Theoretical Analysis, Design and Optimization of Printed Coils for Wireless Power Transmission

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***Abstract*-Printed coils design and optimization are crucial and important for wireless power transmission (WPT). Currently the methods to design printed coils are left with a number of choices that each has advantages and disadvantages in different situations. In this paper, both electromagnetic theory analysis and equivalent circuit modeling have been used together in theoretical analysis, design and optimization of the printed coils system. With these design procedure, we can obtain the optimal parameters like coupling coefficient *K*, the self and mutual Inductance and others in printed coils through electromagnetic theory analysis. And the received power and power transmission efficiency of printed coils system can be achieved by the formula derivation and Matlab simulation more precisely than any previous method for the printed coils system in magnetically resonant coupling WPT system consisting of coils and reactive elements.**

I. INTRODUCTION

Nowadays, the WPT applications are not yet common for the miniature and expensive problem, but statistically indisputable reality is that WPT technology will meet more and more broad market demands and applied prospect because its portability, durative and water tightness advantages. For Magnetically resonant coupling is demonstrated for a steady and feasible approach for WPT [1]-[3], the research on printed coils in Magnetically resonant coupling WPT system is being more and more important. In previous research, when deciding to make printed coils directly, or analysis printed coils just by theory analysis or equivalent circuit modeling, it is imperative to understand the limitations this can place on the overall WPT system because each method has its limitations and the printed coils system consisting of coils and reactive elements.

In this paper, electromagnetic theory analysis and equivalent circuit modeling are presented. By electromagnetic theory analysis, the self and mutual Inductance of coils can be determined though formula derivation in given the number of turns, the width of the metal trace, the turn spacing in coil and the distance between two coils. Furthermore, we can obtain the coupling coefficient *K* and resonant frequency of printed coils through the self and mutual Inductance, which indicates the degree of coupling between the coil pairs.

In equivalent circuit modeling, we can design and optimize whole coils system because the printed coils system consisting of coils and reactive elements what used in magnetically resonant coupling WPT system. And printed coils are mainly presented inductance. The resonant frequency, coupling coefficient *K*, the value of Q, the series resistance and other parameters in equivalent circuit can be obtained through formula derivation and Matlab simulation [4]. Finally, we can determine the optimal working mode of whole printed coils system exactly and embed in WPT systems because these two design approaches complement each other.

II. ELECTROMAGNETIC THEORY AND ANALYSIS

A. *Self and Mutual Inductance*

The conducting loop of wire pass a steady direct current (DC) I, which will produce a magnetic flux density B that circulates about the wire as shown in Figure 1. Besides, Vector magnetic potential A is defined by:

$$B = \nabla \times A \tag{1}$$

So the inductance of the current loop is defined as:

$$L = \frac{\psi}{I} \tag{2}$$

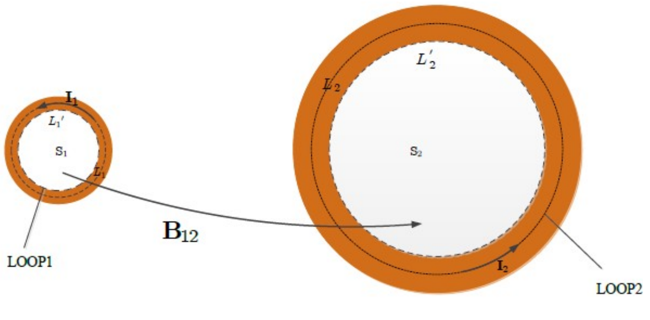

Figure 1. Mutual inductance between two circular loops

When the loop1 carrying a current I_1, the magnetic flux density through loop2 is B_{12}, which is caused by the current of Loop1 as shown in Figure 1. The mutual inductance between the two loops is:

$$M_{12} = \frac{\psi_{12}}{I_1} \tag{3}$$

The vector magnetic potential around the perimeter of that loop1:

$$\psi_{12} = \oint A_{12} \cdot dL_2^{'} \tag{4}$$

Where c_2' is the inner edge of that wire that bounds the surface of the loop, c_1 is the center of the loop. The self inductance of a loop can be determined by letting the two loops be coincident.

$$M_{12} = L_1 = M_{11} = \frac{\mu_0}{4\pi} \oint_{c_2'} \oint_{c_1} \frac{dL_1 \cdot dL_2'}{R_{12}} \tag{5}$$

B. *Theoretical Analysis*

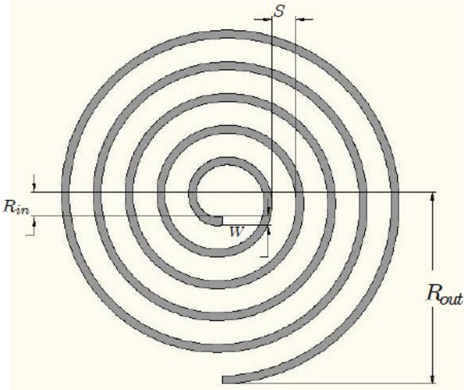

Figure 2. Geometrical parameters of a circular spiral coil

The geometry parameters of the circular spiral inductor are the number of turns *n*, the width of the metal trace W, the turn spacing S, the inner diameter R_{in} and the outer diameter R_{out} as shown in Figure 2.

The number of turns in a spiral coil can then be calculated from:

$$N = \frac{R_{out} - R_{in}}{W + s} + 1 \tag{6}$$

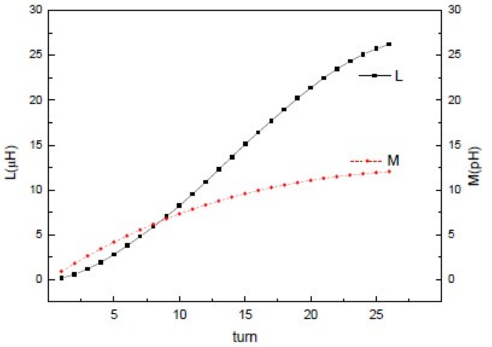

Figure 3. Mutual inductance and self inductance plot for the same R_{out}

Figure 3 shows the self inductance is increased with turn. The parameters of the circular spiral Loop1 is R_{out} =27.28 mm; turn = 1, Loop2 is R_{out} = 27.28 mm, turn from 1 to 26. From the curves, the self inductance is enhanced fast than the mutual inductance.

The coupling coefficient *K* between two coils with the self inductance L_1 and L_2 is defined as:

$$K = \frac{M}{\sqrt{L_1 \cdot L_2}} \tag{7}$$

Where M is mutual inductance of the two coils. So we can achieve maximum received power because the obtained coupling coefficient *K* indicates the degree of coupling between the coil pairs.

In case of spiral, however, the results in Figure 4 showed that as was increased, coupling coefficient *K* on increasing until turn = 9.

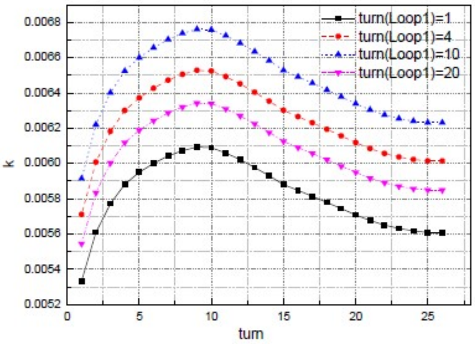

Figure 4. The coupling coefficient k between two spiral

III. EQUIVALENT CIRCUIT AND MODELING

A. *Equivalent Circuit*

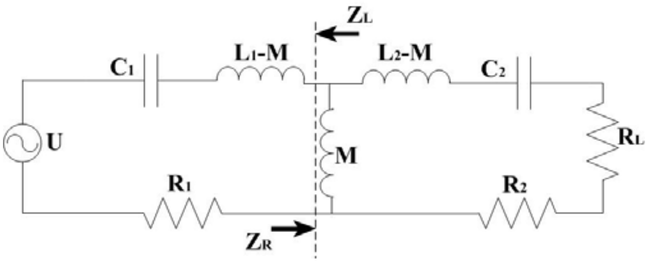

Figure 5. T-type equivalent circuit of WPT system

The printed coils system used in magnetically resonant coupling WPT system consist of coils and reactive elements. So it is incomplete and inaccurate analysis if we design printed coils only by electromagnetic theory analysis. There are four basic types of magnetically resonant coupling circuit designs including SS, PP, PS, SP [5]. We prefer to use SS configuration where the coils and capacitors are connected in series. In order to study this circuit model more conveniently, T-type equivalent circuit is used in Figure 5. Here, L_1 and L_2 represent the inductances of coils. M is mutual inductance. C_1

and C_2 are capacitors what connect to coils in series. R_1 and R_2 are the primary and secondary circuits' losses.

B. Received Power

Some researches of WPT systems focused on the received power [6]. We assume that the internal losses R_1 and R_2 are small enough so that we can ignore their contribution in the equivalent circuit shown in Figure 5.

If we want to maximize the power received by load R_L, Z_R and Z_L must satisfy:

$$Z_R = Z_L^* \tag{8}$$

Where

$$Z_R = \frac{(j\omega(L_2 - M) + \frac{1}{j\omega C_2} + R_L) * j\omega M}{(j\omega(L_2 - M) + \frac{1}{j\omega C_2} + R_L) + j\omega M} \tag{9}$$

$$Z_L = j\omega(L_1 - M) + \frac{1}{j\omega C_2} + R_S \tag{10}$$

We can get inductances of coils that $L_1 = L_2 = 27.6\,\mu H$ from electromagnetic theory analysis and assume capacitors that $C_1 = C_2 = 3.671\,nF$, the self-resonate frequency of primary and secondary circuit is 500 KHz. We can obtain Figure 6 when $R_S = R_L = 3\,ohm$ and Figure 7 when $R_S = R_L = 6\,ohm$ through Matlab simulation.

Equation (8) can be rewritten and becomes (11) and (12).

$$real(Z_L) - real(Z_R) = 0 \tag{11}$$

$$imag(Z_L) + imag(Z_R) = 0 \tag{12}$$

In Figure 6 and Figure 7, the dark red area is the place where Equations (11) and (12) are applied. In this dark red area, received power is the largest when the frequency is 500 KHz.

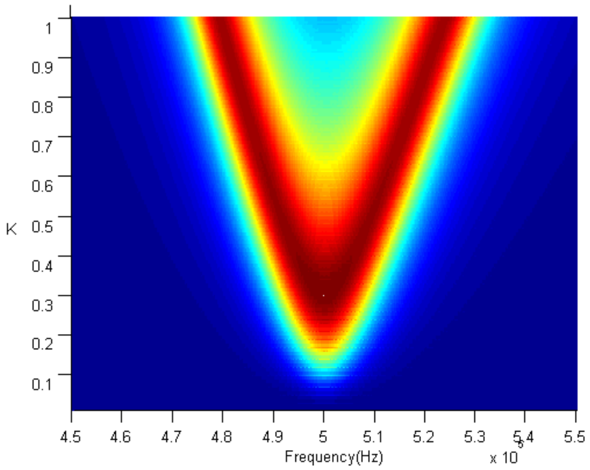

Figure 6. The absolute value of received power versus frequency and K when $R_S = R_L = 3\,ohm$

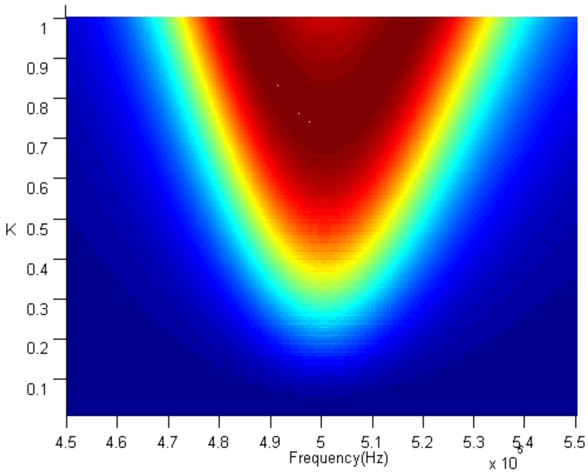

Figure 7. The absolute value of received power versus frequency and K when $R_S = R_L = 6\,ohm$

Considering the system is working at the resonate frequency of 500 KHz, we can achieve that the coupling coefficient K as:

$$K = \sqrt{\frac{Rs + R_1}{\omega L_1}}\sqrt{\frac{R_L + R_2}{\omega L_2}} = \frac{1}{\sqrt{Q_1 Q_2}}. \tag{13}$$

Above all, we can achieve more received power when the Q_1 and Q_2 are lower, which means the series resistance in equivalent circuit become low the further the transmission distance in the WPT system.

C. Power Efficiency

The system may not achieve maximum power transmission efficiency when it leads maximum transmission power. In the current study, most research groups focus on improving the power transmission efficiency, which can eliminate unnecessary energy consume and the heat produced by circuit.

As shown in Figure 5, for the no energy consumption in reactance components like capacitor and inductor, we can achieve power transmission efficiency as follows:

$$\eta = \frac{real(Z_R)}{real(Z_R) + real(Z_L)} \frac{R_L}{R_2 + R_L}. \tag{14}$$

Obviously, the maximum power transmission efficiency is:

$$\eta = \frac{1}{1 + (\frac{1}{K^2 Q_1 Q_2})^2} \frac{R_L}{R_2 + R_L}; \tag{15}$$

When

$$\omega = \frac{1}{\sqrt{L_1 C_1}} = \frac{1}{\sqrt{L_2 C_2}} \tag{16}$$

Where

$$K = \frac{M}{\sqrt{L_1 L_2}},\ Q_1 = \frac{\omega L_1}{R_s + R_1},\ Q_2 = \frac{\omega L_2}{R_L + R_2}. \qquad (17)$$

Similarly, we can get inductances of coils that $L_1 = L_2 = 27.6\ \mu H$ from electromagnetic theory analysis and assume capacitors that $C_1 = C_2 = 3.671\, nF$, the self-resonate frequency of primary and secondary circuit is 500 KHz. We can obtain Figure 8 when $R_1 = R_2 = 0\ ohm$ and Figure 9 when $R_1 = R_2 = 3\ ohm$ through Matlab simulation.

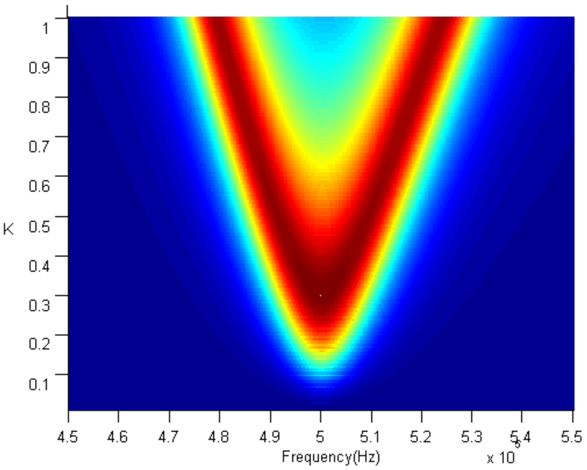

Figure 8. The absolute value of power transmission efficiency versus frequency and *K* when $R_1 = R_2 = 0\ ohm$

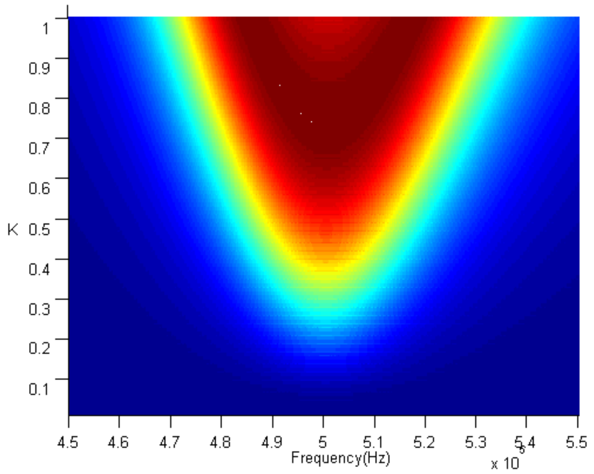

Figure 9. The absolute value of power transmission efficiency versus frequency and *K* when $R_1 = R_2 = 3\, ohm$

In Figure 8, the maximum power transmission efficiency approaches one hundred percent. Besides, the maximum power transmission efficiency approaches thirty-five percent as shown in Figure 9. The dark red area is the place where we can achieve maximum power transmission efficiency in these figures.

Finally, we can achieve maximum power transmission efficiency when the *K* tends to 1 and resonant frequency is 500 KHz.

IV. CONCLUSION

In this paper, we have designed and optimized whole printed coil system base on electromagnetic theory analysis and equivalent circuit modeling. The coupling coefficient *K* of printed coils what indicate the degree of coupling between the coil pairs, the self and mutual Inductance of printed coils are analyzed and optimized by electromagnetic theory analysis. Furthermore, in the process of equivalent circuit modeling, we have taken detailed method that how to achieve received power and power transmission efficiency in printed coils system through the formula derivation and Matlab simulation. In the next stage, how to design and optimize the multi-coils [7] of PCB simultaneously is our major direction.

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