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A Newton-Type Algorithm for Determining Passive Elements Including in Class-E Amplifiers

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Abstract—A Newton-type algorithm for determining the passive elements including the class-E amplifiers is presented. To ensure the convergence of the Newton method, the initial estimation is efficiently provided by the PSO algorithm, where the responses of class-E amplifiers are approximately expressed in a closed form. It is demonstrated in the numerical example that the proposed method determines the passive elements efficiently.

1. Introduction

The class-E switching-mode circuits have become increasingly valuable components in many applications, e.g., radio transmitters, switching mode-dc power supplies, devices of medical applications, and so on. Since the class-E switching, namely, both zero voltage switching (ZVS) and zero derivative switching (ZDS), the class-E switching circuits can achieve high power conversion efficiency at high frequencies. However, the design of the class-E amplifiers is quite difficult because two switching conditions should be satisfied simultaneously on the steady-state of the circuits.

Since the invention of the class-E amplifier, many analytical descriptions of this circuit have been presented [1]-[2]. Although these treatments give useful guidance for designs of the class-E amplifiers, it is impossible for these design methods to consider all the effects of passive/active elements including in these circuits.

A design procedure for the generalized class-E amplifiers was given by using SPICE [3]. In this method, the design of the class-E amplifiers is regarded as determinations of steady-state waveforms and parameters for obtaining the class-E ZVS/ZDS conditions simultaneously. This method allows us to consider all the physical effects of devices within the mathematical model of circuit simulator. In this method, however, the incomplete set of variables is forced into the steady-state conditions. i.e., only the state variables with respect to the external passive elements are considered and the variables for the parasitic elements including in the device model are ignored. Therefore, the steady-state analysis of this method is not accurate. Instead of the Newton method, Particle Swarm Optimization (PSO) was introduced for design of the class-E amplifier [4], where the class-E amplifier is analyzed by a simulator which finds the steady-state directly. Although this method provides a good design of the class-E amplifiers, the stochastic nature of PSO algorithm requires huge number of estimations of objective function, which make the optimization method computationally inefficient.

In this paper, a Newton type algorithm is presented for the optimization of class-E amplifiers, where the circuit responses are calculated by HSPICERF, which is a simulator that finds the steady-state responses of nonlinear circuits. To ensure the convergence of the Newton method, a good initial estimation is required. Thus, the circuit responses of the class-E amplifiers are approximately expressed in a closed from. Next, the PSO algorithm is applied to finding the initial estimation for the Newton method. In the numerical example, it is confirmed that the Newton method with the initial estimation obtained by the PSO provides a good design of class-E amplifiers efficiently.

2. Newton Method for Design of Class-E Amplifier

A circuit topology of the class-E amplifier is shown in Fig. 1(a). The class-E amplifier consists of dc-supply voltage V_D, dc-feed inductor L_C, n-channel MOSFET S, shunt capacitor C_S, and series resonant circuit composed of inductor L_0, capacitor C_0, and output resistor R. For achieving high power conversion efficiency, it is effective that the zero voltage switching (ZVS) and zero derivative switching (ZDS) are fulfilled simultaneously at the turn-on instant of switch. These conditions are called the class-E ZVS and ZDS conditions which are written by

$$\text{ZVS:} \quad v_s(T) = 0, \tag{1}$$

$$\text{ZDS:} \quad \left.\frac{dv_s}{dt}\right|_{t=T} = 0, \tag{2}$$

where T is the switching period and we define that the switch turns on at t = kT for an integer k. Figure 2 shows a typical waveform of the switch voltage v_S when the class-E switching conditions are satisfied. The voltage v_S switches smoothly at T and $2T$ without switching losses. In oder to satisfy the conditions (1) and (2), design parameters such as values of the passive elements and device parameters of the MOSFET S should be adjusted optimally. Moreover, the class-E switching conditions should be satisfied on the steady-state of the circuit, which make design of the class-E amplifier difficult.

The resonant filters in the class-E amplifiers usually have a high Q value, which means that the transition time is long until it reaches the steady-state and the transient analysis of the class-E amplifier requires a large computational cost. It is uncertain how many cycles of input voltage D_r shown in Fig. 2 is necessary for obtaining the steady-state waveform. Hence, a method for finding the steady-state solution directly should be used for analyzing the class-E amplifiers and we use HSPICERF to obtain the responses.

In this paper, we present a procedure for determining the passive elements which satisfy the class-E ZVS and ZDS conditions. First, the nonlinear equations associated with the class-E ZVS and ZDS conditions are written by

$$\boldsymbol{f}(\boldsymbol{x}) \quad = \quad \boldsymbol{0}, \tag{3}$$

where $\boldsymbol{f} = (f_1(\boldsymbol{x}), f_2(\boldsymbol{x}))^T$, $f_1(\boldsymbol{x}) = v_S(T)$, $f_2(\boldsymbol{x}) = i_S(T) = C_S\, dv_s/dt|_{t=T}$, and $\boldsymbol{x}$ is a vector of values of passive elements. It should be noted that C_S is used as a scaling factor to the ZDS condition (2).

The nonlinear equations (3) are solved by the Newton method:

$$\boldsymbol{x}_{n+1} = \boldsymbol{x}_n - \boldsymbol{f}'(\boldsymbol{x}_n)^{-1}\boldsymbol{f}(\boldsymbol{x}_n), \tag{4}$$

where $\boldsymbol{f}'(\boldsymbol{x}_n)$ is the Jacobian matrix. To ensure the convergence of the Newton method, the linear search method with the direction

$$\boldsymbol{d} = -\boldsymbol{f}'(\boldsymbol{x}_n)^{-1}\boldsymbol{f}(\boldsymbol{x}_n) \tag{5}$$

is used. We find the smallest inter $m \geq 0$ such that

$$||\boldsymbol{f}(\boldsymbol{x}_n + 2^{-m}\boldsymbol{d})|| < ||\boldsymbol{f}(\boldsymbol{x}_n)||. \tag{6}$$

The elements of Jacobian matrix are obtained by giving a small perturbation ϵ to the design parameters and running the circuit simulation. For example, when the n-th solution of the Newton method as $\boldsymbol{x}_n = (x_1, x_2, \ldots, x_M)$ [1], $\partial f_1/\partial x_1$ is calculated by

$$\frac{\partial f_1}{\partial x_1} \quad = \quad \frac{f_1(x_1+\epsilon, x_2, \ldots, x_n) - f_1(x_1, x_2, \ldots, x_n)}{\epsilon}. \tag{7}$$

Although this method is similar to one in [3], it has only to determine the design parameters since the steady-state responses are calculated by HSPICERF. The steady-state responses are obtained by the time-domain shooting method on HSPICERF. Therefore, the proposed procedure is a multi-level Newton method.

[1]When the number of values is larger than 2, the generalized inverse matrix is used in (4).

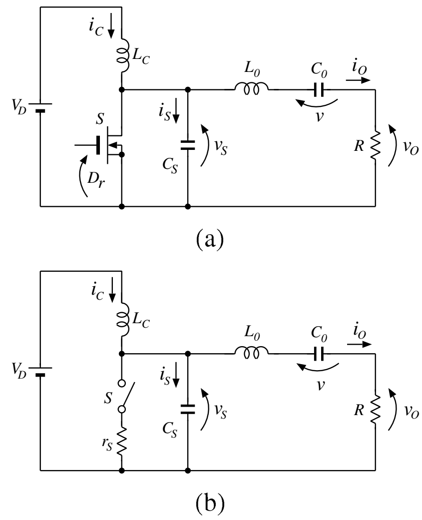


Figure 1: Class-E amplifier. (a)Circuit model with MOSFET. (b)Circuit model with ideal switch.

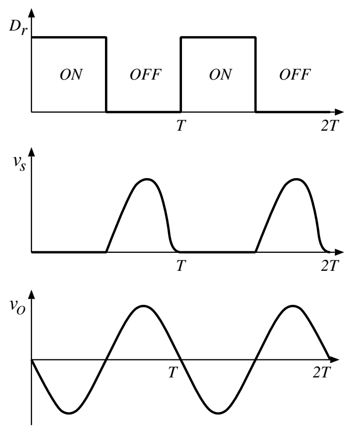
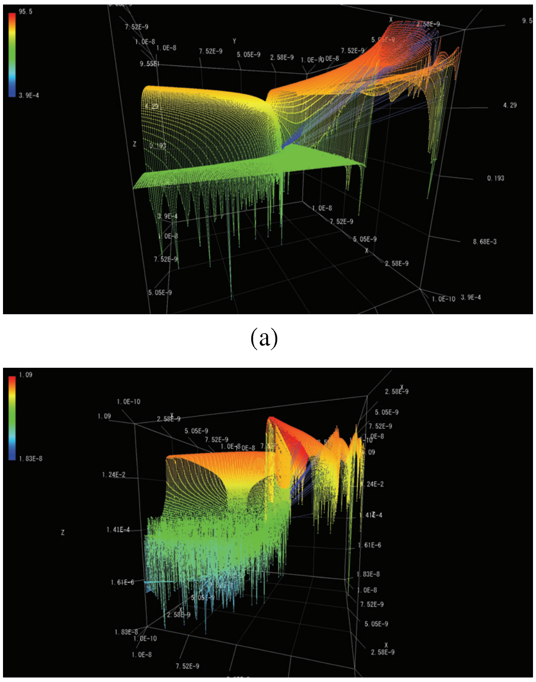


Figure 2: Capacitor voltage of the class-E amplifier.

3. Finding Initial Guess

3.1. Convergence of Newton Method

It is well-known that the Newton method requires a good initial estimation to converge on the solution. The convergence ratio depends on complexity of nonlinear functions. The nonlinear functions of (3) consist of the shunt capacitor voltage $f_1(\boldsymbol{x}) = v_S(T)$ and current $f_2(\boldsymbol{x}) = i_S(T)$. The surfaces of these functions are shown in Figs. 3(a) and 3(b), where there are many local minimums of these functions. Hence, we cannot ensure the convergence of the Newton method without a good initial guess. Since the surfaces are too complicated, a stochastic optimization technique is required to find the estimation. However, this is computationally expensive, because the circuit simulation by a simulator must be run repeatedly. Alternatively, the MOSFET of class-E amplifier shown in Fig. 1(a) is replaced with the



(a)

(b)

Figure 3: Surfaces of nonlinear functions associated with the ZVS and ZDS conditions, where the axes of the bottom plane are C_S and C_0. (a)$|v_S(T)|$. (b)$|i_S(T)|$.

ideal switch. The ZVS and ZDS conditions are expressed in a closed form. Next, finding the initial estimation is defined as an optimization problem and a stochastic method is applied to solving it. The following subsections provide the closed form expression and the optimization method.

3.2. Closed Form Expression

Since the circuit equation of Fig. 1(b) is classified into on the 'on' and 'off' states of the ideal switch S. The circuit equation at the on state is written by

$$\frac{d\boldsymbol{x}_{on}(t)}{dt} = \boldsymbol{\alpha_1}\boldsymbol{x}_{on}(t) + \boldsymbol{\beta}, \quad (0 \le t \le T/2) \quad (8)$$

where $\boldsymbol{x}_{on}(t) = [i_C(t), v_S(t), i_0(t), v(t)]_{on}$ is the state vector at the on state. $\boldsymbol{\alpha}_1$ and $\boldsymbol{\beta}$ are coefficient matrix and a constant vector related with DC voltage supply, respectively. As (8), the circuit equation at the off state is expressed by

$$\frac{d\boldsymbol{x}_{off}(t)}{dt} = \boldsymbol{\alpha}_2\boldsymbol{x}_{off}(t) + \boldsymbol{\beta}, \quad (T/2 \le t \le T) \quad (9)$$

where $\boldsymbol{x}_{off}(t) = [i_C(t), v_S(t), i_0(t), v(t)]_{off}$ is the state vector at the off state.

Equations (8) and (9) are both linear and time invariant. Thus, the solutions are expressed via the eigen decomposition of the coefficient matrix $\boldsymbol{\alpha}$: $\boldsymbol{\alpha}\boldsymbol{S} = \boldsymbol{S}\mathrm{diag}\{\lambda_1, \ldots, \lambda_4\}$, where $\lambda_1, \ldots, \lambda_4$ are the eigen values. The solutions are then written by

$$\boldsymbol{x}_{on}(t) = \boldsymbol{\gamma}_1\boldsymbol{x}(0) + \boldsymbol{\phi}_1, \quad (10)$$

$$\boldsymbol{x}_{off}(t) = \boldsymbol{\gamma}_2\boldsymbol{x}(T/2) + \boldsymbol{\phi}_2, \quad (11)$$

where

$$\boldsymbol{\gamma}_1 = \boldsymbol{S}\mathrm{diag}\{e^{\lambda_1 t}, \ldots, e^{\lambda_4 t}\}\boldsymbol{S}^{-1},$$

$$\boldsymbol{\phi}_1 = \boldsymbol{S}\mathrm{diag}\{\frac{e^{\lambda_1 t}-1}{\lambda_1}, \ldots, \frac{e^{\lambda_4 t}-1}{\lambda_4}\}\boldsymbol{S}^{-1}\boldsymbol{\beta}.$$

$\boldsymbol{\gamma}_2$ and $\boldsymbol{\phi}_2$ can be written similarly to $\boldsymbol{\gamma}_1$ and $\boldsymbol{\phi}_1$. From the steady-state condition; $\boldsymbol{x}_{on}(0) = \boldsymbol{x}_{off}(T)$, the initial conditions which give the steady-state responses are obtained as

$$\boldsymbol{x}(0) = (\boldsymbol{I} - \boldsymbol{\gamma}_2\boldsymbol{\gamma}_1)^{-1}(\boldsymbol{\gamma}_2\boldsymbol{\phi}_1 + \boldsymbol{\phi}_2)\boldsymbol{\beta}, \quad (12)$$

where $\boldsymbol{I}$ is the identity matrix.

In order to obtain the initial estimation for the Newton method provided in Sect. 2, it is defined as an optimization problem. The objective function is given by

$$\hat{f}(\hat{x}_1, \ldots, \hat{x}_n) = \sqrt{|\hat{v}_S(T)|^2 + |\hat{i}_S(T)|^2}, \quad (13)$$

where $\hat{x}_1, \ldots, \hat{x}_n$ are design parameters. $\hat{v}_S(T)$ and $\hat{i}_S$ are respectively the voltage of the shunt capacitor C_S and current flowing though it on the steady-state which are calculated by (12).

3.3. PSO

PSO is used for finding the optimum solution of (4). Let $\boldsymbol{x}_i(k)$ and $\boldsymbol{v}_i(k)$ respectively represent the position and velocity vectors of ith particle $(i = 1, \ldots, m)$ at the iteration step k. The position $\boldsymbol{x}_i(k)$, which is an n-dimension vector, provides the solution of the problem. In the dynamics of the standard PSO, which is called the gbest PSO, two kinds of the best positions, a global best position $\boldsymbol{p}_g(k)$ and a local best position $\boldsymbol{p}_i(k)$, are memorized. If a fitness of a position is better than its previous best position, it is stored as the local best position $\boldsymbol{p}_i(k)$. If the local best position $\boldsymbol{p}_i(k)$ is better than the global best position $\boldsymbol{p}_g(k)$, it is updated by $\boldsymbol{p}_i(k)$ [5]. The dynamics of the gbest PSO used in this paper is written in a set of difference equations:

$$v_{ij}(k+1) = \xi v_{ij}(k) + U(0,\varphi_1)\left(p_{ij}(k) - x_{ij}(k)\right) + U(0,\varphi_2)\left(p_{gj}(k) - x_{ij}(k)\right), \quad (14)$$

$$x_{ij}(k+1) = x_{ij}(k) + v_{ij}(k+1), \quad (15)$$

where $i = 1, \ldots, m$ and $j = 1, \ldots, n$. $U(0, \varphi)$ represents uniformly distributed random numbers in the range of $[0, \varphi]$. φ_1 and φ_2 are acceleration coefficients and ξ is an inertia weight.

Assume that an axis of the optimum solution exists in $[x_{min}, x_{max}]$. If the particles jump to outside of the region, the particles are forced to stay in the area as

$$x_{ij}(k+1) = U(0,1)\left(x_{max} - x_{min}\right) + x_{min}. \quad (16)$$

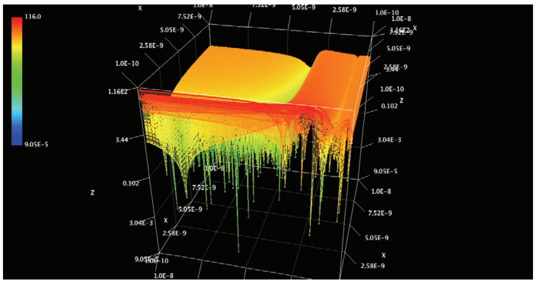
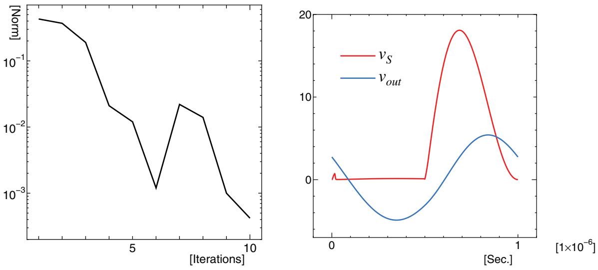


Figure 4: Surface of $|\hat{v}_S(T)|$ which is calculated by the closed form expression (12).



(a) (b)

Figure 5: (a)Convergence of the Newton method. (b)Steady-state responses after optimization.

Therefore, this method belongs to a constrained optimization one.

4. Results

For the design of the class-E amplifier, design specifications are given as $f = 1$MHz, $V_D = 5$V, $R = 5\Omega$, $L_C = 7.96$mH, and $L_0 = 7.96\mu$H. C_S and C_0 are selected as the design parameters. To obtain the initial estimation for the Newton method, $\varphi_1 = \varphi_2 = 1.6$, $\xi = 0.7$, and $m = 24$ are set as the PSO parameters of (14) and (15), and the search area is restricted as (x_{min}, x_{max}) = (1, 10)nF. After 1,000 iterations of the dynamics of PSO, $(6.187575, 3.631032)$nF is given. The Newton method with the initial estimation is applied to solving the solution, where the steady-state responses of Fig. 1(a) are calculated by HSPICERF. Figure 5(a) shows the convergence process of the Newton method, where the linear search method is terminated until 5 iterations. At 10 iterations of the Newton method, $(5.7267, 3.6222)$nF is obtained as an optimum solution. The nonlinear functions of (3) are evaluated totally 46 times for evaluating the convergence and making the Jacobian matrix in the Newton method. The values of the shunt capacitor's voltages are shown in Fig. 4, where the function values are calculated by the closed form expression (12). The surface is so complicated that finding the initial guess for the Newton method is not easy task. Nevertheless, the PSO algorithm in (14) and (15), efficiently finds the optimum solution, since the objective function is calculated by the closed form expression. Therefore, we can say that the Newton method with the initial estimation by the PSO algorithm is superior to the direct application of PSO [4]. Figure 5(b) shows the circuit simulation results after optimization, where the class-E ZVS and ZDS conditions are certainly satisfied.

5. Conclusions

An optimization procedure for design of the class-E amplifiers have been presented in this paper, where the passive elements are efficiently determined by the Newton method with a good initial estimation obtained by the PSO algorithm. Since the objective function for the initial estimation is complicated, the performance of the PSO algorithm is not necessarily high. Hence, we will improve the PSO algorithm in near future.

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References

[1] N. O. Sokal and A. D. Sokal, "Class E-A new class of high-efficiency tuned single-ended switching power amplifiers," *IEEE J. Solid-State Circuits*, vol. SSC-10, no. 3, pp. 168-176, Jun. 1975.

[2] T. Suetsugu and M. K. Kazimierczuk, "Analysis and design of class E amplifier with shunt capacitance composed of nonlinear and linear capacitances," *IEEE Trans. Circuits Syst. I*, vol. 51, no. 7, pp. 1261-1268, Jul. 2004.

[3] H. Sekiya, T. Ezawa, and Y. Tanji, "Design procedure for class E switching circuits allowing implicit circuit equations," *IEEE Trans. Circuits Syst. I*, vol. 55, no. 11, pp. 3688-3696, Dec. 2008.

[4] Y. Tanji, H. Matsushita, and H. Sekiya, "Acceleration of PSO for designing class e amplifier," in Proc. NOLTA'11, pp. 491-494, Sept. 2011.

[5] R. Poli, J. Kennedy, and T. Blackwell, "Particle swarm optimization –an overview," Swarm Intell, vol. 1, pp. 33-57, 2007.