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# Influence of Nonlinear Shunt Capacitance in Harmonic Distortion in Output Envelope of Drain Voltage Modulated Class E Amplifier

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***Abstract*— In this paper, distortion in the output envelope of AM modulated a class E RF power amplifier with a nonlinear capacitance with the grading coefficient $m = 0.5$ is analyzed. The calculation of output amplitude incorporates the influence of power loss versus dc supply voltage. Derivation of half cycle output voltage waveform is done with incorporating output amplitude as a function of dc supply voltage of the class E amplifier. The calculation was performed with Mathcad programming.**

## I. Introduction

The class E amplifier is remarked as next candidate of digital wireless power transmitters [1]-[3]. In the multi-carrier communications such as OFDM and QAM high peak-to-average power ratio (PAPR) is required for power amplifiers. Non switching type power amplifiers suffer from low efficiency due to their inherent power loss in high back off area. Hence, switching power amplifiers such as class D, E, F are remarked as a remedy for improving power efficiency and prolonging battery lifetime of portable devices. In order to modulate the amplitude and phase of output signal of switching amplifier, several methods were proposed. Among them envelope elimination and restoration (EER) modulation is remarked because it does not require power combining circuit at output port of the amplifier. In the EER modulation with a class E amplifier, output amplitude of the class E amplifier is modulated with varying dc supply voltage $V_{DD}$ of the class E amplifier [1]-[3]. It is well known that if the shunt capacitance of class E amplifier is linear, i.e., a constant value, the output amplitude of class E amplifier is in proportional to dc supply voltage $V_{DD}$ [5]. However, if the shunt capacitance is a nonlinear shunt capacitance, i.e., a function of voltage across capacitance, the output amplitude of class E amplifier is not proportional to the dc supply voltage [6] especially when $V_{DD}$ is a low voltage. This nonlinearity cause distortion in the output envelope of EER modulation, and other drain voltage modulation type signals.

In this paper, distortion in the output envelope of drain voltage modulated class E amplifier whose shunt capacitance is a nonlinear capacitance is analyzed. THD of the output envelope is obtained as a function of $V_{DD}$ . The shunt capacitance is supposed to be the output capacitance of MOSFET and it has a grading coefficient value of 0.5. Hence, it is described by following expression:

$$C_{ds} = \frac{C_{j0}}{\sqrt{1 + \frac{v_S}{V_{bi}}}}, \qquad (1)$$

where $C_{j0}$ is the transistor output capacitance at the drain-to-source voltage $v_S = 0$ and $V_{bi}$ is the built-in potential of the MOSFET body diode.

## II. EER Modulation of Class E Amplifier

EER modulation, which is shown in Fig. 1, imposes the varying envelope of the input signal on the power supply of the switching power amplifier, typically using a switching, DC-DC feedback power converter. Phase information is separated from the input signal to be a constant-envelope RF signal. This constant-envelope phase signal is fed to driver

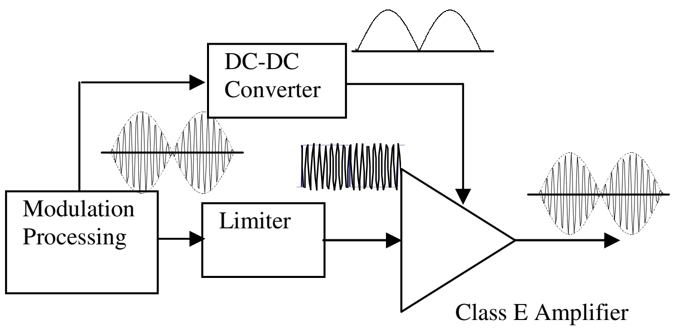


Fig. 1, EER diagram with class E amplifier.

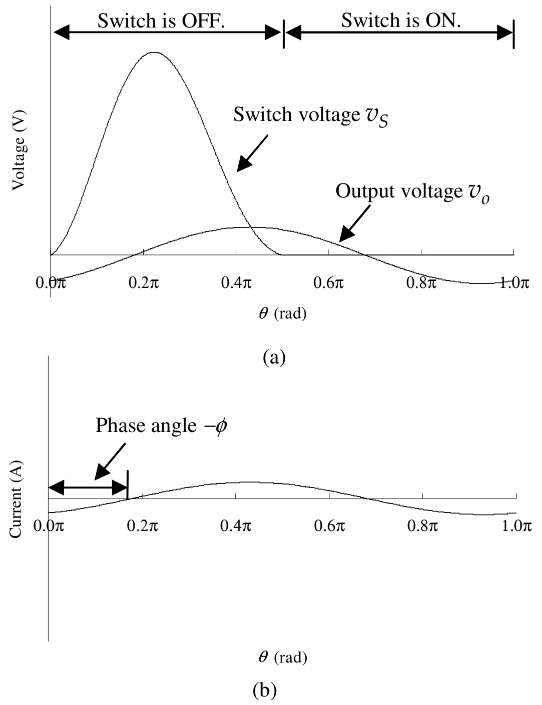


Fig. 3.Typical switch voltage waveform, output voltage waveform, and output current waveform of class E amplifier with nonlinear shunt capacitance in off-nominal operation with $D = 0.5$. (a) Switch voltage waveform $v_S$ and output voltage waveform $v_o$. (b) Output current waveform $i_o$.

port of switching power amplifier. Amplitude modulation of the final stage of the power amplifier restores the envelope to the phase-modulated carrier signal creating an amplitude replica of the input signal. Linearity in power supply of the power amplifier is easily obtained due to feedback control of DC-DC converter. However, linearity of output amplitude of power amplifier to input envelope has not been taken care well because influence of nonlinear shunt capacitance was not investigated yet. In [6], it was found that influence of nonlinear shunt capacitance is higher at low dc supply voltage. Hence, output envelope of the power amplifier distorted at zero cross point of envelope waveform.

## III. CLASS E AMPLIFIER WITH NONLINEAR SHUNT CAPACITANCE

Fig. 2 shows the zero-voltage slope switching type class E amplifier, which is analyzed in this paper. The choke inductor $L_{RFC}$ limits input current to be a constant current. The switch device turns on and off periodically and makes periodical voltage excitation across the switch. The switch voltage waveform $v_S$ is shown in Fig. 3. The switch is OFF for $0 < \theta \le \pi$ and ON for $\pi < \theta \le 2\pi$, namely the duty ratio is assumed to be 0.5, $\theta = \omega t$, and $\omega = 2\pi / T$ is the angular switching frequency. The output $LC$ circuit extracts the fundamental frequency component from the switch voltage waveform and the outputs sinusoidal voltage wave across the load resistance $R$. The waveforms of the output voltage $v_o$ and the output current $i_o$ are shown in Fig. 3. The output current is a sinusoidal wave, which can be expressed as

$$i_o = I_m \sin(\theta + \phi). \qquad (2)$$

In this paper, it is assumed that

1. The inductance of the choke coil $L_{RFC}$ is large enough to neglect its current ripple.
2. The internal resistance of the choke coil is zero; therefore, the DC voltage drop across the choke is very low so that it can be considered as zero.
3. The loaded quality factor $Q$ of the output resonant circuit is high enough so that the output current can be considered as a sine wave.
4. The load resistance includes parasitic resistances of the series resonant circuit, i.e., the resonant circuit is considered to be a pure reactance.
5. The MOSFET on-resistance is very low so that it can be considered as zero.
6. The MOSFET turns on and off very fast so that it can be considered to turn instantly.

## IV. STEADY-STATE RESPONSE OUTSIDE DESIGNED CONDITION

The switch current during switch on-state is described as

$$i_S = I_{DD} - I_m \sin(\theta + \phi), \qquad (3)$$

where $I_{DD}$ is the input current, $I_m$ is the amplitude of the output current, and $\phi$ is the phase angle of output current. The switch voltage waveform of the class E amplifier with nonlinear shunt capacitance for the grading coefficient 0.5 was derived in [7][8] and it is given by

$$v_S(\theta) = V_{bi}\left\langle \left\{ \frac{I_{DD}\theta + I_m[\cos(\theta+\phi) - \cos\phi]}{2V_{bi}\omega C_{j0}} + 1 \right\}^2 - 1 \right\rangle \text{ for } 0 < \theta \le \theta_d \qquad (4)$$

$$v_S(\theta) = 0 \quad \text{for } \theta_d < \theta \le 2\pi \qquad (5)$$

where $\theta_d$ is the phase angle at the body diode turning on. Hence, $\theta_d < \pi$ when $V_{DD} > V_{DDinitial}$ and $\theta_d = \pi$ when $V_{DD} \le V_{DDinitial}$. In this equation, the power loss due to the

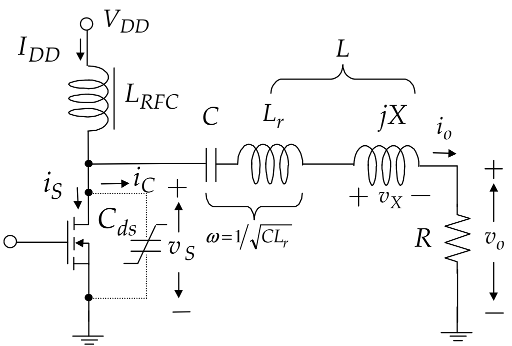


Fig. 2. Basic circuit of a class E amplifier with nonlinear shunt capacitance.

MOSFET on-resistance and the power loss due to the voltage drop of the body diode are neglected. (In the numerical results in following section, the power losses due to the on-resistance and the diode voltage drop are very low and they do not affect much operation of the class E amplifier.) The circuit behavior of the class E amplifier, which satisfies the assumptions are governed by the following three equations [5], valid not only for nominal conditions, but also for any conditions:

$$V_{DD} = \frac{1}{T}\int_0^T v_S dt \qquad (6)$$

$$RI_m = \frac{2}{T}\int_0^T v_S \sin(\omega t + \phi)dt \qquad (7)$$

$$XI_m = \frac{2}{T}\int_0^T v_S \cos(\omega t + \phi)dt\,, \qquad (8)$$

where $T$ is the switching period and $\phi$ is the phase angle between the switch and the output current shown in Fig. 3. Eq. (6) indicates that the dc voltage drop across the choke inductor is zero. Eq. (7) indicates that the real part of the fundamental frequency component of the switch voltage is equal to the voltage amplitude across the load resistance. Eq. (8) indicates that the imaginary part of the fundamental component of the switch voltage is equal to the fundamental component of the voltage across the reactance of the output $LC$ circuit.

Substituting (4) into (6)-(8) and solving the three equations, the solutions for three unknowns $I_{DD}$, $I_m$, $\phi$ are obtained. In this paper, we avoid long and erroneous hand manipulations, we introduced numerical analysis using mathematical software to obtain the solution.

In prior to calculating the circuit behavior outside the designed conditions, the initial values of circuit parameters should be determined. The initial values are the parameters for the designed state. Using the papers [5], [9], the initial circuit parameters are determined as

$$\phi = -\tan^{-1}\frac{2}{\pi} \qquad (9)$$

$$I_{DD} = \frac{2V_{DD}}{R_L}\sin^2\phi \qquad (10)$$

$$I_m = -\frac{I_{DD}}{\sin\phi} \qquad (11)$$

$$C_{j0} = \frac{12V_{bi}\sin^2\phi - \sin\phi\sqrt{6V_{bi}\left[\left(12V_{bi} - 24\pi^2 V_{DD} + \pi^4 V_{DD}\right)\sin^2\phi + 6\pi^2 V_{DD}\right]}}{12\pi V_{bi}\omega R_L} \qquad (12)$$

$$\phi_1 = \tan^{-1}\frac{2\left[\left(6\pi^2 - 48\right)V_{bi}\omega C_{j0} + \pi I_{DD}\right]}{24\pi V_{bi}\omega C_{j0} + \left(5\pi^2 - 32\right)I_{DD}} \qquad (13)$$

$$X = R_L\tan(\phi_1 - \phi). \qquad (14)$$

## V. RESULTS

With Mathcad calculation, $I_m$ versus $V_{DD}$ is obtained. Fig. 4 shows . $V_o = RI_m$ as a function of $V_{DD}$. As seen in Fig. 4, $V_o$ increases almost proportional to $V_{DD}$. However, it is slightly lower than $V_{DD}$ when $V_{DD} < 20\,\text{V}$ and slightly higher when $V_{DD} > 20\,\text{V}$ . In this calculation, design conditions were $V_{DD} = 20\,\text{V}$ , $f = 4\,\text{MHz}$ , $R = 88.8\,\Omega$ , and $V_{bi} = 0.7\,\text{V}$ . Then, the calculated circuit parameters were $L = 35.3\mu\text{H}$ , $C = 51.4\,\text{pF}$ , and $C_{j0} = 408\,\text{pF}$ . Fig.5 shows estimated output envelope in solid line and input envelope in dot line when $V_{DD}(t)$ is a sinusoidal waveform

$$V_{DD}(t) = V_{DDm}\left|\sin 2\pi f_0 t\right| \qquad (15)$$

where frequency of envelope $f_0$ is enough lower than carrier frequency $f$ . Output waveform is described as

$$v_o(t) = RI_m\left(V_{DDm}\left|\sin 2\pi f_0 t\right|\right)\sin(\omega t + \phi) \qquad (16)$$

Hence, the envelope of output voltage is described as

$$V_o(t) = RI_m\left(V_{DDm}\left|\sin 2\pi f_0 t\right|\right) \qquad (17)$$

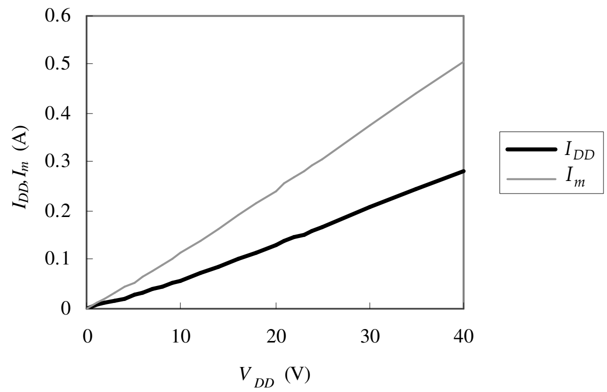


Fig. 4. Input current $I_{DD}$ and output current amplitude $I_m$ versus supply voltage $V_{DD}$ .

## VI. CONCLUSIONS

In this paper, distortion of output voltage of class E amplifier when nonlinear shunt capacitance is considered was analyzed. Distortion of output voltage is higher when dc supply voltage is low. Therefore, influence of nonlinear distortion is relatively low when amplitude of input signal is high.

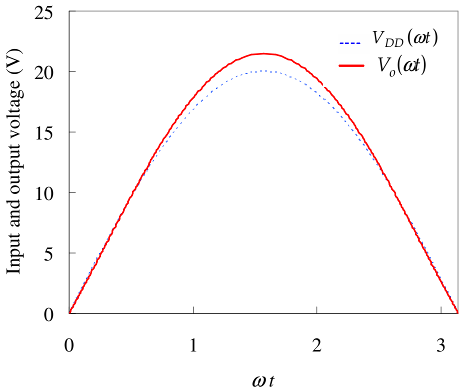


Fig. 5.Calculated half cycle waveform of sinusoidal input voltage $V_{DD}$ and envelope of corresponding output voltage $V_o$.

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