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Yutaro Yamashita, Hiroyuki Torikai

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# Bifurcation-based learning of a PWC spiking neuron model

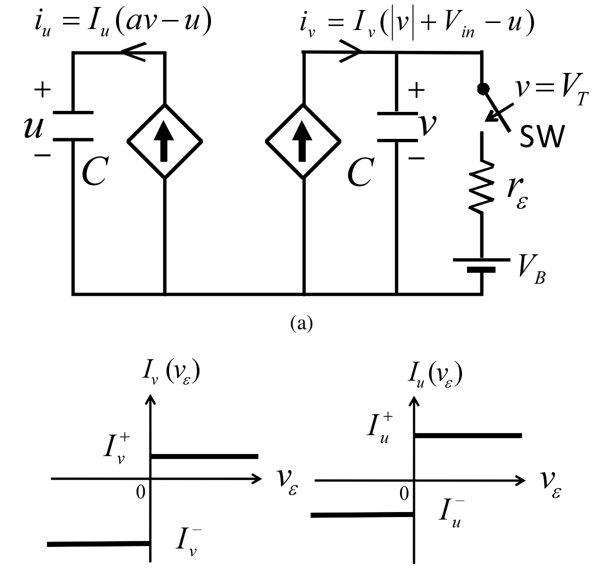
Yutaro Yamashita[†] and Hiroyuki Torikai[‡]

Graduate School of Engineering Science, Osaka University
1–3 Machikaneyama, Toyonaka, Osaka, 560–8531 Japan
Email: †yutaro@hopf.sys.es.osaka-u.ac.jp, ‡torikai@sys.es.osaka-u.ac.jp

**Abstract**—A *piece-wise constant* (ab. PWC) spiking neuron model (ab. PWN), which can reproduce various bifurcations observed in standard neuron models, is introduced. Using knowledge of bifurcations of the PWN, a heuristic but powerful learning method for the PWN is proposed. It is shown that the PWN can learn a typical response of the Izhikevich model which is also observed in not only other standard neuron models but also biological neurons.

## 1. Introduction

Neurons exhibit various responses depending on stimulation inputs and parameter values. Many mathematical neuron models and related analog circuit implementations have been studied intensively, where most models are described by continuous *ordinary differential equations* (ab. ODEs) like the Hodgkin-Huxley model, continuous ODEs with state-dependent resets like the Izhikevich model, and piece-wise smooth ODEs with/without resets [1]-[13]. Recently, an alternative hardware-oriented neuron modeling approach by using a *piece-wise constant* (ab. PWC) ODE with a state-dependent reset has been proposed [14]-[16]. It has been confirmed that the PWC spiking neuron model (ab. PWN) can reproduce a variety of neuron-like responses and related bifurcation phenomena. The bifurcation phenomena have been analyzed so far, and the sufficient parameter spaces for existence of some responses have been derived. In this paper, we propose a heuristic but powerful learning method for the PWN that utilizes the knowledge of the bifurcation phenomena. It is shown that the PWN can learn a response of the Izhikevich model which is also observed in not only other standard neuron models but also biological neurons. Novelties and significances of this paper include the following points. **(1)** This paper shows the learning method for the PWN for the first time. **(2)** The neural prosthesis is a recent hot topic, where a typical approach is to prosthesize a damaged part of neural systems by a digital processor [17, 18]. On the other hand, sensory neurons should be prosthesized by analog circuits since sensory neurons accept analog signals and it is not so efficient to utilize digital processor neurons together with analog-to-digital converters to implement them. The learning method for the PWN will be useful for parameter tuning before implementation. Note that different kinds of neurons have different parameter values.

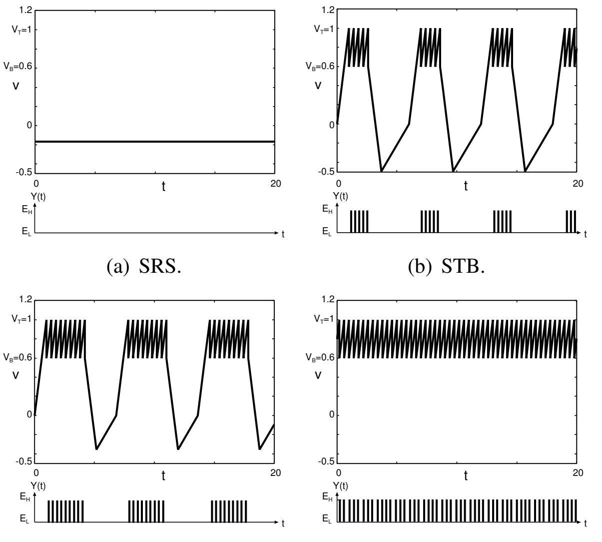



| PWC spiking neuron | Meaning as a neuron model |
|---|---|
| Capacitor voltage $v$ | Membrane potential |
| Capacitor voltage $u$ | Recovery variable |
| Constant voltage $V_T$ | Spiking threshold |
| Pulse $Y = E_H$ | Firing spike (Action potential) |
| Voltage input $V_{in}$ | Stimulation input |

Figure 1: (a) The generalized piece-wise constant spiking neuron model (ab. PWN). (b) Characteristics of the *voltage-controlled current source* (ab. VCCS).

## 2. Piece-wise Constant Spiking Neuron Model

A *piece-wise constant* spiking neuron model (ab. PWN) is introduced in Fig.1(a). The PWN consists of two capacitors whose capacitances are $C$ and $C$, two *voltage-controlled current sources* (ab. VCCSs) which are described by functions $I_v$ and $I_u$, a state-dependent switch $SW$ with an internal resistor $r_\varepsilon$, a voltage source $V_B$, and an output $Y$. From a neuron model's viewpoint, the capacitor voltages $v$ and $u$ can be regarded as a *membrane potential* and a *recovery variable* [1], respectively, as explained in the table in Fig.1. Fig.1(b) shows characteristics of each VCCS: it outputs a constant current if the control voltage $v_\varepsilon$ is positive and outputs another constant current if $v_\varepsilon$ is neg-



(a) SRS. (b) STB.

(c) STB. (d) STS.

Figure 2: Burst-related bifurcation phenomena of the PWN. The parameters of the PWN are $a = 5.0$, $I_v^+ = 1.0$, $I_v^- = -1.0$, $I_u^+ = 0.3$, $I_u^- = -0.3$, $V_T = 1.0$, $V_B = 0.6$, and $C = 1.0$. (a) *Stable resting state*. $V_{in} = -1$. The PWN doesn't oscillate and output any spikes. (b) *Stable tonic bursting*. $V_{in} = 1$. The PWN oscillates and output bursting spikes periodically. (c) *Stable tonic bursting*. $V_{in} = 3$. The PWN oscillates and output bursting spikes periodically. (d) *Stable tonic spiking*. $V_{in} = 5$. The PWN oscillates and output single spikes periodically The changes of phenomena between (a) and (b), and (c) and (d) are *border-collision bifurcations* [19].

ative. If the membrane potential $v$ is below a constant voltage $V_T$, the state-dependent switch $SW$ is opened. From a neuron model's viewpoint, the constant voltage $V_T$ can be regarded as a *spiking threshold*. If the membrane potential $v$ reaches the spiking threshold $V_T$, the switch $SW$ is closed for a short time duration $t_\varepsilon$ and is opened again. In this paper, the time duration $t_\varepsilon$ is assumed to be much shorter than $C(V_T - V_B)/I_v^+$ and $C(V_T - V_B)/I_v^-$, and the time constant $r_\varepsilon C$ is assumed to be much shorter than the time duration $t_\varepsilon$. Under these assumptions, the membrane potential $v$ is approximated to exhibit an instantaneous jump to the constant voltage $V_B$ when $v$ reaches the spiking threshold $V_T$, where $V_B$ is referred to as a *reset base* in this paper. When the switch $SW$ is opened (closed), the PWN outputs a constant voltage $Y = E_L$ (an instantaneous pulse $Y = E_H$). From a neuron model's viewpoint, the pulse $Y = E_H$ can be regarded as a firing spike or an action potential as explained in the table in Fig.1. Also, the PWN accepts a voltage input $V_{in}$ that can be regarded as a *stimulation input*. As a result, the dynamics of the PWN is described by the following equation.

$$
\begin{aligned}
&\left\{ \begin{array}{l} C\dot{v} = I_v(|v| + V_{in} - u) \\ C\dot{u} = I_u(av - u) \end{array} \right. && \text{if } v < V_T, \\
&v(t^+) = V_B && \text{if } v(t) = V_T, \\
&I_v(v_\varepsilon) = \left\{ \begin{array}{ll} I_v^+ & \text{if } v_\varepsilon > 0, \\ I_v^- & \text{if } v_\varepsilon < 0, \end{array} \right. \\
&I_u(v_\varepsilon) = \left\{ \begin{array}{ll} I_u^+ & \text{if } v_\varepsilon > 0, \\ I_u^- & \text{if } v_\varepsilon < 0, \end{array} \right. \\
&Y(t) = \left\{ \begin{array}{ll} E_H & \text{if } v(t) = V_T, \\ E_L & \text{if } v(t) < V_T, \end{array} \right.
\end{aligned}
\tag{1}
$$

where the dot "˙" represents the time derivative "$d/dt$", the symbol "$t^+$" represents the moment "$\lim_{\varepsilon \to +0}(t + \varepsilon)$" just after $t$ hereafter, $a$, $V_B$, $I_v^+$, $I_v^-$, $I_u^+$, $I_u^-$ are parameters, $V_T > V_B$ and $v(0) < V_T$ are assumed, and the spiking threshold $V_T$ can be normalized to $V_T = 1$ without loss of generality. $E_H$ and $E_L$ have no effects against the dynamics of the PWN. Note that the currents $I_v(v_\varepsilon)$ and $I_u(v_\varepsilon)$ are assumed to be multivalued functions with respect to the voltages $v_\varepsilon$, i.e., $I_v(v_\varepsilon)$ for $v_\varepsilon = 0$ ($I_u(v_\varepsilon)$ for $v_\varepsilon = 0$) takes a value in $(I_v^-, I_v^+)$ (a value in $(I_u^-, I_u^+)$) that is determined not only by the voltage $v_\varepsilon$ but also by other voltages and currents in the circuit in Fig.1(a). The dynamics for the cases of $v_\varepsilon = 0$ are omitted in this paper due to the page length limitation [14]–[16].

## 3. Bifurcation-based Learning

In this section, we propose a heuristic learning method for the PWN that utilizes the knowledge of bifurcation phenomena. In this paper, the *teacher* neuron is the Izhikevich model described by the following equation.

$$
\left\{ \begin{array}{l} \dot{v} = 0.04v^2 + 5v + 140 - u + I, \\ \dot{u} = a(bv - u), \end{array} \right. \tag{2}
$$
$$
\text{if } v \geq 30\text{mV}, \text{ then } \left\{ \begin{array}{l} v \leftarrow c, \\ u \leftarrow u + d, \end{array} \right.
$$

where $(a, b, c, d)$ are parameters. The stimulation input $V_{in}$ of the PWN (*student* neuron) is converted as follows by considering the difference between the Izhikevich model and the PWN.

$$
V_{in} = rI + V_{bias}, \tag{3}
$$

where $r$ and $V_{bias}$ are regarded as a sensitivity and bias voltage, respectively. Also, in this paper,

$$
v' = v + \frac{Y - E_L}{E_H - E_L} K \tag{4}
$$

is used to show neuron-like waveforms, i.e., spiking waveforms of $v'$ are regarded as action potentials. The learning parameters of the PWN (student neuron) are

$$
r, V_{bias}, a, V_B, I_v^+, I_v^-, I_u^+, I_u^-. \tag{5}
$$

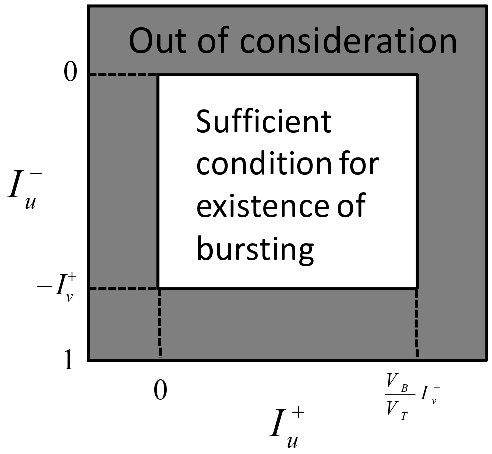



Figure 3: The sufficient condition in Equation (6) for existence of the burst-related bifurcations. $1 < a, 0 < I_v^+, 0 < V_T, V_B > 0$.

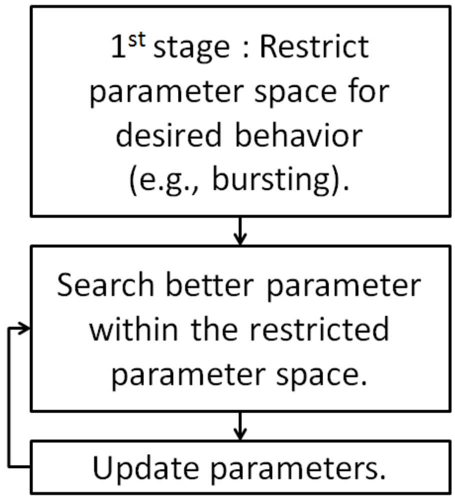



Figure 4: Learning algorithm.

**1st stage: Rough parameter setting**

A variety of bifurcation phenomena of the PWN have been analyzed so far, and the sufficient parameter spaces for existence of some responses have been led [14]-[16]. For example, the PWN can reproduce burst-related bifurcation scenario in Fig.2, where the sufficient parameter space is given as follows (see also Fig.3) [15].

$$1 < a,\ 0 < I_v^+,\ 0 < V_T,\ 0 < I_u^+/I_v^+ < V_B/V_T,$$
$$-1 < I_u^-/I_v^+ < 0,\quad V_B > 0. \qquad (6)$$

In order for the teacher neuron to exhibit a desired response (e.g., bursting), the learning parameters of the student neuron are restricted in the corresponding sufficient parameter space.

**2nd stage: Fine parameter tuning**

As shown in Fig.4, a learning algorithm is proposed. The learning algorithm searches better parameters within the restricted parameter space, updates parameters, and repeats these "restricted 2-opt like searches" finite times like [20, 21]. A heuristic evaluation function of the learning algorithm checks numbers of spikes, inter-spike intervals, and inter-burst-intervals. The detailed expressions of the evaluation function is shown at the conference as well as in our future paper. In Fig.5(a), the Izhikevich model (teacher neuron), whose parameters are $(a, b, c, d) = (0.02, 0.2, -50, 2)$, exhibits a bursting response. Hence, the learning parameters of the PWN (student neuron) is restricted in the parameter space (6) in the 1st stage. In Fig.5(b), at the initial state, the PWN (student neuron) doesn't exhibit bursting response. In Fig.5(c), after 1000 times learning, the PWN (student neuron) learns to exhibit bursting response which is very similar to Fig.5(a) of the Izhikevich model (teacher neuron). In conclusion, as is shown above, the PWN (student neuron) can learn the response of the Izhikevich model (teacher neuron) which is also observed in not only other standard neuron models but also biological neurons.

## 4. Conclusions

The PWN, which can reproduce various bifurcations observed in standard neuron models, has been introduced. The heuristic but powerful learning method for the PWN that utilizes knowledge of bifurcations of the PWN has been proposed. It has been shown that the knowledge of the bifurcations enables the PWN to learn the bursting response of the Izhikevich model efficiently. Future problems include : (a) implementation of the learning method in an actual hardware, and (b) proposal of more efficient learning methods.

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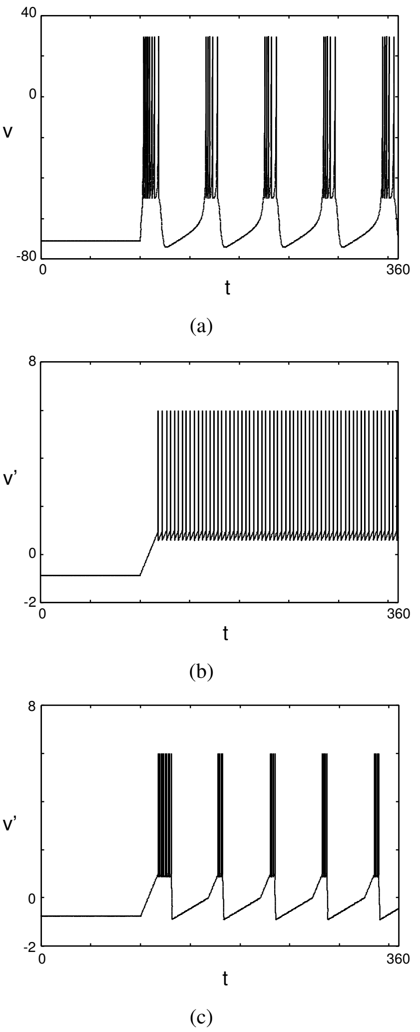


Figure 5: An example of learning. (a) The Izhikevich model (teacher neuron). $(a, b, c, d) = (0.02, 0.2, -50, 2)$. (b) The PWN (student neuron) at the initial sate. (c) The PWN (student neuron) after 1000 times learning iterations.

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