Theoretical and Experimental Study on Statistics of Electromagnetic Fields above Concrete Random Rough Surface Model

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1. Introduction

Recently, a rapid progress has been made in the area of wireless sensor networks to gather physical data and to control natural environment [1]. Sensor devices are usually distributed on dessert, hilly and sea surfaces. They are assumed to be random rough surfaces, and thus it is important to investigate propagation characteristics of electromagnetic waves traveling along rough surfaces and its statistical properties above rough surface since the sensor devices located on rough surface are much influenced by scattered waves from random rough surface.

In this paper we investigate statistics of electromagnetic field above random rough surfaces from a view point of experiment and numerical simulation based on the Finite Volume Time Domain (FVTD) method [2]. Experiments using a concrete rough surface model are carried out in a radio anechoic chamber. Using a probability paper, we consider whether the statistics of electromagnetic fields agree with a Nakagami-Rice distribution which is well known as statistics of electromagnetic fields in urban area.

2. FVTD Method

The FVTD method has widely been applied to the numerical computation of the electromagnetic fields [2]. However, it requires much computer memory. As a result, when the computational size is much larger than the wavelength, it is difficult to perform the FVTD computations by use of a small computer such as a PC. We select extrapolated absorbing boundary condition (EABC) [3] as boundary condition. The accuracy of EABC is a little worse than PML but it is better than Mur's ABC. From a view point of computer memory, however, its performance is much better than PML.

We first normalize the magnetic field as follows:

$$\tilde{\boldsymbol{H}} = Z_0 \boldsymbol{H} = \sqrt{\mu/\epsilon}\boldsymbol{H}. \tag{1}$$

where permitivity $\epsilon = \epsilon_0\epsilon_r$, permeability $\mu = \mu_0\mu_r$. In this paper, we investigate 2D propagation problem above 1D random rough surfaces. Therefore, electromagnetic fields can be decomposed to two different types of independent polarization, E and H waves. The E wave has a polarization described as $\tilde{\boldsymbol{H}} = (\tilde{H}_x, \tilde{H}_y, 0)$ and $\boldsymbol{E} = (0, 0, E_z)$. 2D FVTD equations for E-wave are given as follows [2]:

$$\begin{aligned} \tilde{H}_x^{n'+1}(i,j) &= \tilde{H}_x^{n'}(i,j) - \Lambda_y[E_z^n(i,j+1) - E_z^n(i,j-1)] \\ \tilde{H}_y^{n'+1}(i,j) &= \tilde{H}_y^{n'}(i,j) + \Lambda_x[E_z^n(i+1,j) - E_z^n(i-1,j)] \\ E_z^{n+1}(i,j) &= E_z^n(i,j) \\ &\quad + \Gamma_x[\tilde{H}_y^{n'+1}(i+1,j) - \tilde{H}_y^{n'+1}(i-1,j)] - \Gamma_y[\tilde{H}_x^{n'+1}(i,j+1) - \tilde{H}_x^{n'+1}(i,j-1)] \end{aligned} \tag{2}$$

where the step parameters are defined by

$$\begin{aligned} \Lambda_{x,y,z} &= (c\Delta t)/(2\mu_r\Delta_{x,y,z}) \\ \Gamma_{x,y,z} &= (c\Delta t)/(2\epsilon_r\Delta_{x,y,z}) \ . \end{aligned} \tag{3}$$

On the other hand, in case of H-wave with polarization described as $\tilde{\boldsymbol{H}} = (0, 0, \tilde{H}_z)$ and $\boldsymbol{E} = (E_x, E_y, 0)$, 2D FVTD equations are given as follows [2]:

$$
\begin{aligned}
\tilde{H}_z^{n'+1}(i,j) &= \tilde{H}_z^{n'}(i,j) \\
&\quad + \Lambda_y[E_x^n(i,j+1) - E_x^n(i,j-1)] - \Lambda_x[E_y^n(i+1,j) - E_y^n(i-1,j)] \\
E_x^{n+1}(i,j) &= E_x^n(i,j) + \Gamma_y[\tilde{H}_z^{n'+1}(i,j+1) - \tilde{H}_z^{n'+1}(i,j-1)] \\
E_y^{n+1}(i,j) &= E_y^n(i,j) - \Gamma_x[\tilde{H}_z^{n'+1}(i+1,j) - \tilde{H}_z^{n'+1}(i-1,j)].
\end{aligned}
\tag{4}
$$

3. Nakagami-Rice

Since the propagations in a urban area are described well by multi path model which takes into account many rays, incident, reflection, transmission and diffraction rays, the statistical properties of the field envelopes are given by the Nakagami-Rice probability density and distribution functions [4]. They are written as follows:

$$p(r, \sigma, s) = \frac{r}{\sigma^2} exp\left(-\frac{r^2 + s^2}{2\sigma^2}\right) I_0\left(\frac{sr}{\sigma^2}\right) \tag{5}$$

$$F(r, \sigma, s) = \int_0^r \frac{r}{\sigma^2} exp\left(-\frac{r^2 + s^2}{2\sigma^2}\right) I_0\left(\frac{sr}{\sigma^2}\right) dr \tag{6}$$

where $s^2/2$ is the averaged incident power, $\sigma^2/2$ is the averaged scattered power and $I_0(x)$ is the modified Bessel function of the first kind. These probability density and distribution functions are well-known in case of line of sight (LOS) in urban or suburban. When the receiver is non line of sight (NLOS), the incident power vanishes ($s \to 0$), and Eqs.(5) and (6) are reduced to the following well-known Rayleigh density and distribution functions:

$$p(r, \sigma) = \frac{r}{\sigma^2} exp\left(-\frac{r^2}{2\sigma^2}\right) \tag{7}$$

$$F(r, \sigma) = \int_0^r \frac{r}{\sigma^2} exp\left(-\frac{r^2}{2\sigma^2}\right) dr \quad . \tag{8}$$

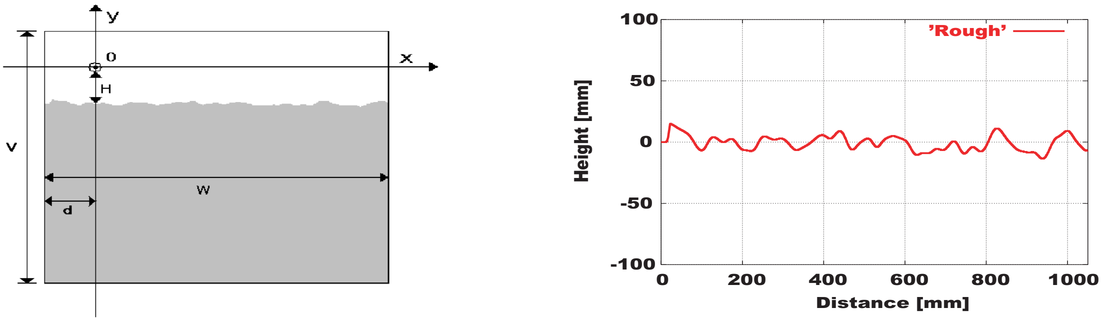
It is enough to consider only Nakagami-Rice distribution because the Rayleigh distribution is a special form of the Nakagami-Rice distribution.

In the present numerical simulations, we first gather the field intensity distribution data by using the FVTD. Second we obtain a discrete histogram from the data, and finally we have discrete probability density and distribution functions. The probability distribution of a sampled data should be compared with the theoretical one using estimated parameters by using the least mean square method. The data obtained by experiment is also carried out same as the numerical simulation.

It is convenient to use a probability paper to check the statistical property of the sampled data, because it is enough only to check whether the data lies on a straight line or not.

4. Comparison of Experiment and Numerical Simulation

Figure 1 shows a concrete random rough surface model. Experiment has been carried out for polarization E-wave in an anechoic chamber, using a monopole antenna with 10[GHz]. Source point is set on the origin as d= 5λ, and its height is $H = 150[mm]$ from the average height of rough surface. In the experiment, the measurement size for vertical is v= 35λ. The observation points are determined such as region I ($10\lambda < x < 15\lambda$, $2.5\lambda < y < 7.5\lambda$), region II ($16\lambda < x < 21\lambda$, $2.5\lambda < y < 7.5\lambda$) and region III ($23\lambda < x < 28\lambda$, $2.5\lambda < y < 7.5\lambda$). The length of concrete random rough surface model is w= $1050[mm]$, and its height deviation is $h = 5.9[mm]$. When we estimate the correlation length (cl), we need a random rough surface with length more than 458 times of correlation length [5]. In this concrete model, it is difficult to make such a long rough surface. Therefore, we have approximately determined



(a) Geometry of the problem.

(b) Surface profile of concrete rough surface model.

Figure 1: Concrete random rough surface model.

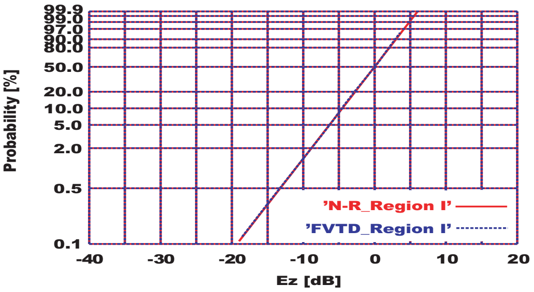


Figure 2: Comparison of numerical simulation with Nakagami-Rice distribution (Region I).

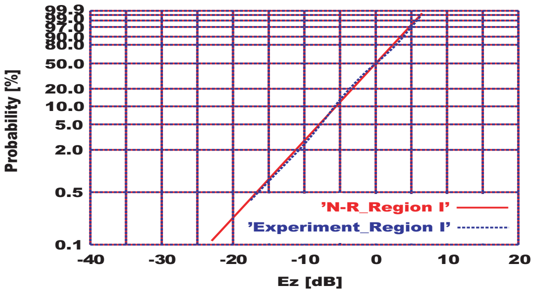


Figure 3: Comparison of experiment data with Nakagami-Rice distribution (Region I).

the correlation length as $cl = 24[mm]$ by evaluating the auto correlation corresponding to $1/e$. In the numerical simulations, we have selected parameters as frequency $f = 10.0[GHz]$, dielectric constant $\epsilon_r = 4.5$ and conductivity $\sigma = 0.1[S/m]$.

Figure 2 shows a comparison of field distribution computed by FVTD simulation in region I with Nakagami-Rice distribution on a probability paper. It is shown that simulated data is in line with the Nakagami-Rice distribution. Figure 3 shows a comparison of field distribution obtained by experiment in region I with Nakagami-Rice distribution on a probability paper. It is also shown that experimental data is in line with the Nakagami-Rice distribution. It is found in these figures that the field distributions above random rough surface agree with Nakagami-Rice distribution.

Figure 4 shows a comparison of field distribution computed by FVTD simulation in region II with Nakagami-Rice distribution on a probability paper. It is shown that simulated data is in line with the Nakagami-Rice distribution. Figure 5 shows a comparison of field distribution obtained by experiment in region II with Nakagami-Rice distribution on a probability paper. It is also shown that experimental data is in line with the Nakagami-Rice distribution.

Figure 6 shows a comparison of field distribution computed by FVTD simulation in region III with Nakagami-Rice distribution on a probability paper. It is shown that simulated data is in line with the Nakagami-Rice distribution. Figure 7 shows a comparison of field distribution obtained by experiment in region III with Nakagami-Rice distribution on a probability paper. It is demonstrated that experimental data is slightly different from the Nakagami-Rice distribution, however, we can neglect that error.

5. Conclusion

We have investigated statistical properties of electromagnetic fields above random rough surface from view point of experiment and numerical simulation based on the FVTD method. In the numerical simulation, we have used EABC as absorbing boundary condition to save much computer memory. Experiments using concrete random rough surface model were carried out in an anechoic chamber. It

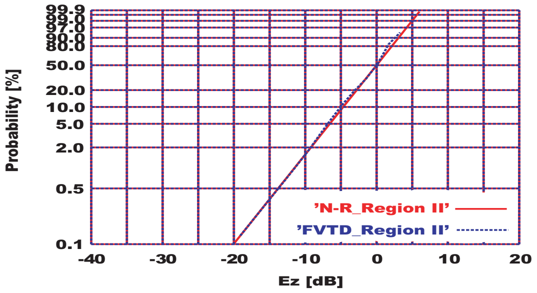


Figure 4: Comparison of numerical simulation with Nakagami-Rice distribution (Region II).

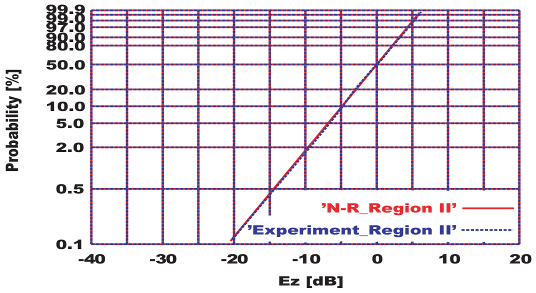


Figure 5: Comparison of experiment data with Nakagami-Rice distribution (Region II).

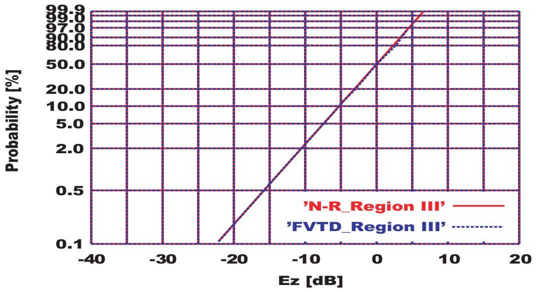


Figure 6: Comparison of numerical simulation with Nakagami-Rice distribution (Region III).

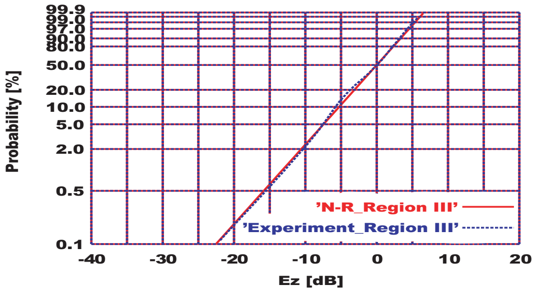


Figure 7: Comparison of experiment data with Nakagami-Rice distribution (Region III).

has been found that the field distribution computed by FVTD are in good agreement with Nakagami-Rice distribution, and it has been also shown that the field distributions obtained by experiment are in good agreement with Nakagami-Rice distribution.

In the present numerical simulation we have treated 1D random rough surface because of limitation of computer memory. We need to investigate statistics of electromagnetic fields above 2D random rough surfaces, and we have to compare numerical data with experimental data. This study will be our near future work.

Acknowledgments

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