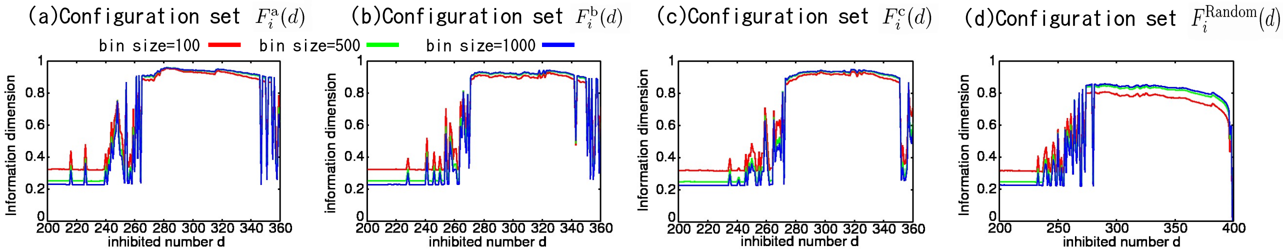

Figure 6: Information dimension without in-coming connections from the fragments and with connections.

sults, the model without in-coming connections from the fragments takes much larger values of Lyapunov dimension and information dimension than the model with the connections, suggesting much higher developed chaos.

For the configurations without the in-coming connections, the success ratio takes larger value and the accessing time becomes shorter as Lyapunov dimension and information dimension are larger. Thus, the instability of the chaotic orbit would realize the sensitive response to memory pattern fragments. On the other hand, for the configuration with the connections, we can not observe the correlation between the sensitivity and the instability of the orbit. We consider that different types of chaos would occur whether the configuration includes the inhibited in-coming synaptic connections from the elements of memory pattern fragments or not.

4. Conclusion

In this paper, we investigate the dependence of the sensitive responses of the chaotic wandering states to memory pattern fragments on the configuration of the inhibited in-coming synaptic connections in Nara & Davis chaotic neural network model. With the configuration which does not include the connections from the elements of memory pattern fragments, the model has quite higher potentiality in the accessing time and the success ratio than the configuration which includes the connections. In addition, for the configuration which does not include, the success ratio takes larger value and the accessing time becomes shorter as Lyapunov dimension and information dimension are larger. Thus, the instability of the chaotic orbit would realize the sensitive response to memory pattern fragments.

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