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The integration/segregation phenomena in evolving complex networks

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Abstract—We study the integration/segregation problem from the viewpoint of complex networks, but considering that the network topology is not static but there is an adaptive mechanism acting on the links. Our goal is to identify under which conditions network synchronization occurs and what structural properties are present in the network topology when this happens. In particular, we experimentally compute the main descriptive structural properties of the network when it has been modified with the proposed mechanism, and it is elucidated the relationship between these results and the observed synchronization at both the local and global scale. Our main finding is that modularity, a global feature, can naturally emerge in a network when evolving links are considered, that is, by means of dynamical properties at the local scale.

1. Introduction

The majority of articles on complex networks have focused, until recently, on either the local scale structure of real complex systems or their macroscopic properties. However, neither of these descriptions can adequately describe the important features that complex systems exhibit due to their organization in modules. Since the seminal work of Girvan and Newman [1] on uncovering the modular network structure in social and biological networks, it was evident that nature exhibits, in many cases, *communities* (groups of highly interconnected nodes that are sparsely connected to the rest of the network). Such a modular organization has only recently been recognized to be crucial in the way in which a complex system works [2].

Among the questions that must be addressed in order to provide a full understanding of modularity, one of the most important is that of growth and formation of such mesoscales in complex systems. Most existing models for network growth introduce modularity through topological arguments [3]. Yet, very little attention has been paid to mechanisms based on the dynamical behavior of the components, a common feature in many real systems. The modeling of growing mechanisms that enable the description of the real evolutionary processes involved in the formation of mesoscales is necessary.

On the other hand, another important open problem is the role of mesoscales in the production of a collective and coordinated dynamics. It is evident that the existence of communities in a task-performing network is closely related to the coexistence of two seemingly but not fundamentally opposite phenomena: the establishment of collective subtasks in the network (*segregation* of the network) and the coordination of those subtasks at a global scale (*integration*). The hierarchical nature of the function of complex systems is not yet fully elucidated; a thorough treatment of the relation between network structure and its dynamics at mesoscale level is also of interest.

A nice example of this open topic can be found in the neural system [4, 5, 6], for both the visual and audio areas, in which it is observed a functional segregation of anatomically and physiologically different areas and a global integration resulting in a unique perception. It is, then, needed to examine the relationship between the structural substrate of a complex system (its local scale) and its effective dynamics (its global scale) taking into account the connectivity patterns at the mesoscale.

In particular, a basic coordination task is *synchronization*: an ensemble of individuals adjust their inner dynamics to that of the others. This is found in neural ensembles but other examples include fireflies flashing at the same rhythm, the muscle cells in the heart simultaneously beating, and an audience of a concert all applauding in unison to convince the artist to perform an encore [7].

In this paper, we present a model for complex networks in which the formation of the mesoscale is dynamically driven by a simple adaptive rule. More precisely, we consider a network formed by Kuramoto oscillators, coupled through evolving links. The links are defined to be bistable; those coupling nodes with similar dynamics are reinforced, while those linking non-synchronous nodes are weakened. We show that the presence of modularity is related to the emergence of collective subtasks, which, in turn, are globally coordinated. The rest of the paper is structured as follows: Section 2 presents the iterative decentralized mechanism for adapting the network topology, Section 3 provides the experimental results, and finally in Section 4 we conclude the present work.

2. Topological Adaptation under a Synchronization Process

We study the dynamics of synchronization on an undirected network through a *Kuramoto model* [8, 9]: each node $n \in \{1, \ldots, N\}$ is characterized by a *phase* θ_n and an *intrinsic frequency* ω_n. The underlying network topology is a complete graph: each node is linked to all the other nodes; a *coupling strength* parameter σ determines how strongly the phase of a node is affected by those of the other nodes. In this work we regard σ as a global constant.

We additionally introduce a *weight* for each link, thus producing a weighted network; we denote by $w_{mn} \in [0, 1]$ the weight of the link between the nodes m and n. Links that are beneficial to an improvement in the degree of synchronization are reinforced, whereas links that damage it will be weakened. Such an evolution of the link weights in response to the network synchronization is carried out in a bistable fashion: each weight will asymptotically tend to one of the values in $\{0, 1\}$, thus giving rise to an unweighted final network topology.

2.1. Weight Evolution

Using the established notation and with the introduction of the weight term, the individual dynamics over time for the node n in our modified Kuramoto model are expressed as

$$\dot{\theta}_n = \omega_n + \frac{\sigma}{N} \sum_{m=1}^{N} w_{mn} \sin(\theta_m - \theta_n). \qquad (1)$$

We propose an evolution of the link weights, parametrized by a constant p_c, to be carried out in the following manner:

$$\dot{w}_{mn} = (p_{mn} - p_c) w_{mn} (1 - w_{mn}), \qquad (2)$$

where p_{mn} is the *phase correlation* of the nodes m and n,

$$p_{mn} := \frac{1}{2} \left| e^{i\theta_m(t)} + e^{i\theta_n(t)} \right|. \qquad (3)$$

The phase correlation yields one for pairs of nodes that have equals phases and zero for pairs that have opposite phases, $\theta_n = \theta_m \pm \pi$. The parameter p_c is the *correlation threshold*: a link weight will be reinforced when $p_{mn} > p_c$ and weakened when $p_{mn} < p_c$; for $p_{mn} = p_c$ the derivative will be zero and no change is made.

2.2. Measures of Global and Local Synchronization

It is important to note that the equality of phases is merely an instantaneous measure and does not imply long-term synchronization as each node has its individual intrinsic frequency. The degree of long-term *global synchronization* in the Kuramoto model is traditionally measured in [0, 1] as

$$R_g := \left\langle \frac{1}{N} \left| \sum_{n=1}^{N} e^{i\theta_n(t)} \right| \right\rangle_t. \qquad (4)$$

When $R_g \approx 1$, the network is considered synchronized, whereas $R_g \approx 0$ is a sign of asynchronous behavior. However, a network may be globally unsynchronized and yet have groups of nodes that — locally — are highly synchronized. Measuring the degree of local synchronization requires another approach. We measure the degree of *local synchronization* at a node n as

$$R_\ell^n := \left\langle \left| \frac{\sum_{m=1}^{N} w_{mn} e^{i\theta_m}}{\sum_{m=1}^{N} w_{mn}} \right| \right\rangle_t \qquad (5)$$

and then average over this measure to obtain the degree of local synchronization in the network as a whole at a given time:

$$R_\ell := \frac{1}{N} \sum_{n=1}^{N} R_\ell^n. \qquad (6)$$

2.3. Modularity

After the weight evolution mechanism of Section 2.1 converges to a final unweighted topology, we measure structural properties of the resulting network. In the following, we still refer to the link weights as w_{mn}, but now they only take on values in $\{0, 1\}$, where zero means that the link is not present in the network and one means that it is.

The presence of communities can be quantified in terms of *modularity*, as proposed by Newman [10],

$$\mathcal{M} = \frac{1}{2M} \sum_{m,n} \left(w_{mn} - \frac{k_m k_n}{2M} \right) \delta(c_m, c_n), \qquad (7)$$

in which the variable c_n indicates the *cluster* to which node n belongs, δ is the Kronecker delta function, and M is the number of links in the network. We compute the modularity based on a community structure obtained by means of the algorithm of Blondel *et al.* [11]. Notice that this community structure is not necessarily the one optimizing modularity, as this constitutes a computationally demanding problem in itself.

3. Results

In this section, we present numerical experiments on the proposed model for adaptive topology creation by weight evolution in a modified Kuramoto model. We begin by discussing the parameter selection and then present results of the global and local degree of synchronization and the values of the structural properties measured, as defined in the previous section.

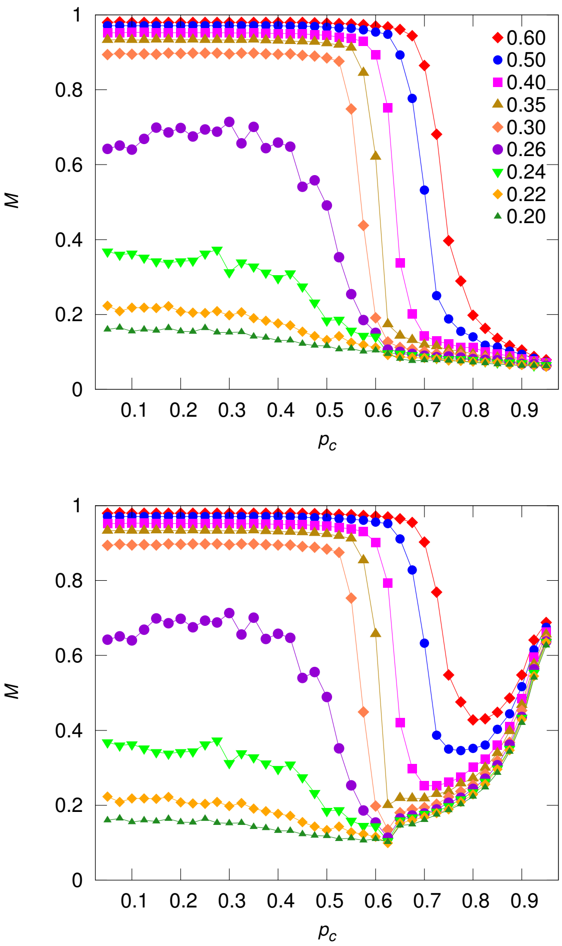

Figure 1: On the top, the global synchronization is depicted using Eq. 4. At the bottom, it is the local synchronization measured with Eq. 5. In both plots, the horizontal axis is the correlation threshold p_c and each curve corresponds to a different value of σ.

3.1. Parameter Selection

The reported experiments were carried out with complete networks of $N = 300$, meaning that the initial topology has $N(N-1)/2 = 44850$ links. The intrinsic frequency ω_n of each node was chosen independently and uniformly at random in the interval $[0.8, 1.2]$. This was chosen to balance the time scales of the node and link dynamics as to keep the link dynamics slower, letting the nodes complete several cycles before the links converge. Kuramoto points out that the initial phases are not relevant for the final state of synchronization in the original model [7, 9]; hence we choose each θ_n uniformly at random in the interval $[0, 2\pi)$. The initial values for the link weights w_{mn} were chosen uniformly at random in the interval $(0, 1)$.

The first parameter varied in the experiments is the correlation threshold p_c from 0 to 1 with increments of 0.025. The second parameter was σ running from 0.2 to 0.6. Our initial experiments revealed that the mechanism was less sensitive in some ranges, thus we used the following values of σ: $\{0.20, 0.22, 0.24, 0.26, 0.30, 0.35, 0.40, 0.50, 0.60\}$. For each combination of p_c and σ, we made 100 repetitions to report the average values over all sets.

3.2. Results Regarding the Degree of Synchronization

Figure 1 shows the degree of global and local synchronization of Eq. 4 and Eq. 5, respectively, in the final topology weight evolution under different values of σ and p_c. A transition is observed at $\sigma \approx 0.26$ for values of p_c approximately from 0.05 up to 0.70. In this transition we move from having global and local synchronization (the region on the left) to have no global synchronization but local synchronization (the region on the right). Interestingly, the critical coupling strength coincides with the one predicted for the phase transition of the Kuramoto model [8, 9].

3.3. Results Regarding Structural Properties

Modularity, shown in Figure 2 experiences a rapid transition at $p_c \approx 0.65$ over the combinations (p_c, σ). We observe that for $p_c > 0.650$, the final topology is increasingly modular (using the definition of Eq. 7), regardless of the value of σ.

We find that the resulting network for p_c below the transition is a completely connected graph and, as p_c is increased, we observe that the final topology has less and less links, giving rise to communities and, consequently, incrementing the modularity. The specific value of p_c for which transition appears depends on the initial distribution of the natural frequencies, $g(\Omega)$. However, in all cases, there is a transition to a modular network.

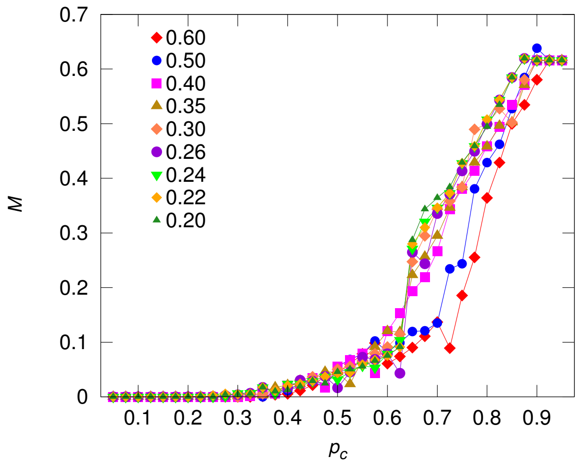

Figure 2: Modularity as defined in Eq. 7. The plot, covering the (p_c, σ) combinations described in the text, has the correlation threshold p_c on the horizontal axis and modularity on the vertical axis, for the different values of the coupling strength σ.

4. Conclusions

We have proposed an adaptive, bistable model for network synchronization. We implement both local and global synchronization measures. The proposed rule for link-weight evolution avoids having to define an initial network topology instead of having to choose a generation model, and then optimizes the network topology for easier synchronization. We also note that even in the absence of global synchronization, a high-degree local synchronization can prevail. The experimental results show that the proposed model is adequate for studying synchronization in conjunction with topology formation, due to its bistable nature.

We characterize the resulting network topologies in terms of modularity and observe that synchronization is improved in networks that are modular. This is of interest as existing literature focuses on studying synchronization under pre-imposed topologies; our approach allow the study of how synchronization and alike processes affect the formation of communities, to complement existing work on how the presence of communities affects synchronization.

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